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Average Value Theorems for Real Functions of One Variable [1]

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Summary. Three basic theorems in differential calculus of one variable functions are presented: Rolle Theorem, Lagrange Theorem and Cauchy Theorem. There are also direct conclusions.

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The terminology and notation used here have been introduced in the following papers: [2], [1], [3], [4], [5], [8], [6], and [7]. We adopt the following rules: g, r, s, p, t, x, x_0, x_1 will denote real numbers and f, f_1, f_2 will denote partial functions from $\mathbb{R}$ to $\mathbb{R}$. We now state a number of propositions:

(1) For all p, g such that $p < g$ for every f such that f is continuous on $[p,g]$ and $f(p) = f(g)$ and f is differentiable on $]p,g[$ there exists x_0 such that $x_0 \in]p,g[$ and $f'(x_0) = 0$.

(2) Given x, t. Suppose $0 < t$. Then for every f such that f is continuous on $[x, x+t]$ and $f(x) = f(x+t)$ and f is differentiable on $]x, x+t[$ there exists s such that $0 < s$ and $s < 1$ and $f'(x + s \cdot t) = 0$.

(3) For all p, g such that $p < g$ for every f such that f is continuous on $[p,g]$ and f is differentiable on $]p,g[$ there exists x_0 such that $x_0 \in]p,g[$ and $f'(x_0) = \frac{f(g)-f(p)}{g-p}$.

(4) Given x, t. Suppose $0 < t$. Then for every f such that f is continuous on $[x, x+t]$ and f is differentiable on $]x, x+t[$ there exists s such that $0 < s$ and $s < 1$ and $f(x+t) = f(x) + t \cdot (f'(x + s \cdot t))$.

(5) Given p, g. Suppose $p < g$. Given f_1, f_2. Suppose f_1 is continuous on $[p,g]$ and f_1 is differentiable on $]p,g[$ and f_2 is continuous on $[p,g]$ and f_2 is differentiable on $]p,g[$. Then there exists x_0 such that $x_0 \in]p,g[$ and $(f_1(g) - f_1(p)) \cdot (f_2'(x_0)) = (f_2(g) - f_2(p)) \cdot (f_1'(x_0))$.

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(6) Given x, t. Suppose $0 < t$. Given f_1, f_2. Suppose f_1 is continuous on $[x, x+t]$ and f_1 is differentiable on $]x, x+t[$ and f_2 is continuous on $[x, x+t]$ and f_2 is differentiable on $]x, x+t[$ and for every x_1 such that $x_1 \in]x, x+t[$ holds $f_2'(x_1) \neq 0$. Then there exists s such that $0 < s$ and $s < 1$ and $\frac{f_1(x+t)-f_1(x)}{f_2(x+t)-f_2(x)} = \frac{f_1'(x+s \cdot t)}{f_2'(x+s \cdot t)}$.

(7) For all p, g such that $p < g$ for every f such that f is differentiable on $]p, g[$ and for every x such that $x \in]p, g[$ holds $f'(x) = 0$ holds f is a constant on $]p, g[$.

(8) Given p, g. Suppose $p < g$. Given f_1, f_2. Suppose f_1 is differentiable on $]p, g[$ and f_2 is differentiable on $]p, g[$ and for every x such that $x \in]p, g[$ holds $f_1'(x) = f_2'(x)$. Then $f_1 - f_2$ is a constant on $]p, g[$ and there exists r such that for every x such that $x \in]p, g[$ holds $f_1(x) = f_2(x) + r$.

(9) For all p, g such that $p < g$ for every f such that f is differentiable on $]p, g[$ and for every x such that $x \in]p, g[$ holds $0 < f'(x)$ holds f is increasing on $]p, g[$.

(10) For all p, g such that $p < g$ for every f such that f is differentiable on $]p, g[$ and for every x such that $x \in]p, g[$ holds $f'(x) < 0$ holds f is decreasing on $]p, g[$.

(11) For all p, g such that $p < g$ for every f such that f is differentiable on $]p, g[$ and for every x such that $x \in]p, g[$ holds $0 \leq f'(x)$ holds f is non-decreasing on $]p, g[$.

(12) For all p, g such that $p < g$ for every f such that f is differentiable on $]p, g[$ and for every x such that $x \in]p, g[$ holds $f'(x) \leq 0$ holds f is non-increasing on $]p, g[$.

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