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Desargues Theorem In Projective 3-Space

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Summary. Proof of the Desargues theorem in Fanoian projective at least 3-dimensional space.

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The notation and terminology used in this paper are introduced in the following papers: [5], [1], [2], [3], and [4]. We follow a convention: F_1 will be an at least 3-dimensional projective space defined in terms of collinearity and a, a', b, b', c, c', d, d', o, p, q, r, s, t, u, x will be elements of the points of F_1. One can prove the following propositions:

(1) If a, b and c are collinear, then b, c and a are collinear and c, a and b are collinear and b, a and c are collinear and a, c and b are collinear and c, b and a are collinear.

(2) If $a \neq b$ and a, b and c are collinear and a, b and d are collinear, then a, c and d are collinear.

(3) If $p \neq q$ and a, b and p are collinear and a, b and q are collinear and p, q and r are collinear, then a, b and r are collinear.

(4) If $p \neq q$, then there exists r such that p, q and r are not collinear.

(5) There exist q, r such that p, q and r are not collinear.

(6) If a, b and c are not collinear and a, b and b' are collinear and $a \neq b'$, then a, b' and c are not collinear.

(7) If a, b and c are not collinear and a, b and d are collinear and a, c and d are collinear, then $a = d$.

(8) If o, a and d are not collinear and o, d and d' are collinear and a, d and s are collinear and $d \neq d'$ and a', d' and s are collinear and o, a and a' are collinear and $o \neq a'$, then $s \neq d$.

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Let us consider F_1, a, b, c, d. We say that a, b, c, d are coplanar if and only if:

(Def.1) there exists an element x of the points of F_1 such that a, b and x are collinear and c, d and x are collinear.

One can prove the following propositions:

(10)[2] If a, b and c are collinear or b, c and d are collinear or c, d and a are collinear or d, a and b are collinear, then a, b, c, d are coplanar.

(11) Suppose a, b, c, d are coplanar. Then b, c, d, a are coplanar and c, d, a, b are coplanar and d, a, b, c are coplanar and b, a, c, d are coplanar and c, b, d, a are coplanar and d, c, a, b are coplanar and a, d, b, c are coplanar and a, c, d, b are coplanar and b, d, a, c are coplanar and c, a, b, d are coplanar and d, b, c, a are coplanar and c, a, d, b are coplanar and d, b, a, c are coplanar and a, c, b, d are coplanar and b, d, c, a are coplanar and a, b, d, c are coplanar and a, d, c, b are coplanar and b, c, a, d are coplanar and b, a, d, c are coplanar and c, b, a, d are coplanar and c, d, b, a are coplanar and d, a, c, b are coplanar and d, c, b, a are coplanar.

(12) If a, b and c are not collinear and a, b, c, p are coplanar and a, b, c, q are coplanar and a, b, c, r are coplanar and a, b, c, s are coplanar, then p, q, r, s are coplanar.

(13) If p, q and r are not collinear and a, b, c, p are coplanar and a, b, c, r are coplanar and a, b, c, q are coplanar and p, q, r, s are coplanar, then a, b, c, s are coplanar.

(14) If $p \neq q$ and p, q and r are collinear and a, b, c, p are coplanar and a, b, c, q are coplanar, then a, b, c, r are coplanar.

(15) If a, b and c are not collinear and a, b, c, p are coplanar and a, b, c, q are coplanar and a, b, c, r are coplanar and a, b, c, s are coplanar, then there exists x such that p, q and x are collinear and r, s and x are collinear.

(16) There exist a, b, c, d such that a, b, c, d are not coplanar.

(17) If p, q and r are not collinear, then there exists s such that p, q, r, s are not coplanar.

(18) If $a = b$ or $a = c$ or $b = c$ or $a = d$ or $b = d$ or $d = c$, then a, b, c, d are coplanar.

(19) If a, b, c, o are not coplanar and o, a and a' are collinear and $a \neq a'$, then a, b, c, a' are not coplanar.

(20) Suppose that

- (i) a, b and c are not collinear,
- (ii) a', b' and c' are not collinear,
- (iii) a, b, c, p are coplanar,
- (iv) a, b, c, q are coplanar,
- (v) a, b, c, r are coplanar,

[2]The proposition (9) was either repeated or obvious.

(vi) a', b', c', p are coplanar,
(vii) a', b', c', q are coplanar,
(viii) a', b', c', r are coplanar,
(ix) a, b, c, a' are not coplanar.
Then p, q and r are collinear.

(21) Suppose that
(i) $a \neq a'$,
(ii) o, a and a' are collinear,
(iii) a, b, c, o are not coplanar,
(iv) a', b' and c' are not collinear,
(v) a, b and p are collinear,
(vi) a', b' and p are collinear,
(vii) b, c and q are collinear,
(viii) b', c' and q are collinear,
(ix) a, c and r are collinear,
(x) a', c' and r are collinear.
Then p, q and r are collinear.

(22) If a, b, c, d are not coplanar and a, b, c, o are coplanar and a, b and o are not collinear, then a, b, d, o are not coplanar.

(23) If a, b, c, o are not coplanar and o, a and a' are collinear and o, b and b' are collinear and o, c and c' are collinear and $o \neq a'$ and $o \neq b'$ and $o \neq c'$, then a', b' and c' are not collinear and a', b', c', o are not coplanar.

(24) Suppose that
(i) a, b, c, o are coplanar,
(ii) a, b, c, d are not coplanar,
(iii) a, b, d, o are not coplanar,
(iv) b, c, d, o are not coplanar,
(v) a, c, d, o are not coplanar,
(vi) o, d and d' are collinear,
(vii) o, a and a' are collinear,
(viii) o, b and b' are collinear,
(ix) o, c and c' are collinear,
(x) a, d and s are collinear,
(xi) a', d' and s are collinear,
(xii) b, d and t are collinear,
(xiii) b', d' and t are collinear,
(xiv) c, d and u are collinear,
(xv) $o \neq a'$,
(xvi) $o \neq d'$,
(xvii) $d \neq d'$,
(xviii) $o \neq b'$.
Then s, t and u are not collinear.

Let us consider F_1, o, a, b, c. We say that o, a, b, and c constitute a quadrangle if and only if:

(Def.2) a, b and c are not collinear and o, a and b are not collinear and o, b and c are not collinear and o, c and a are not collinear.

The following propositions are true:

(26)[3] Suppose that
(i) o, a and b are not collinear,
(ii) o, b and c are not collinear,
(iii) o, a and c are not collinear,
(iv) o, a and a' are collinear,
(v) o, b and b' are collinear,
(vi) o, c and c' are collinear,
(vii) a, b and p are collinear,
(viii) a', b' and p are collinear,
(ix) $a \neq a'$,
(x) b, c and r are collinear,
(xi) b', c' and r are collinear,
(xii) a, c and q are collinear,
(xiii) $b \neq b'$,
(xiv) a', c' and q are collinear,
(xv) $o \neq a'$,
(xvi) $o \neq b'$,
(xvii) $o \neq c'$.
Then r, q and p are collinear.

(27) For every at least 3-dimensional projective space C_1 defined in terms of collinearity holds C_1 is a Desarguesian at least 3-dimensional projective space defined in terms of collinearity.

We see that the at least 3-dimensional projective space defined in terms of collinearity is a Desarguesian at least 3-dimensional projective space defined in terms of collinearity.

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[3]The proposition (25) was either repeated or obvious.