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A First-Order Predicate Calculus

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Summary. A continuation of [3], with an axiom system of first-order predicate theory. The consequence Cn of a set of formulas X is defined as the intersection of all theories containing X and some basic properties of it has been proved (monotonicity, idempotency, completness etc.). The notion of a proof of given formula is also introduced and it is shown that CnX = $\{ p : p$ has a proof w.r.t. $X\}$. First 14 theorems are rather simply facts. I just wanted them to be included in the data base.

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The papers [11], [10], [9], [8], [4], [6], [1], [5], [2], [7], and [3] provide the terminology and notation for this paper. In the sequel i, j, n, k, l will be natural numbers. One can prove the following propositions:

(1) If $n \leq 0$, then $n = 0$.

(2) If $n \leq 1$, then $n = 0$ or $n = 1$.

(3) If $n \leq 2$, then $n = 0$ or $n = 1$ or $n = 2$.

(4) If $n \leq 3$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$.

(5) If $n \leq 4$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$ or $n = 4$.

(6) If $n \leq 5$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$ or $n = 4$ or $n = 5$.

(7) If $n \leq 6$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$ or $n = 4$ or $n = 5$ or $n = 6$.

(8) If $n \leq 7$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$ or $n = 4$ or $n = 5$ or $n = 6$ or $n = 7$.

(9) If $n \leq 8$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$ or $n = 4$ or $n = 5$ or $n = 6$ or $n = 7$ or $n = 8$.

(10) If $n \leq 9$, then $n = 0$ or $n = 1$ or $n = 2$ or $n = 3$ or $n = 4$ or $n = 5$ or $n = 6$ or $n = 7$ or $n = 8$ or $n = 9$.

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Next we state two propositions:

(11) $\{k : k \leq n+1\} = \{i : i \leq n\} \cup \{n+1\}$.

(12) For every n holds $\{k : k \leq n\}$ is finite.

In the sequel X, Y, Z denote sets. One can prove the following two propositions:

(13) If X is finite and $X \subseteq [\!: Y, Z :\!]$, then there exist sets A, B such that A is finite and $A \subseteq Y$ and B is finite and $B \subseteq Z$ and $X \subseteq [\!: A, B :\!]$.

(14) If X is finite and Z is finite and $X \subseteq [\!: Y, Z :\!]$, then there exists a set A such that A is finite and $A \subseteq Y$ and $X \subseteq [\!: A, Z :\!]$.

For simplicity we adopt the following convention: T, S, X, Y will be subsets of $\mathrm{WFF}_{\mathrm{CQC}}$, p, q, r, t, F will be elements of $\mathrm{WFF}_{\mathrm{CQC}}$, s will be a formula, and x, y will be bound variables. Let us consider T. We say that T is a theory if and only if:

(i) $\mathrm{VERUM} \in T$,

(ii) for all p, q, r, s, x, y holds $(\neg p \Rightarrow p) \Rightarrow p \in T$ and $p \Rightarrow (\neg p \Rightarrow q) \in T$ and $(p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r)) \in T$ and $p \wedge q \Rightarrow q \wedge p \in T$ but if $p \in T$ and $p \Rightarrow q \in T$, then $q \in T$ and $\forall_x p \Rightarrow p \in T$ but if $p \Rightarrow q \in T$ and $x \notin \mathrm{snb}(p)$, then $p \Rightarrow \forall_x q \in T$ but if $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $s(x) \in T$, then $s(y) \in T$.

Next we state a number of propositions:

(15) Suppose that

(i) $\mathrm{VERUM} \in T$,

(ii) for all p, q, r, s, x, y holds $(\neg p \Rightarrow p) \Rightarrow p \in T$ and $p \Rightarrow (\neg p \Rightarrow q) \in T$ and $(p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r)) \in T$ and $p \wedge q \Rightarrow q \wedge p \in T$ but if $p \in T$ and $p \Rightarrow q \in T$, then $q \in T$ and $\forall_x p \Rightarrow p \in T$ but if $p \Rightarrow q \in T$ and $x \notin \mathrm{snb}(p)$, then $p \Rightarrow \forall_x q \in T$ but if $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $s(x) \in T$, then $s(y) \in T$.

Then T is a theory.

(16) If T is a theory, then $\mathrm{VERUM} \in T$.

(17) If T is a theory, then $(\neg p \Rightarrow p) \Rightarrow p \in T$.

(18) If T is a theory, then $p \Rightarrow (\neg p \Rightarrow q) \in T$.

(19) If T is a theory, then $(p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r)) \in T$.

(20) If T is a theory, then $p \wedge q \Rightarrow q \wedge p \in T$.

(21) If T is a theory and $p \in T$ and $p \Rightarrow q \in T$, then $q \in T$.

(22) If T is a theory, then $\forall_x p \Rightarrow p \in T$.

(23) If T is a theory and $p \Rightarrow q \in T$ and $x \notin \mathrm{snb}(p)$, then $p \Rightarrow \forall_x q \in T$.

(24) If T is a theory and $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $s(x) \in T$, then $s(y) \in T$.

Let us consider T, S. Then $T \cup S$ is a subset of $\mathrm{WFF}_{\mathrm{CQC}}$. Then $T \cap S$ is a subset of $\mathrm{WFF}_{\mathrm{CQC}}$. Then $T \setminus S$ is a subset of $\mathrm{WFF}_{\mathrm{CQC}}$.

Let us consider p. Then $\{p\}$ is a subset of $\mathrm{WFF}_{\mathrm{CQC}}$.

Next we state the proposition

(25) If T is a theory and S is a theory, then $T \cap S$ is a theory.

Let us consider X. The functor $\mathrm{Cn}\, X$ yielding a subset of $\mathrm{WFF}_{\mathrm{CQC}}$ is defined as follows:

$t \in \mathrm{Cn}\, X$ if and only if for every T such that T is a theory and $X \subseteq T$ holds $t \in T$.

We now state a number of propositions:

(26) $Y = \mathrm{Cn}\, X$ if and only if for every t holds $t \in Y$ if and only if for every T such that T is a theory and $X \subseteq T$ holds $t \in T$.

(27) $\mathrm{VERUM} \in \mathrm{Cn}\, X$.

(28) $(\neg p \Rightarrow p) \Rightarrow p \in \mathrm{Cn}\, X$.

(29) $p \Rightarrow (\neg p \Rightarrow q) \in \mathrm{Cn}\, X$.

(30) $(p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r)) \in \mathrm{Cn}\, X$.

(31) $p \wedge q \Rightarrow q \wedge p \in \mathrm{Cn}\, X$.

(32) If $p \in \mathrm{Cn}\, X$ and $p \Rightarrow q \in \mathrm{Cn}\, X$, then $q \in \mathrm{Cn}\, X$.

(33) $\forall_x p \Rightarrow p \in \mathrm{Cn}\, X$.

(34) If $p \Rightarrow q \in \mathrm{Cn}\, X$ and $x \notin \mathrm{snb}(p)$, then $p \Rightarrow \forall_x q \in \mathrm{Cn}\, X$.

(35) If $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $s(x) \in \mathrm{Cn}\, X$, then $s(y) \in \mathrm{Cn}\, X$.

(36) $\mathrm{Cn}\, X$ is a theory.

(37) If T is a theory and $X \subseteq T$, then $\mathrm{Cn}\, X \subseteq T$.

(38) $X \subseteq \mathrm{Cn}\, X$.

(39) If $X \subseteq Y$, then $\mathrm{Cn}\, X \subseteq \mathrm{Cn}\, Y$.

(40) $\mathrm{Cn}(\mathrm{Cn}\, X) = \mathrm{Cn}\, X$.

(41) T is a theory if and only if $\mathrm{Cn}\, T = T$.

The non-empty set $\mathbb{K}$ is defined by:

$\mathbb{K} = \{k : k \leq 9\}$.

Next we state three propositions:

(42) $\mathbb{K} = \{k : k \leq 9\}$.

(43) $0 \in \mathbb{K}$ and $1 \in \mathbb{K}$ and $2 \in \mathbb{K}$ and $3 \in \mathbb{K}$ and $4 \in \mathbb{K}$ and $5 \in \mathbb{K}$ and $6 \in \mathbb{K}$ and $7 \in \mathbb{K}$ and $8 \in \mathbb{K}$ and $9 \in \mathbb{K}$.

(44) $\mathbb{K}$ is finite.

In the sequel f, g are finite sequences of elements of $[\!:\mathrm{WFF}_{\mathrm{CQC}}, \mathbb{K}:\!]$. The following proposition is true

(45) Suppose $1 \leq n$ and $n \leq \mathrm{len}\, f$. Then

(i) $(f(n))_{\mathbf{2}} = 0$, or

(ii) $(f(n))_{\mathbf{2}} = 1$, or

(iii) $(f(n))_{\mathbf{2}} = 2$, or

(iv) $(f(n))_{\mathbf{2}} = 3$, or

(v) $(f(n))_{\mathbf{2}} = 4$, or

(vi) $(f(n))_{\mathbf{2}} = 5$, or

(vii) $(f(n))_{\mathbf{2}} = 6$, or

(viii) $(f(n))_{\mathbf{2}} = 7$, or
(ix) $(f(n))_{\mathbf{2}} = 8$, or
(x) $(f(n))_{\mathbf{2}} = 9$.

Let PR be a finite sequence of elements of $[:\mathrm{WFF}_{\mathrm{CQC}}, \mathbb{K}:]$, and let us consider n, X. Let us assume that $1 \leq n$ and $n \leq \mathrm{len}\, PR$. We say that $PR(n)$ is a correct proof step w.r.t. X if and only if:

$(PR(n))_{\mathbf{1}} \in X$ if $(PR(n))_{\mathbf{2}} = 0$, $(PR(n))_{\mathbf{1}} = \mathrm{VERUM}$ if $(PR(n))_{\mathbf{2}} = 1$, there exists p such that $(PR(n))_{\mathbf{1}} = (\neg p \Rightarrow p) \Rightarrow p$ if $(PR(n))_{\mathbf{2}} = 2$, there exist p, q such that $(PR(n))_{\mathbf{1}} = p \Rightarrow (\neg p \Rightarrow q)$ if $(PR(n))_{\mathbf{2}} = 3$, there exist p, q, r such that $(PR(n))_{\mathbf{1}} = (p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r))$ if $(PR(n))_{\mathbf{2}} = 4$, there exist p, q such that $(PR(n))_{\mathbf{1}} = p \wedge q \Rightarrow q \wedge p$ if $(PR(n))_{\mathbf{2}} = 5$, there exist p, x such that $(PR(n))_{\mathbf{1}} = \forall_x p \Rightarrow p$ if $(PR(n))_{\mathbf{2}} = 6$, there exist i, j, p, q such that $1 \leq i$ and $i < n$ and $1 \leq j$ and $j < i$ and $p = (PR(j))_{\mathbf{1}}$ and $q = (PR(n))_{\mathbf{1}}$ and $(PR(i))_{\mathbf{1}} = p \Rightarrow q$ if $(PR(n))_{\mathbf{2}} = 7$, there exist i, p, q, x such that $1 \leq i$ and $i < n$ and $(PR(i))_{\mathbf{1}} = p \Rightarrow q$ and $x \notin \mathrm{snb}(p)$ and $(PR(n))_{\mathbf{1}} = p \Rightarrow \forall_x q$ if $(PR(n))_{\mathbf{2}} = 8$, there exist i, x, y, s such that $1 \leq i$ and $i < n$ and $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $s(x) = (PR(i))_{\mathbf{1}}$ and $s(y) = (PR(n))_{\mathbf{1}}$ if $(PR(n))_{\mathbf{2}} = 9$.

The following propositions are true:

(46) If $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 0$, then $f(n)$ is a correct proof step w.r.t. X if and only if $(f(n))_{\mathbf{1}} \in X$.

(47) If $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 1$, then $f(n)$ is a correct proof step w.r.t. X if and only if $(f(n))_{\mathbf{1}} = \mathrm{VERUM}$.

(48) If $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 2$, then $f(n)$ is a correct proof step w.r.t. X if and only if there exists p such that $(f(n))_{\mathbf{1}} = (\neg p \Rightarrow p) \Rightarrow p$.

(49) If $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 3$, then $f(n)$ is a correct proof step w.r.t. X if and only if there exist p, q such that $(f(n))_{\mathbf{1}} = p \Rightarrow (\neg p \Rightarrow q)$.

(50) If $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 4$, then $f(n)$ is a correct proof step w.r.t. X if and only if there exist p, q, r such that $(f(n))_{\mathbf{1}} = (p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r))$.

(51) If $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 5$, then $f(n)$ is a correct proof step w.r.t. X if and only if there exist p, q such that $(f(n))_{\mathbf{1}} = p \wedge q \Rightarrow q \wedge p$.

(52) If $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 6$, then $f(n)$ is a correct proof step w.r.t. X if and only if there exist p, x such that $(f(n))_{\mathbf{1}} = \forall_x p \Rightarrow p$.

(53) Suppose $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 7$. Then $f(n)$ is a correct proof step w.r.t. X if and only if there exist i, j, p, q such that $1 \leq i$ and $i < n$ and $1 \leq j$ and $j < i$ and $p = (f(j))_{\mathbf{1}}$ and $q = (f(n))_{\mathbf{1}}$ and $(f(i))_{\mathbf{1}} = p \Rightarrow q$.

(54) Suppose $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 8$. Then $f(n)$ is a correct proof step w.r.t. X if and only if there exist i, p, q, x such that $1 \leq i$ and $i < n$ and $(f(i))_{\mathbf{1}} = p \Rightarrow q$ and $x \notin \mathrm{snb}(p)$ and $(f(n))_{\mathbf{1}} = p \Rightarrow \forall_x q$.

(55) Suppose $1 \leq n$ and $n \leq \mathrm{len}\, f$ and $(f(n))_{\mathbf{2}} = 9$. Then $f(n)$ is a correct proof step w.r.t. X if and only if there exist i, x, y, s such that $1 \leq i$

and $i < n$ and $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $s(x) = (f(i))_{\mathbf{1}}$ and $(f(n))_{\mathbf{1}} = s(y)$.

Let us consider X, f. We say that f is a proof w.r.t. X if and only if:

$f \neq \varepsilon$ and for every n such that $1 \leq n$ and $n \leq \mathrm{len}\, f$ holds $f(n)$ is a correct proof step w.r.t. X.

The following propositions are true:

(56) f is a proof w.r.t. X if and only if $f \neq \varepsilon$ and for every n such that $1 \leq n$ and $n \leq \mathrm{len}\, f$ holds $f(n)$ is a correct proof step w.r.t. X.

(57) If f is a proof w.r.t. X, then $\mathrm{rng}\, f \neq \emptyset$.

(58) If f is a proof w.r.t. X, then $1 \leq \mathrm{len}\, f$.

(59) Suppose f is a proof w.r.t. X. Then $(f(1))_{\mathbf{2}} = 0$ or $(f(1))_{\mathbf{2}} = 1$ or $(f(1))_{\mathbf{2}} = 2$ or $(f(1))_{\mathbf{2}} = 3$ or $(f(1))_{\mathbf{2}} = 4$ or $(f(1))_{\mathbf{2}} = 5$ or $(f(1))_{\mathbf{2}} = 6$.

(60) If $1 \leq n$ and $n \leq \mathrm{len}\, f$, then $f(n)$ is a correct proof step w.r.t. X if and only if $f \frown g(n)$ is a correct proof step w.r.t. X.

(61) If $1 \leq n$ and $n \leq \mathrm{len}\, g$ and $g(n)$ is a correct proof step w.r.t. X, then $f \frown g(n + \mathrm{len}\, f)$ is a correct proof step w.r.t. X.

(62) If f is a proof w.r.t. X and g is a proof w.r.t. X, then $f \frown g$ is a proof w.r.t. X.

(63) If f is a proof w.r.t. X and $X \subseteq Y$, then f is a proof w.r.t. Y.

(64) If f is a proof w.r.t. X and $1 \leq l$ and $l \leq \mathrm{len}\, f$, then $(f(l))_{\mathbf{1}} \in \mathrm{Cn}\, X$.

Let us consider f. Let us assume that $f \neq \varepsilon$. The functor $\mathrm{Effect}\, f$ yields an element of $\mathrm{WFF}_{\mathrm{CQC}}$ and is defined as follows:

$\mathrm{Effect}\, f = (f(\mathrm{len}\, f))_{\mathbf{1}}$.

The following propositions are true:

(65) If $f \neq \varepsilon$, then $\mathrm{Effect}\, f = (f(\mathrm{len}\, f))_{\mathbf{1}}$.

(66) If f is a proof w.r.t. X, then $\mathrm{Effect}\, f \in \mathrm{Cn}\, X$.

(67) $X \subseteq \{F : \bigvee_f [\, f$ is a proof w.r.t. $X \wedge \mathrm{Effect}\, f = F]\}$.

(68) For every X such that $Y = \{p : \bigvee_f [\, f$ is a proof w.r.t. $X \wedge \mathrm{Effect}\, f = p]\}$ holds Y is a theory.

(69) For every X holds $\{p : \bigvee_f [\, f$ is a proof w.r.t. $X \wedge \mathrm{Effect}\, f = p]\} = \mathrm{Cn}\, X$.

(70) $p \in \mathrm{Cn}\, X$ if and only if there exists f such that f is a proof w.r.t. X and $\mathrm{Effect}\, f = p$.

(71) If $p \in \mathrm{Cn}\, X$, then there exists Y such that $Y \subseteq X$ and Y is finite and $p \in \mathrm{Cn}\, Y$.

The subset $\emptyset_{\mathrm{CQC}}$ of $\mathrm{WFF}_{\mathrm{CQC}}$ is defined by:

$\emptyset_{\mathrm{CQC}} = \emptyset_{\mathrm{WFF}_{\mathrm{CQC}}}$.

We now state the proposition

(72) $\emptyset_{\mathrm{CQC}} = \emptyset_{\mathrm{WFF}_{\mathrm{CQC}}}$.

The subset Taut of $\mathrm{WFF}_{\mathrm{CQC}}$ is defined as follows:

$\mathrm{Taut} = \mathrm{Cn}\, \emptyset_{\mathrm{CQC}}$.

One can prove the following propositions:

(73) $\mathrm{Taut} = \mathrm{Cn}\, \emptyset_{\mathrm{CQC}}$.

(74) If T is a theory, then $\mathrm{Taut} \subseteq T$.

(75) $\mathrm{Taut} \subseteq \mathrm{Cn}\, X$.

(76) Taut is a theory.

(77) $\mathrm{VERUM} \in \mathrm{Taut}$.

(78) $(\neg p \Rightarrow p) \Rightarrow p \in \mathrm{Taut}$.

(79) $p \Rightarrow (\neg p \Rightarrow q) \in \mathrm{Taut}$.

(80) $(p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r)) \in \mathrm{Taut}$.

(81) $p \wedge q \Rightarrow q \wedge p \in \mathrm{Taut}$.

(82) If $p \in \mathrm{Taut}$ and $p \Rightarrow q \in \mathrm{Taut}$, then $q \in \mathrm{Taut}$.

(83) $\forall_x p \Rightarrow p \in \mathrm{Taut}$.

(84) If $p \Rightarrow q \in \mathrm{Taut}$ and $x \notin \mathrm{snb}(p)$, then $p \Rightarrow \forall_x q \in \mathrm{Taut}$.

(85) If $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $s(x) \in \mathrm{Taut}$, then $s(y) \in \mathrm{Taut}$.

Let us consider X, s. The predicate $X \vdash s$ is defined as follows:
$s \in \mathrm{Cn}\, X$.

Next we state a number of propositions:

(86) $X \vdash s$ if and only if $s \in \mathrm{Cn}\, X$.

(87) $X \vdash \mathrm{VERUM}$.

(88) $X \vdash (\neg p \Rightarrow p) \Rightarrow p$.

(89) $X \vdash p \Rightarrow (\neg p \Rightarrow q)$.

(90) $X \vdash (p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r))$.

(91) $X \vdash p \wedge q \Rightarrow q \wedge p$.

(92) If $X \vdash p$ and $X \vdash p \Rightarrow q$, then $X \vdash q$.

(93) $X \vdash \forall_x p \Rightarrow p$.

(94) If $X \vdash p \Rightarrow q$ and $x \notin \mathrm{snb}(p)$, then $X \vdash p \Rightarrow \forall_x q$.

(95) If $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $X \vdash s(x)$, then $X \vdash s(y)$.

Let us consider s. The predicate $\vdash s$ is defined as follows:
$\emptyset_{\mathrm{CQC}} \vdash s$.

Next we state two propositions:

(96) $\vdash s$ if and only if $\emptyset_{\mathrm{CQC}} \vdash s$.

(97) $\vdash s$ if and only if $s \in \mathrm{Taut}$.

Let us consider s. Let us note that one can characterize the predicate $\vdash s$ by the following (equivalent) condition: $s \in \mathrm{Taut}$.

We now state a number of propositions:

(98) If $\vdash p$, then $X \vdash p$.

(99) $\vdash \mathrm{VERUM}$.

(100) $\vdash (\neg p \Rightarrow p) \Rightarrow p$.
(101) $\vdash p \Rightarrow (\neg p \Rightarrow q)$.
(102) $\vdash (p \Rightarrow q) \Rightarrow (\neg(q \wedge r) \Rightarrow \neg(p \wedge r))$.
(103) $\vdash p \wedge q \Rightarrow q \wedge p$.
(104) If $\vdash p$ and $\vdash p \Rightarrow q$, then $\vdash q$.
(105) $\vdash \forall_x p \Rightarrow p$.
(106) If $\vdash p \Rightarrow q$ and $x \notin \mathrm{snb}(p)$, then $\vdash p \Rightarrow \forall_x q$.
(107) If $s(x) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $s(y) \in \mathrm{WFF}_{\mathrm{CQC}}$ and $x \notin \mathrm{snb}(s)$ and $\vdash s(x)$, then $\vdash s(y)$.

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