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Homotheties and Shears in Affine Planes [1]

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Summary. We study connections between Major Desargues Axiom and the transitivity of group of homotheties. A formal proof of the theorem which establishes an equivalence of these two properties of affine planes is given. We also study connections between the trapezium version of Major Desargues Axiom and the existence of the shears in affine planes. The article contains investigations on "Scherungssatz".

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The papers [9], [1], [2], [10], [3], [4], [6], [7], [5], and [8] provide the terminology and notation for this paper. For simplicity we adopt the following rules: A_1 will be an affine plane, a, b, o, p, p', q, q', x, y will be elements of the points of A_1, M, K will be subsets of the points of A_1, and f will be a permutation of the points of A_1. We now state four propositions:

(1) Suppose that
(i) not $\mathbf{L}(o,a,p)$,
(ii) $\mathbf{L}(o,a,b)$,
(iii) $\mathbf{L}(o,a,x)$,
(iv) $\mathbf{L}(o,a,y)$,
(v) $\mathbf{L}(o,p,p')$,
(vi) $\mathbf{L}(o,p,q)$,
(vii) $\mathbf{L}(o,p,q')$,
(viii) $p \neq q$,
(ix) $a \neq x$,
(x) $o \neq q$,
(xi) $o \neq x$,
(xii) $a,p \parallel b,p'$,
(xiii) $a,q \parallel b,q'$,
(xiv) $x,p \parallel y,p'$,

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(xv) A_1 satisfies **DES**.

Then $x, q \parallel y, q'$.

(2) If for all o, a, b such that $o \neq a$ and $o \neq b$ and $\mathbf{L}(o, a, b)$ there exists f such that f is a dilatation and $f(o) = o$ and $f(a) = b$, then A_1 satisfies **DES**.

(3) If A_1 satisfies **DES**, then for all o, a, b such that $o \neq a$ and $o \neq b$ and $\mathbf{L}(o, a, b)$ there exists f such that f is a dilatation and $f(o) = o$ and $f(a) = b$.

(4) A_1 satisfies **DES** if and only if for all o, a, b such that $o \neq a$ and $o \neq b$ and $\mathbf{L}(o, a, b)$ there exists f such that f is a dilatation and $f(o) = o$ and $f(a) = b$.

Let us consider A_1, f, K. We say that f is Sc K if and only if:

(Def.1) f is a collineation and K is a line and for every x such that $x \in K$ holds $f(x) = x$ and for every x holds $x, f(x) \parallel K$.

One can prove the following propositions:

(5) If f is Sc K and $f(p) = p$ and $p \notin K$, then $f = \mathrm{id}_{\text{the points of } A_1}$.

(6) If for all a, b, K such that $a, b \parallel K$ and $a \notin K$ there exists f such that f is Sc K and $f(a) = b$, then A_1 satisfies **TDES**.

(7) Suppose that

(i) $K \parallel M$,

(ii) $p \in K$,

(iii) $q \in K$,

(iv) $p' \in K$,

(v) $q' \in K$,

(vi) A_1 satisfies **TDES**,

(vii) $a \in M$,

(viii) $b \in M$,

(ix) $x \in M$,

(x) $y \in M$,

(xi) $a \neq b$,

(xii) $q \neq p$,

(xiii) $p, a \parallel p', x$,

(xiv) $p, b \parallel p', y$,

(xv) $q, a \parallel q', x$.

Then $q, b \parallel q', y$.

(8) If $a, b \parallel K$ and $a \notin K$ and A_1 satisfies **TDES**, then there exists f such that f is Sc K and $f(a) = b$.

(9) A_1 satisfies **TDES** if and only if for all a, b, K such that $a, b \parallel K$ and $a \notin K$ there exists f such that f is Sc K and $f(a) = b$.

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