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Finite Sets

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Summary. The article contains the definition of a finite set based on the notion of finite sequence. Some theorems about properties of finite sets and finite families of sets are proved.

The terminology and notation used here are introduced in the following papers: [5], [6], [4], [2], [1], and [3]. Let A have the type set. The predicate

A is_finite is defined by **ex** p **being** FinSequence **st** rng $p = A$.

For simplicity we adopt the following convention: A, B, C, D, X, Y have the type set; x, y, z, $x1$, $x2$, $x3$, $x4$, $x5$, $x6$, $x7$, $x8$ have the type Any; f has the type Function; n has the type Nat. The following propositions are true:

(1) A is_finite **iff** **ex** p **being** FinSequence **st** rng $p = A$,

(2) **for** p **being** FinSequence **holds** rng p is_finite ,

(3) Seg n is_finite ,

(4) $\emptyset$ is_finite ,

(5) $\{x\}$ is_finite ,

(6) $\{x, y\}$ is_finite ,

(7) $\{x, y, z\}$ is_finite ,

(8) $\{x1,x2,x3,x4\}$ is_finite ,

(9) $\{x1,x2,x3,x4,x5\}$ is_finite ,

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(10) $\{x1,x2,x3,x4,x5,x6\}$ is_finite ,

(11) $\{x1,x2,x3,x4,x5,x6,x7\}$ is_finite ,

(12) $\{x1,x2,x3,x4,x5,x6,x7,x8\}$ is_finite ,

(13) $A \subseteq B$ & B is_finite **implies** A is_finite ,

(14) A is_finite & B is_finite **implies** $A \cup B$ is_finite ,

(15) A is_finite **implies** $A \cap B$ is_finite & $B \cap A$ is_finite ,

(16) A is_finite **implies** $A \setminus B$ is_finite ,

(17) A is_finite **implies** $f\,^\circ\,A$ is_finite ,

(18) A is_finite **implies for** X **being** Subset-Family **of** A **st** $X \neq \emptyset$ **ex** x **being** set **st** $x \in X$ & **for** B **being** set **st** $B \in X$ **holds** $x \subseteq B$ **implies** $B = x$.

The scheme *Finite* deals with a constant $\mathcal{A}$ that has the type set and a unary predicate $\mathcal{P}$ and states that the following holds

$$\mathcal{P}[\mathcal{A}]$$

provided the parameters satisfy the following conditions:

- $\mathcal{A}$ is_finite ,
- $\mathcal{P}[\emptyset]$,
- **for** x,B **being** set **st** $x \in \mathcal{A}$ & $B \subseteq \mathcal{A}$ & $\mathcal{P}[B]$ **holds** $\mathcal{P}[B \cup \{x\}]$.

We now state several propositions:

(19) A is_finite & B is_finite **implies** $[:A, B:]$ is_finite ,

(20) A is_finite & B is_finite & C is_finite **implies** $[:A, B, C:]$ is_finite ,

(21) A is_finite & B is_finite & C is_finite & D is_finite **implies** $[:A, B, C, D:]$ is_finite ,

(22) $B \neq \emptyset$ & $[:A, B:]$ is_finite **implies** A is_finite ,

(23) $A \neq \emptyset$ & $[:A, B:]$ is_finite **implies** B is_finite ,

(24) A is_finite **iff** bool A is_finite ,

(25) A is_finite & (**for** X **st** $X \in A$ **holds** X is_finite) **iff** $\bigcup A$ is_finite ,

(26) dom f is_finite **implies** rng f is_finite ,

(27) $Y \subseteq$ rng f & $f^{-1}\,Y$ is_finite **implies** Y is_finite .

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