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Affine Localizations of Desargues Axiom [1]

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Summary. Several affine localizations of Major Desargues Axiom together with its indirect forms are introduced. Logical relationships between these formulas and between them and the classical Desargues Axiom are demonstrated.

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The articles [1], [3], and [2] provide the notation and terminology for this paper. We follow a convention: AP denotes an affine plane, a, a', b, b', c, c', o, p, q denote elements of the points of AP, and A, C, P denote subsets of the points of AP. Let us consider AP. We say that AP satisfies **DES1** if and only if:

Given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,
(iii) C is a line,
(iv) $P \neq A$,
(v) $P \neq C$,
(vi) $A \neq C$,
(vii) $o \in A$,
(viii) $a \in A$,
(ix) $a' \in A$,
(x) $o \in P$,
(xi) $b \in P$,
(xii) $b' \in P$,

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(xiii) $o \in C$,
(xiv) $c \in C$,
(xv) $c' \in C$,
(xvi) $o \neq a$,
(xvii) $o \neq b$,
(xviii) $o \neq c$,
(xix) $p \neq q$,
(xx) not $\mathbf{L}(b, a, c)$,
(xxi) not $\mathbf{L}(b', a', c')$,
(xxii) $a \neq a'$,
(xxiii) $\mathbf{L}(b, a, p)$,
(xxiv) $\mathbf{L}(b', a', p)$,
(xxv) $\mathbf{L}(b, c, q)$,
(xxvi) $\mathbf{L}(b', c', q)$,
(xxvii) $a, c \parallel a', c'$.

Then $a, c \parallel p, q$.

We now state the proposition

(1) Given AP. Then AP satisfies **DES1** if and only if for all A, P, C, o, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $o \in P$ and $b \in P$ and $b' \in P$ and $o \in C$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $a \neq a'$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel a', c'$ holds $a, c \parallel p, q$.

Let us consider AP. We say that AP satisfies $\mathbf{DES1_1}$ if and only if:

Given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,
(iii) C is a line,
(iv) $P \neq A$,
(v) $P \neq C$,
(vi) $A \neq C$,
(vii) $o \in A$,
(viii) $a \in A$,
(ix) $a' \in A$,
(x) $o \in P$,
(xi) $b \in P$,
(xii) $b' \in P$,
(xiii) $o \in C$,
(xiv) $c \in C$,
(xv) $c' \in C$,
(xvi) $o \neq a$,
(xvii) $o \neq b$,
(xviii) $o \neq c$,

(xix) $p \neq q$,
(xx) $c \neq q$,
(xxi) not $\mathbf{L}(b, a, c)$,
(xxii) not $\mathbf{L}(b', a', c')$,
(xxiii) $\mathbf{L}(b, a, p)$,
(xxiv) $\mathbf{L}(b', a', p)$,
(xxv) $\mathbf{L}(b, c, q)$,
(xxvi) $\mathbf{L}(b', c', q)$,
(xxvii) $a, c \parallel p, q$.

Then $a, c \parallel a', c'$.

The following proposition is true

(2) Given AP. Then AP satisfies $\mathbf{DES1_1}$ if and only if for all A, P, C, o, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $o \in P$ and $b \in P$ and $b' \in P$ and $o \in C$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and $c \neq q$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel p, q$ holds $a, c \parallel a', c'$.

Let us consider AP. We say that AP satisfies $\mathbf{DES1_2}$ if and only if:

Given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,
(iii) C is a line,
(iv) $P \neq A$,
(v) $P \neq C$,
(vi) $A \neq C$,
(vii) $o \in A$,
(viii) $a \in A$,
(ix) $a' \in A$,
(x) $o \in P$,
(xi) $b \in P$,
(xii) $b' \in P$,
(xiii) $c \in C$,
(xiv) $c' \in C$,
(xv) $o \neq a$,
(xvi) $o \neq b$,
(xvii) $o \neq c$,
(xviii) $p \neq q$,
(xix) not $\mathbf{L}(b, a, c)$,
(xx) not $\mathbf{L}(b', a', c')$,
(xxi) $c \neq c'$,
(xxii) $\mathbf{L}(b, a, p)$,
(xxiii) $\mathbf{L}(b', a', p)$,
(xxiv) $\mathbf{L}(b, c, q)$,

(xxv) $\mathbf{L}(b', c', q)$,
(xxvi) $a, c \parallel a', c'$,
(xxvii) $a, c \parallel p, q$.

Then $o \in C$.

Next we state the proposition

(3) Given AP. Then AP satisfies $\mathbf{DES1_2}$ if and only if for all A, P, C, o, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $o \in P$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $c \neq c'$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel a', c'$ and $a, c \parallel p, q$ holds $o \in C$.

Let us consider AP. We say that AP satisfies $\mathbf{DES1_3}$ if and only if:

Given A, P, C, o, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,
(iii) C is a line,
(iv) $P \neq A$,
(v) $P \neq C$,
(vi) $A \neq C$,
(vii) $o \in A$,
(viii) $a \in A$,
(ix) $a' \in A$,
(x) $b \in P$,
(xi) $b' \in P$,
(xii) $o \in C$,
(xiii) $c \in C$,
(xiv) $c' \in C$,
(xv) $o \neq a$,
(xvi) $o \neq b$,
(xvii) $o \neq c$,
(xviii) $p \neq q$,
(xix) not $\mathbf{L}(b, a, c)$,
(xx) not $\mathbf{L}(b', a', c')$,
(xxi) $b \neq b'$,
(xxii) $a \neq a'$,
(xxiii) $\mathbf{L}(b, a, p)$,
(xxiv) $\mathbf{L}(b', a', p)$,
(xxv) $\mathbf{L}(b, c, q)$,
(xxvi) $\mathbf{L}(b', c', q)$,
(xxvii) $a, c \parallel a', c'$,
(xxviii) $a, c \parallel p, q$.

Then $o \in P$.

Next we state the proposition

(4) Given AP. Then AP satisfies $\mathbf{DES1_3}$ if and only if for all A, P, C, o, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $P \neq A$ and $P \neq C$ and $A \neq C$ and $o \in A$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $o \in C$ and $c \in C$ and $c' \in C$ and $o \neq a$ and $o \neq b$ and $o \neq c$ and $p \neq q$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $b \neq b'$ and $a \neq a'$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel a', c'$ and $a, c \parallel p, q$ holds $o \in P$.

Let us consider AP. We say that AP satisfies **DES2** if and only if:

Given A, P, C, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,
(iii) C is a line,
(iv) $A \neq P$,
(v) $A \neq C$,
(vi) $P \neq C$,
(vii) $a \in A$,
(viii) $a' \in A$,
(ix) $b \in P$,
(x) $b' \in P$,
(xi) $c \in C$,
(xii) $c' \in C$,
(xiii) $A \parallel P$,
(xiv) $A \parallel C$,
(xv) not $\mathbf{L}(b, a, c)$,
(xvi) not $\mathbf{L}(b', a', c')$,
(xvii) $p \neq q$,
(xviii) $a \neq a'$,
(xix) $\mathbf{L}(b, a, p)$,
(xx) $\mathbf{L}(b', a', p)$,
(xxi) $\mathbf{L}(b, c, q)$,
(xxii) $\mathbf{L}(b', c', q)$,
(xxiii) $a, c \parallel a', c'$.

Then $a, c \parallel p, q$.

We now state the proposition

(5) Given AP. Then AP satisfies **DES2** if and only if for all A, P, C, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \parallel P$ and $A \parallel C$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $p \neq q$ and $a \neq a'$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel a', c'$ holds $a, c \parallel p, q$.

Let us consider AP. We say that AP satisfies $\mathbf{DES2_1}$ if and only if:

Given A, P, C, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,

(iii) C is a line,
(iv) $A \neq P$,
(v) $A \neq C$,
(vi) $P \neq C$,
(vii) $a \in A$,
(viii) $a' \in A$,
(ix) $b \in P$,
(x) $b' \in P$,
(xi) $c \in C$,
(xii) $c' \in C$,
(xiii) $A \parallel P$,
(xiv) $A \parallel C$,
(xv) not $\mathbf{L}(b, a, c)$,
(xvi) not $\mathbf{L}(b', a', c')$,
(xvii) $p \neq q$,
(xviii) $\mathbf{L}(b, a, p)$,
(xix) $\mathbf{L}(b', a', p)$,
(xx) $\mathbf{L}(b, c, q)$,
(xxi) $\mathbf{L}(b', c', q)$,
(xxii) $a, c \parallel p, q$.

Then $a, c \parallel a', c'$.

We now state the proposition

(6) Given AP. Then AP satisfies $\mathbf{DES2_1}$ if and only if for all A, P, C, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \parallel P$ and $A \parallel C$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $p \neq q$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel p, q$ holds $a, c \parallel a', c'$.

Let us consider AP. We say that AP satisfies $\mathbf{DES2_2}$ if and only if:
Given A, P, C, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,
(iii) C is a line,
(iv) $A \neq P$,
(v) $A \neq C$,
(vi) $P \neq C$,
(vii) $a \in A$,
(viii) $a' \in A$,
(ix) $b \in P$,
(x) $b' \in P$,
(xi) $c \in C$,
(xii) $c' \in C$,
(xiii) $A \parallel C$,
(xiv) not $\mathbf{L}(b, a, c)$,

(xv) not $\mathbf{L}(b', a', c')$,
(xvi) $p \neq q$,
(xvii) $a \neq a'$,
(xviii) $\mathbf{L}(b, a, p)$,
(xix) $\mathbf{L}(b', a', p)$,
(xx) $\mathbf{L}(b, c, q)$,
(xxi) $\mathbf{L}(b', c', q)$,
(xxii) $a, c \parallel a', c'$,
(xxiii) $a, c \parallel p, q$.

Then $A \parallel P$.

Next we state the proposition

(7) Given AP. Then AP satisfies $\mathbf{DES2_2}$ if and only if for all A, P, C, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \parallel C$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $p \neq q$ and $a \neq a'$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel a', c'$ and $a, c \parallel p, q$ holds $A \parallel P$.

Let us consider AP. We say that AP satisfies $\mathbf{DES2_3}$ if and only if:

Given A, P, C, a, a', b, b', c, c', p, q. Suppose that

(i) A is a line,
(ii) P is a line,
(iii) C is a line,
(iv) $A \neq P$,
(v) $A \neq C$,
(vi) $P \neq C$,
(vii) $a \in A$,
(viii) $a' \in A$,
(ix) $b \in P$,
(x) $b' \in P$,
(xi) $c \in C$,
(xii) $c' \in C$,
(xiii) $A \parallel P$,
(xiv) not $\mathbf{L}(b, a, c)$,
(xv) not $\mathbf{L}(b', a', c')$,
(xvi) $p \neq q$,
(xvii) $c \neq c'$,
(xviii) $\mathbf{L}(b, a, p)$,
(xix) $\mathbf{L}(b', a', p)$,
(xx) $\mathbf{L}(b, c, q)$,
(xxi) $\mathbf{L}(b', c', q)$,
(xxii) $a, c \parallel a', c'$,
(xxiii) $a, c \parallel p, q$.

Then $A \parallel C$.

We now state a number of propositions:

(8) Given AP. Then AP satisfies $\mathbf{DES2_3}$ if and only if for all A, P, C, a, a', b, b', c, c', p, q such that A is a line and P is a line and C is a line and $A \neq P$ and $A \neq C$ and $P \neq C$ and $a \in A$ and $a' \in A$ and $b \in P$ and $b' \in P$ and $c \in C$ and $c' \in C$ and $A \parallel P$ and not $\mathbf{L}(b, a, c)$ and not $\mathbf{L}(b', a', c')$ and $p \neq q$ and $c \neq c'$ and $\mathbf{L}(b, a, p)$ and $\mathbf{L}(b', a', p)$ and $\mathbf{L}(b, c, q)$ and $\mathbf{L}(b', c', q)$ and $a, c \parallel a', c'$ and $a, c \parallel p, q$ holds $A \parallel C$.

(9) If AP satisfies **DES1**, then AP satisfies $\mathbf{DES1_1}$.

(10) If AP satisfies $\mathbf{DES1_1}$, then AP satisfies **DES1**.

(11) If AP satisfies **DES**, then AP satisfies **DES1**.

(12) If AP satisfies **DES**, then AP satisfies $\mathbf{DES1_2}$.

(13) If AP satisfies $\mathbf{DES1_2}$, then AP satisfies $\mathbf{DES1_3}$.

(14) If AP satisfies $\mathbf{DES1_2}$, then AP satisfies **DES**.

(15) If AP satisfies $\mathbf{DES2_1}$, then AP satisfies **DES2**.

(16) AP satisfies $\mathbf{DES2_1}$ if and only if AP satisfies $\mathbf{DES2_3}$.

(17) AP satisfies **DES2** if and only if AP satisfies $\mathbf{DES2_2}$.

(18) If AP satisfies $\mathbf{DES1_3}$, then AP satisfies $\mathbf{DES2_1}$.

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