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Domains and Their Cartesian Products

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Summary. The article includes: theorems related to domains, theorems related to Cartesian products presented earlier in various articles and simplified here by substituting domains for sets and omitting the assumption that the sets involved must not be empty. Several schemes and theorems related to Fraenkel operator are given. We also redefine subset yielding functions such as the pair of elements of a set and the union of two subsets of a set.

The terminology and notation used in this paper have been introduced in the following articles: [2], [5], [1], [4], and [3]. For simplicity we adopt the following convention: a, b, c, d will have the type Any; A, B will have the type set; D, $X1$, $X2$, $X3$, $X4$, $Y1$, $Y2$, $Y3$, $Y4$ will have the type DOMAIN; $x1$, $y1$, $z1$ will have the type Element **of** $X1$; $x2$ will have the type Element **of** $X2$; $x3$ will have the type Element **of** $X3$; $x4$ will have the type Element **of** $X4$. The following three propositions are true:

$$A \textbf{ is } \mathrm{DOMAIN} \textbf{ iff } A \neq \emptyset, \tag{1}$$

$$D \neq \emptyset, \tag{2}$$

$$a \textbf{ is } \mathrm{Element} \textbf{ of } D \textbf{ implies } a \in D. \tag{3}$$

In the sequel $A1$, $B1$ will denote objects of the type Subset **of** $X1$. One can prove the following propositions:

$$A1 = B1^{\mathrm{c}} \textbf{ iff for } x1 \textbf{ holds } x1 \in A1 \textbf{ iff not } x1 \in B1, \tag{4}$$

$$A1 = B1^{\mathrm{c}} \textbf{ iff for } x1 \textbf{ holds not } x1 \in A1 \textbf{ iff } x1 \in B1, \tag{5}$$

$$A1 = B1^{\mathrm{c}} \textbf{ iff for } x1 \textbf{ holds not } (x1 \in A1 \textbf{ iff } x1 \in B1), \tag{6}$$

$$\langle x1, x2 \rangle \in [: X1, X2 :], \tag{7}$$

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$$\langle x1,x2\rangle \textbf{ is } \text{Element } \textbf{of } [:X1,X2:], \tag{8}$$

$$a \in [:X1,X2:] \textbf{ implies } \textbf{ex } x1,x2 \textbf{ st } a = \langle x1,x2\rangle. \tag{9}$$

In the sequel x denotes an object of the type Element **of** $[:X1,X2:]$. One can prove the following propositions:

$$x = \langle x_{\mathbf{1}},x_{\mathbf{2}}\rangle, \tag{10}$$

$$x \neq x_{\mathbf{1}} \ \&\ x \neq x_{\mathbf{2}}, \tag{11}$$

$$\textbf{for } x,y \textbf{ being } \text{Element } \textbf{of } [:X1,X2:] \textbf{ st } x_{\mathbf{1}} = y_{\mathbf{1}} \ \&\ x_{\mathbf{2}} = y_{\mathbf{2}} \textbf{ holds } x = y, \tag{12}$$

$$[:A, D:] \subseteq [:B, D:] \textbf{ or } [:D, A:] \subseteq [:D, B:] \textbf{ implies } A \subseteq B, \tag{13}$$

$$[:X1,X2:] = [:A, B:] \textbf{ implies } X1 = A \ \&\ X2 = B. \tag{14}$$

Let us consider $X1$, $X2$, $x1$, $x2$. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$\langle x1,x2\rangle \quad \text{is} \quad \text{Element } \textbf{of } [:X1,X2:].$$

The arguments of the notions defined below are the following: $X1$, $X2$ which are objects of the type reserved above; x which is an object of the type Element **of** $[:X1,X2:]$. Let us note that it makes sense to consider the following functors on restricted areas. Then

$$x_{\mathbf{1}} \quad \text{is} \quad \text{Element } \textbf{of } X1,$$

$$x_{\mathbf{2}} \quad \text{is} \quad \text{Element } \textbf{of } X2.$$

One can prove the following propositions:

$$a \in [:X1,X2,X3:] \textbf{ iff } \textbf{ex } x1,x2,x3 \textbf{ st } a = \langle x1,x2,x3\rangle, \tag{15}$$

$$\begin{gathered}(\textbf{for } a \textbf{ holds } a \in D \textbf{ iff } \textbf{ex } x1,x2,x3 \textbf{ st } a = \langle x1,x2,x3\rangle)\\ \textbf{implies } D = [:X1,X2,X3:],\end{gathered} \tag{16}$$

$$D = [:X1,X2,X3:] \textbf{ iff for } a \textbf{ holds } a \in D \textbf{ iff } \textbf{ex } x1,x2,x3 \textbf{ st } a = \langle x1,x2,x3\rangle, \tag{17}$$

$$[:X1,X2,X3:] = [:Y1,Y2,Y3:] \textbf{ implies } X1 = Y1 \ \&\ X2 = Y2 \ \&\ X3 = Y3. \tag{18}$$

In the sequel x, y will have the type Element **of** $[:X1,X2,X3:]$. Next we state several propositions:

$$x = \langle a, b, c\rangle \textbf{ implies } x_{\mathbf{1}} = a \ \&\ x_{\mathbf{2}} = b \ \&\ x_{\mathbf{3}} = c, \tag{19}$$

$$x = \langle x_{\mathbf{1}},x_{\mathbf{2}},x_{\mathbf{3}}\rangle, \tag{20}$$

$$x_{\mathbf{1}} = (x \textbf{ qua } \text{Any})_{\mathbf{1}\,\mathbf{1}} \ \&\ x_{\mathbf{2}} = (x \textbf{ qua } \text{Any})_{\mathbf{1}\,\mathbf{2}} \ \&\ x_{\mathbf{3}} = (x \textbf{ qua } \text{Any})_{\mathbf{2}}, \tag{21}$$

(22) $$x \neq x_{\mathbf{1}} \;\&\; x \neq x_{\mathbf{2}} \;\&\; x \neq x_{\mathbf{3}}\,,$$

(23) $$\langle x1,x2,x3\rangle \in [:X1,X2,X3:].$$

Let us consider $X1$, $X2$, $X3$, $x1$, $x2$, $x3$. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$\langle x1,x2,x3\rangle \quad \text{is} \quad \text{Element } \mathbf{of}\ [:X1,X2,X3:].$$

The arguments of the notions defined below are the following: $X1$, $X2$, $X3$ which are objects of the type reserved above; x which is an object of the type Element **of** $[:X1,X2,X3:]$. Let us note that it makes sense to consider the following functors on restricted areas. Then

$$x_{\mathbf{1}} \quad \text{is} \quad \text{Element } \mathbf{of}\ X1,$$

$$x_{\mathbf{2}} \quad \text{is} \quad \text{Element } \mathbf{of}\ X2,$$

$$x_{\mathbf{3}} \quad \text{is} \quad \text{Element } \mathbf{of}\ X3.$$

The following propositions are true:

(24) $$a = x_{\mathbf{1}} \ \mathbf{iff}\ \mathbf{for}\ x1,x2,x3\ \mathbf{st}\ x = \langle x1,x2,x3\rangle\ \mathbf{holds}\ a = x1,$$

(25) $$b = x_{\mathbf{2}} \ \mathbf{iff}\ \mathbf{for}\ x1,x2,x3\ \mathbf{st}\ x = \langle x1,x2,x3\rangle\ \mathbf{holds}\ b = x2,$$

(26) $$c = x_{\mathbf{3}} \ \mathbf{iff}\ \mathbf{for}\ x1,x2,x3\ \mathbf{st}\ x = \langle x1,x2,x3\rangle\ \mathbf{holds}\ c = x3,$$

(27) $$\langle x_{\mathbf{1}}, x_{\mathbf{2}}, x_{\mathbf{3}}\rangle = x,$$

(28) $$x_{\mathbf{1}} = y_{\mathbf{1}} \;\&\; x_{\mathbf{2}} = y_{\mathbf{2}} \;\&\; x_{\mathbf{3}} = y_{\mathbf{3}}\ \mathbf{implies}\ x = y,$$

(29) $$\langle x1,x2,x3\rangle_{\mathbf{1}} = x1 \;\&\; \langle x1,x2,x3\rangle_{\mathbf{2}} = x2 \;\&\; \langle x1,x2,x3\rangle_{\mathbf{3}} = x3,$$

(30) $$\mathbf{for}\ x\ \mathbf{being}\ \text{Element } \mathbf{of}\ [:X1,X2,X3:], y\ \mathbf{being}\ \text{Element } \mathbf{of}\ [:Y1,Y2,Y3:]$$
$$\mathbf{holds}\ x = y\ \mathbf{implies}\ x_{\mathbf{1}} = y_{\mathbf{1}} \;\&\; x_{\mathbf{2}} = y_{\mathbf{2}} \;\&\; x_{\mathbf{3}} = y_{\mathbf{3}}\,,$$

(31) $$a \in [:X1,X2,X3,X4:]\ \mathbf{iff}\ \mathbf{ex}\ x1,x2,x3,x4\ \mathbf{st}\ a = \langle x1,x2,x3,x4\rangle,$$

(32) $$(\mathbf{for}\ a\ \mathbf{holds}\ a \in D\ \mathbf{iff}\ \mathbf{ex}\ x1,x2,x3,x4\ \mathbf{st}\ a = \langle x1,x2,x3,x4\rangle)$$
$$\mathbf{implies}\ D = [:X1,X2,X3,X4:],$$

(33) $$D = [:X1,X2,X3,X4:]$$
$$\mathbf{iff}\ \mathbf{for}\ a\ \mathbf{holds}\ a \in D\ \mathbf{iff}\ \mathbf{ex}\ x1,x2,x3,x4\ \mathbf{st}\ a = \langle x1,x2,x3,x4\rangle.$$

In the sequel x denotes an object of the type Element **of** $[:X1,X2,X3,X4:]$. The following propositions are true:

(34) $$[:X1,X2,X3,X4:] = [:Y1,Y2,Y3,Y4:]$$
$$\mathbf{implies}\ X1 = Y1 \;\&\; X2 = Y2 \;\&\; X3 = Y3 \;\&\; X4 = Y4,$$

(35) $$x = \langle a, b, c, d\rangle \textbf{ implies } x_{\mathbf{1}} = a \ \&\ x_{\mathbf{2}} = b \ \&\ x_{\mathbf{3}} = c \ \&\ x_{\mathbf{4}} = d,$$

(36) $$x = \langle x_{\mathbf{1}}, x_{\mathbf{2}}, x_{\mathbf{3}}, x_{\mathbf{4}}\rangle,$$

(37) $$x_{\mathbf{1}} = (x \textbf{ qua } \mathrm{Any})_{\mathbf{1}\,\mathbf{1}\,\mathbf{1}}$$
$$\&\ x_{\mathbf{2}} = (x \textbf{ qua } \mathrm{Any})_{\mathbf{1}\,\mathbf{1}\,\mathbf{2}} \ \&\ x_{\mathbf{3}} = (x \textbf{ qua } \mathrm{Any})_{\mathbf{1}\,\mathbf{2}} \ \&\ x_{\mathbf{4}} = (x \textbf{ qua } \mathrm{Any})_{\mathbf{2}},$$

(38) $$x \neq x_{\mathbf{1}} \ \&\ x \neq x_{\mathbf{2}} \ \&\ x \neq x_{\mathbf{3}} \ \&\ x \neq x_{\mathbf{4}},$$

(39) $$\langle x1, x2, x3, x4\rangle \in [\!:X1, X2, X3, X4:\!].$$

Let us consider $X1$, $X2$, $X3$, $X4$, $x1$, $x2$, $x3$, $x4$. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$\langle x1, x2, x3, x4\rangle \quad \text{is} \quad \text{Element } \textbf{of } [\!:X1, X2, X3, X4:\!].$$

The arguments of the notions defined below are the following: $X1$, $X2$, $X3$, $X4$ which are objects of the type reserved above; x which is an object of the type Element **of** $[\!:X1, X2, X3, X4:\!]$. Let us note that it makes sense to consider the following functors on restricted areas. Then

$$x_{\mathbf{1}} \quad \text{is} \quad \text{Element } \textbf{of } X1,$$

$$x_{\mathbf{2}} \quad \text{is} \quad \text{Element } \textbf{of } X2,$$

$$x_{\mathbf{3}} \quad \text{is} \quad \text{Element } \textbf{of } X3,$$

$$x_{\mathbf{4}} \quad \text{is} \quad \text{Element } \textbf{of } X4.$$

The following propositions are true:

(40) $$a = x_{\mathbf{1}} \textbf{ iff for } x1, x2, x3, x4 \textbf{ st } x = \langle x1, x2, x3, x4\rangle \textbf{ holds } a = x1,$$

(41) $$b = x_{\mathbf{2}} \textbf{ iff for } x1, x2, x3, x4 \textbf{ st } x = \langle x1, x2, x3, x4\rangle \textbf{ holds } b = x2,$$

(42) $$c = x_{\mathbf{3}} \textbf{ iff for } x1, x2, x3, x4 \textbf{ st } x = \langle x1, x2, x3, x4\rangle \textbf{ holds } c = x3,$$

(43) $$d = x_{\mathbf{4}} \textbf{ iff for } x1, x2, x3, x4 \textbf{ st } x = \langle x1, x2, x3, x4\rangle \textbf{ holds } d = x4,$$

(44) $$\textbf{for } x \textbf{ being } \text{Element } \textbf{of } [\!:X1, X2, X3, X4:\!] \textbf{ holds } \langle x_{\mathbf{1}}, x_{\mathbf{2}}, x_{\mathbf{3}}, x_{\mathbf{4}}\rangle = x,$$

(45) $$\textbf{for } x, y \textbf{ being } \text{Element } \textbf{of } [\!:X1, X2, X3, X4:\!]$$
$$\textbf{st } x_{\mathbf{1}} = y_{\mathbf{1}} \ \&\ x_{\mathbf{2}} = y_{\mathbf{2}} \ \&\ x_{\mathbf{3}} = y_{\mathbf{3}} \ \&\ x_{\mathbf{4}} = y_{\mathbf{4}} \textbf{ holds } x = y,$$

(46) $$\langle x1, x2, x3, x4\rangle_{\mathbf{1}} = x1$$
$$\&\ \langle x1, x2, x3, x4\rangle_{\mathbf{2}} = x2 \ \&\ \langle x1, x2, x3, x4\rangle_{\mathbf{3}} = x3 \ \&\ \langle x1, x2, x3, x4\rangle_{\mathbf{4}} = x4,$$

(47) $$\textbf{for } x \textbf{ being } \text{Element } \textbf{of } [\!:X1, X2, X3, X4:\!], y \textbf{ being } \text{Element } \textbf{of } [\!:Y1, Y2, Y3, Y4:\!]$$
$$\textbf{holds } x = y \textbf{ implies } x_{\mathbf{1}} = y_{\mathbf{1}} \ \&\ x_{\mathbf{2}} = y_{\mathbf{2}} \ \&\ x_{\mathbf{3}} = y_{\mathbf{3}} \ \&\ x_{\mathbf{4}} = y_{\mathbf{4}}.$$

In the sequel $A2$ will denote an object of the type Subset **of** $X2$; $A3$ will denote an object of the type Subset **of** $X3$; $A4$ will denote an object of the type Subset **of** $X4$. In the article we present several logical schemes. The scheme *Fraenkel1* deals with a unary predicate $\mathcal{P}$ states that the following holds

$$\textbf{for } X1 \textbf{ holds } \{\, x1 : \mathcal{P}[x1] \,\} \textbf{ is } \mathrm{Subset} \textbf{ of } X1$$

for all values of the parameter.

The scheme *Fraenkel2* deals with a binary predicate $\mathcal{P}$ states that the following holds

$$\textbf{for } X1,X2 \textbf{ holds } \{\, \langle x1,x2 \rangle : \mathcal{P}[x1,x2] \,\} \textbf{ is } \mathrm{Subset} \textbf{ of } [\!:X1,X2:\!]$$

for all values of the parameter.

The scheme *Fraenkel3* concerns a ternary predicate $\mathcal{P}$ states that the following holds

$$\textbf{for } X1,X2,X3 \textbf{ holds } \{\, \langle x1,x2,x3 \rangle : \mathcal{P}[x1,x2,x3] \,\} \textbf{ is } \mathrm{Subset} \textbf{ of } [\!:X1,X2,X3:\!]$$

for all values of the parameter.

The scheme *Fraenkel4* deals with a 4-ary predicate $\mathcal{P}$ states that the following holds

$$\begin{gathered}\textbf{for } X1,X2,X3,X4 \\ \textbf{holds } \{\, \langle x1,x2,x3,x4 \rangle : \mathcal{P}[x1,x2,x3,x4] \,\} \textbf{ is } \mathrm{Subset} \textbf{ of } [\!:X1,X2,X3,X4:\!]\end{gathered}$$

for all values of the parameter.

The scheme *Fraenkel5* concerns a unary predicate $\mathcal{P}$ and a unary predicate $\mathcal{Q}$ and states that the following holds

$$\textbf{for } X1 \textbf{ st for } x1 \textbf{ holds } \mathcal{P}[x1] \textbf{ implies } \mathcal{Q}[x1] \textbf{ holds } \{\, y1 : \mathcal{P}[y1] \,\} \subseteq \{\, z1 : \mathcal{Q}[z1] \,\}$$

for all values of the parameters.

The scheme *Fraenkel6* deals with a unary predicate $\mathcal{P}$ and a unary predicate $\mathcal{Q}$ and states that the following holds

$$\textbf{for } X1 \textbf{ st for } x1 \textbf{ holds } \mathcal{P}[x1] \textbf{ iff } \mathcal{Q}[x1] \textbf{ holds } \{\, y1 : \mathcal{P}[y1] \,\} = \{\, z1 : \mathcal{Q}[z1] \,\}$$

for all values of the parameters.

Next we state several propositions:

(48) $$X1 = \{\, x1 : \textbf{not contradiction} \,\},$$

(49) $$[\!:X1,X2:\!] = \{\, \langle x1,x2 \rangle : \textbf{not contradiction} \,\},$$

(50) $$[\!:X1,X2,X3:\!] = \{\, \langle x1,x2,x3 \rangle : \textbf{not contradiction} \,\},$$

(51) $$[\!:X1,X2,X3,X4:\!] = \{\, \langle x1,x2,x3,x4 \rangle : \textbf{not contradiction} \,\},$$

(52) $$A1 = \{\, x1 : x1 \in A1 \,\}.$$

Let us consider $X1$, $X2$, $A1$, $A2$. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$[:A1,A2:] \quad \text{is} \quad \text{Subset } \mathbf{of}\ [:X1,X2:].$$

Next we state a proposition

$$[:A1,A2:] = \{\, \langle x1,x2 \rangle : x1 \in A1\ \&\ x2 \in A2 \,\}. \tag{53}$$

Let us consider $X1$, $X2$, $X3$, $A1$, $A2$, $A3$. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$[:A1,A2,A3:] \quad \text{is} \quad \text{Subset } \mathbf{of}\ [:X1,X2,X3:].$$

Next we state a proposition

$$[:A1,A2,A3:] = \{\, \langle x1,x2,x3 \rangle : x1 \in A1\ \&\ x2 \in A2\ \&\ x3 \in A3 \,\}. \tag{54}$$

Let us consider $X1$, $X2$, $X3$, $X4$, $A1$, $A2$, $A3$, $A4$. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$[:A1,A2,A3,A4:] \quad \text{is} \quad \text{Subset } \mathbf{of}\ [:X1,X2,X3,X4:].$$

Next we state a number of propositions:

$$\begin{aligned}(55)\qquad & [:A1,A2,A3,A4:] \\ & = \{\, \langle x1,x2,x3,x4 \rangle : x1 \in A1\ \&\ x2 \in A2\ \&\ x3 \in A3\ \&\ x4 \in A4 \,\},\end{aligned}$$

$$\emptyset X1 = \{\, x1 : \mathbf{contradiction} \,\}, \tag{56}$$

$$A1^{\mathrm{c}} = \{\, x1 : \mathbf{not}\, x1 \in A1 \,\}, \tag{57}$$

$$A1 \cap B1 = \{\, x1 : x1 \in A1\ \&\ x1 \in B1 \,\}, \tag{58}$$

$$A1 \cup B1 = \{\, x1 : x1 \in A1\ \mathbf{or}\ x1 \in B1 \,\}, \tag{59}$$

$$A1 \setminus B1 = \{\, x1 : x1 \in A1\ \&\ \mathbf{not}\, x1 \in B1 \,\}, \tag{60}$$

$$A1 \dot{-} B1 = \{\, x1 : x1 \in A1\ \&\ \mathbf{not}\, x1 \in B1\ \mathbf{or}\ \mathbf{not}\, x1 \in A1\ \&\ x1 \in B1 \,\}, \tag{61}$$

$$A1 \dot{-} B1 = \{\, x1 : \mathbf{not}\, x1 \in A1\ \mathbf{iff}\ x1 \in B1 \,\}, \tag{62}$$

$$A1 \dot{-} B1 = \{\, x1 : x1 \in A1\ \mathbf{iff}\ \mathbf{not}\, x1 \in B1 \,\}, \tag{63}$$

$$A1 \dot{-} B1 = \{\, x1 : \mathbf{not}\,(x1 \in A1\ \mathbf{iff}\ x1 \in B1) \,\}. \tag{64}$$

In the sequel $x1$, $x2$, $x3$, $x4$, $x5$, $x6$, $x7$, $x8$ will have the type Element **of** D. We now state several propositions:

$$\{x1\}\ \mathbf{is}\ \text{Subset } \mathbf{of}\ D, \tag{65}$$

(66) $\{x1,x2\}$ **is** Subset **of** D,

(67) $\{x1,x2,x3\}$ **is** Subset **of** D,

(68) $\{x1,x2,x3,x4\}$ **is** Subset **of** D,

(69) $\{x1,x2,x3,x4,x5\}$ **is** Subset **of** D,

(70) $\{x1,x2,x3,x4,x5,x6\}$ **is** Subset **of** D,

(71) $\{x1,x2,x3,x4,x5,x6,x7\}$ **is** Subset **of** D,

(72) $\{x1,x2,x3,x4,x5,x6,x7,x8\}$ **is** Subset **of** D.

Let us consider D. Let $x1$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$\{x1\}$ is Subset **of** D.

Let $x2$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$\{x1,x2\}$ is Subset **of** D.

Let $x3$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$\{x1,x2,x3\}$ is Subset **of** D.

Let $x4$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$\{x1,x2,x3,x4\}$ is Subset **of** D.

Let $x5$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$\{x1,x2,x3,x4,x5\}$ is Subset **of** D.

Let $x6$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$\{x1,x2,x3,x4,x5,x6\}$ is Subset **of** D.

Let $x7$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$\{x1,x2,x3,x4,x5,x6,x7\}$ is Subset **of** D.

Let $x8$ have the type Element **of** D. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$\{x1, x2, x3, x4, x5, x6, x7, x8\} \qquad \text{is} \qquad \text{Subset } \textbf{of } D.$$

Let us consider $X1$, $A1$. Let us note that it makes sense to consider the following functor on a restricted area. Then

$$A1^{\mathrm{c}} \qquad \text{is} \qquad \text{Subset } \textbf{of } X1.$$

Let us consider $B1$. Let us note that it makes sense to consider the following functors on restricted areas. Then

$$A1 \cup B1 \qquad \text{is} \qquad \text{Subset } \textbf{of } X1,$$

$$A1 \cap B1 \qquad \text{is} \qquad \text{Subset } \textbf{of } X1,$$

$$A1 \setminus B1 \qquad \text{is} \qquad \text{Subset } \textbf{of } X1,$$

$$A1 \dot{-} B1 \qquad \text{is} \qquad \text{Subset } \textbf{of } X1.$$

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