FORMALIZED MATHEMATICS
Vol.2, No.1, January–February 1991
Université Catholique de Louvain

Schemes of Existence of some Types of Functions

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Summary. We prove some useful shemes of existence of real sequences, partial functions from a domain into a domain, partial functions from a set to a set and functions from a domain into a domain. At the begining we prove some related auxiliary theorems to the article [1].

MML Identifier: `SCHEME1`.

The notation and terminology used here are introduced in the following articles: [9], [5], [1], [2], [3], [8], [6], [4], and [7]. We adopt the following convention: x, y will be arbitrary, n, m will denote natural numbers, and r will denote a real number. Next we state four propositions:

(1) For every n there exists m such that $n = 2 \cdot m$ or $n = 2 \cdot m + 1$.

(2) For every n there exists m such that $n = 3 \cdot m$ or $n = 3 \cdot m + 1$ or $n = 3 \cdot m + 2$.

(3) For every n there exists m such that $n = 4 \cdot m$ or $n = 4 \cdot m + 1$ or $n = 4 \cdot m + 2$ or $n = 4 \cdot m + 3$.

(4) For every n there exists m such that $n = 5 \cdot m$ or $n = 5 \cdot m + 1$ or $n = 5 \cdot m + 2$ or $n = 5 \cdot m + 3$ or $n = 5 \cdot m + 4$.

In this article we present several logical schemes. The scheme *ExRealSubseq* concerns a sequence of real numbers $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

there exists a sequence of real numbers q such that q is a subsequence of $\mathcal{A}$ and for every n holds $\mathcal{P}[q(n)]$ and for every n such that for every r such that $r = \mathcal{A}(n)$ holds $\mathcal{P}[r]$ there exists m such that $\mathcal{A}(n) = q(m)$

provided the following requirement is met:

- for every n there exists m such that $n \leq m$ and $\mathcal{P}[\mathcal{A}(m)]$.

[1] Supported by RPBP.III-24.C8

ISSN 0777–4028

The scheme *ExRealSeq2* deals with a unary functor $\mathcal{F}$ yielding a real number and a unary functor $\mathcal{G}$ yielding a real number and states that:

there exists a sequence of real numbers s such that for every n holds $s(2 \cdot n) = \mathcal{F}(n)$ and $s(2 \cdot n + 1) = \mathcal{G}(n)$

for all values of the parameters.

The scheme *ExRealSeq3* deals with a unary functor $\mathcal{F}$ yielding a real number, a unary functor $\mathcal{G}$ yielding a real number, and a unary functor $\mathcal{H}$ yielding a real number and states that:

there exists a sequence of real numbers s such that for every n holds $s(3 \cdot n) = \mathcal{F}(n)$ and $s(3 \cdot n + 1) = \mathcal{G}(n)$ and $s(3 \cdot n + 2) = \mathcal{H}(n)$

for all values of the parameters.

The scheme *ExRealSeq4* deals with a unary functor $\mathcal{F}$ yielding a real number, a unary functor $\mathcal{G}$ yielding a real number, a unary functor $\mathcal{H}$ yielding a real number, and a unary functor $\mathcal{I}$ yielding a real number and states that:

there exists a sequence of real numbers s such that for every n holds $s(4 \cdot n) = \mathcal{F}(n)$ and $s(4 \cdot n + 1) = \mathcal{G}(n)$ and $s(4 \cdot n + 2) = \mathcal{H}(n)$ and $s(4 \cdot n + 3) = \mathcal{I}(n)$

for all values of the parameters.

The scheme *ExRealSeq5* deals with a unary functor $\mathcal{F}$ yielding a real number, a unary functor $\mathcal{G}$ yielding a real number, a unary functor $\mathcal{H}$ yielding a real number, a unary functor $\mathcal{I}$ yielding a real number, and a unary functor $\mathcal{J}$ yielding a real number and states that:

there exists a sequence of real numbers s such that for every n holds $s(5 \cdot n) = \mathcal{F}(n)$ and $s(5 \cdot n + 1) = \mathcal{G}(n)$ and $s(5 \cdot n + 2) = \mathcal{H}(n)$ and $s(5 \cdot n + 3) = \mathcal{I}(n)$ and $s(5 \cdot n + 4) = \mathcal{J}(n)$

for all values of the parameters.

The scheme *PartFuncExD2* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, and two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds $c \in \mathrm{dom}\, f$ if and only if $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ and for every element c of $\mathcal{A}$ such that $c \in \mathrm{dom}\, f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$

provided the following condition is met:

- for every element c of $\mathcal{A}$ such that $\mathcal{P}[c]$ holds not $\mathcal{Q}[c]$.

The scheme *PartFuncExD2'* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, and two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds $c \in \mathrm{dom}\, f$ if and only if $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ and for every element c of $\mathcal{A}$ such that $c \in \mathrm{dom}\, f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$

provided the following requirement is met:

- for every element c of $\mathcal{A}$ such that $\mathcal{P}[c]$ and $\mathcal{Q}[c]$ holds $\mathcal{F}(c) = \mathcal{G}(c)$.

The scheme *PartFuncExD2"* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, and a unary predicate $\mathcal{P}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that f is total and for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if not $\mathcal{P}[c]$, then $f(c) = \mathcal{G}(c)$
for all values of the parameters.

The scheme *PartFuncExD3* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{H}$ yielding an element of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ if and only if $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ and for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ but if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$
provided the parameters satisfy the following condition:

- for every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$ but if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ but if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$.

The scheme *PartFuncExD3'* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{H}$ yielding an element of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ if and only if $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ and for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ but if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$
provided the following requirement is met:

- for every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$ and $\mathcal{Q}[c]$, then $\mathcal{F}(c) = \mathcal{G}(c)$ but if $\mathcal{P}[c]$ and $\mathcal{R}[c]$, then $\mathcal{F}(c) = \mathcal{H}(c)$ but if $\mathcal{Q}[c]$ and $\mathcal{R}[c]$, then $\mathcal{G}(c) = \mathcal{H}(c)$.

The scheme *PartFuncExD4* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{H}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{I}$ yielding an element of $\mathcal{B}$, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and $\mathcal{S}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds $c \in \operatorname{dom} f$ if and only if $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ or $\mathcal{S}[c]$ and for every element c of $\mathcal{A}$ such that $c \in \operatorname{dom} f$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ but if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$ but if $\mathcal{S}[c]$, then $f(c) = \mathcal{I}(c)$
provided the parameters satisfy the following condition:

- for every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$ but if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ but if $\mathcal{P}[c]$, then not $\mathcal{S}[c]$ but if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$ but if $\mathcal{Q}[c]$, then not $\mathcal{S}[c]$ but if $\mathcal{R}[c]$, then not $\mathcal{S}[c]$.

The scheme *PartFuncExS2* deals with a set $\mathcal{A}$, a set $\mathcal{B}$, a unary functor $\mathcal{F}$, a unary functor $\mathcal{G}$, and two unary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every x holds $x \in \operatorname{dom} f$ if and only if $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$ and for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ but if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$

provided the parameters satisfy the following conditions:

- for every x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$.

The scheme *PartFuncExS3* deals with a set $\mathcal{A}$, a set $\mathcal{B}$, a unary functor $\mathcal{F}$, a unary functor $\mathcal{G}$, a unary functor $\mathcal{H}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every x holds $x \in \operatorname{dom} f$ if and only if $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$ or $\mathcal{R}[x]$ and for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ but if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$ but if $\mathcal{R}[x]$, then $f(x) = \mathcal{H}(x)$

provided the parameters meet the following conditions:

- for every x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$ but if $\mathcal{P}[x]$, then not $\mathcal{R}[x]$ but if $\mathcal{Q}[x]$, then not $\mathcal{R}[x]$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{R}[x]$ holds $\mathcal{H}(x) \in \mathcal{B}$.

The scheme *PartFuncExS4* deals with a set $\mathcal{A}$, a set $\mathcal{B}$, a unary functor $\mathcal{F}$, a unary functor $\mathcal{G}$, a unary functor $\mathcal{H}$, a unary functor $\mathcal{I}$, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and $\mathcal{S}$, and states that:

there exists a partial function f from $\mathcal{A}$ to $\mathcal{B}$ such that for every x holds $x \in \operatorname{dom} f$ if and only if $x \in \mathcal{A}$ but $\mathcal{P}[x]$ or $\mathcal{Q}[x]$ or $\mathcal{R}[x]$ or $\mathcal{S}[x]$ and for every x such that $x \in \operatorname{dom} f$ holds if $\mathcal{P}[x]$, then $f(x) = \mathcal{F}(x)$ but if $\mathcal{Q}[x]$, then $f(x) = \mathcal{G}(x)$ but if $\mathcal{R}[x]$, then $f(x) = \mathcal{H}(x)$ but if $\mathcal{S}[x]$, then $f(x) = \mathcal{I}(x)$

provided the parameters meet the following requirements:

- for every x such that $x \in \mathcal{A}$ holds if $\mathcal{P}[x]$, then not $\mathcal{Q}[x]$ but if $\mathcal{P}[x]$, then not $\mathcal{R}[x]$ but if $\mathcal{P}[x]$, then not $\mathcal{S}[x]$ but if $\mathcal{Q}[x]$, then not $\mathcal{R}[x]$ but if $\mathcal{Q}[x]$, then not $\mathcal{S}[x]$ but if $\mathcal{R}[x]$, then not $\mathcal{S}[x]$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{P}[x]$ holds $\mathcal{F}(x) \in \mathcal{B}$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{Q}[x]$ holds $\mathcal{G}(x) \in \mathcal{B}$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{R}[x]$ holds $\mathcal{H}(x) \in \mathcal{B}$,
- for every x such that $x \in \mathcal{A}$ and $\mathcal{S}[x]$ holds $\mathcal{I}(x) \in \mathcal{B}$.

The scheme *PartFuncExC_D2* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{C}$, and two binary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a partial function f from $[\!: \mathcal{A}, \mathcal{B} :\!]$ to $\mathcal{C}$ such that for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d \rangle \in \operatorname{dom} f$ if and only if $\mathcal{P}[c, d]$ or $\mathcal{Q}[c, d]$ and for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\langle c, d \rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c, d]$, then $f(\langle c, d \rangle) = \mathcal{F}(c, d)$ but if $\mathcal{Q}[c, d]$, then $f(\langle c, d \rangle) = \mathcal{G}(c, d)$

provided the parameters meet the following requirement:

- for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\mathcal{P}[c, d]$ holds not $\mathcal{Q}[c, d]$.

The scheme *PartFuncExC_D3* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{C}$, a binary functor $\mathcal{H}$ yielding an element of $\mathcal{C}$, and three binary predicates $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$, and states that:

there exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d\rangle \in \operatorname{dom} f$ if and only if $\mathcal{P}[c, d]$ or $\mathcal{Q}[c, d]$ or $\mathcal{R}[c, d]$ and for every element c of $\mathcal{A}$ and for every element r of $\mathcal{B}$ such that $\langle c, r\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c, r]$, then $f(\langle c, r\rangle) = \mathcal{F}(c, r)$ but if $\mathcal{Q}[c, r]$, then $f(\langle c, r\rangle) = \mathcal{G}(c, r)$ but if $\mathcal{R}[c, r]$, then $f(\langle c, r\rangle) = \mathcal{H}(c, r)$

provided the following requirement is met:

- for every element c of $\mathcal{A}$ and for every element s of $\mathcal{B}$ holds if $\mathcal{P}[c, s]$, then not $\mathcal{Q}[c, s]$ but if $\mathcal{P}[c, s]$, then not $\mathcal{R}[c, s]$ but if $\mathcal{Q}[c, s]$, then not $\mathcal{R}[c, s]$.

The scheme *PartFuncExC_S2* concerns a set $\mathcal{A}$, a set $\mathcal{B}$, a set $\mathcal{C}$, a binary functor $\mathcal{F}$, a binary functor $\mathcal{G}$, and two binary predicates $\mathcal{P}$ and $\mathcal{Q}$, and states that:

there exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that for all x, y holds $\langle x, y\rangle \in \operatorname{dom} f$ if and only if $x \in \mathcal{A}$ and $y \in \mathcal{B}$ but $\mathcal{P}[x, y]$ or $\mathcal{Q}[x, y]$ and for all x, y such that $\langle x, y\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[x, y]$, then $f(\langle x, y\rangle) = \mathcal{F}(x, y)$ but if $\mathcal{Q}[x, y]$, then $f(\langle x, y\rangle) = \mathcal{G}(x, y)$

provided the following conditions are met:

- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x, y]$, then not $\mathcal{Q}[x, y]$,
- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x, y]$ holds $\mathcal{F}(x, y) \in \mathcal{C}$,
- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{Q}[x, y]$ holds $\mathcal{G}(x, y) \in \mathcal{C}$.

The scheme *PartFuncExC_S3* concerns a set $\mathcal{A}$, a set $\mathcal{B}$, a set $\mathcal{C}$, a binary functor $\mathcal{F}$, a binary functor $\mathcal{G}$, a binary functor $\mathcal{H}$, and three binary predicates $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$, and states that:

there exists a partial function f from $[:\mathcal{A}, \mathcal{B}:]$ to $\mathcal{C}$ such that for all x, y holds $\langle x, y\rangle \in \operatorname{dom} f$ if and only if $x \in \mathcal{A}$ and $y \in \mathcal{B}$ but $\mathcal{P}[x, y]$ or $\mathcal{Q}[x, y]$ or $\mathcal{R}[x, y]$ and for all x, y such that $\langle x, y\rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[x, y]$, then $f(\langle x, y\rangle) = \mathcal{F}(x, y)$ but if $\mathcal{Q}[x, y]$, then $f(\langle x, y\rangle) = \mathcal{G}(x, y)$ but if $\mathcal{R}[x, y]$, then $f(\langle x, y\rangle) = \mathcal{H}(x, y)$

provided the following conditions are met:

- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x, y]$, then not $\mathcal{Q}[x, y]$ but if $\mathcal{P}[x, y]$, then not $\mathcal{R}[x, y]$ but if $\mathcal{Q}[x, y]$, then not $\mathcal{R}[x, y]$,
- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{P}[x, y]$, then $\mathcal{F}(x, y) \in \mathcal{C}$,
- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{Q}[x, y]$, then $\mathcal{G}(x, y) \in \mathcal{C}$,
- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ holds if $\mathcal{R}[x, y]$, then $\mathcal{H}(x, y) \in \mathcal{C}$.

The scheme *ExFuncD3* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{H}$ yielding an element of $\mathcal{B}$, and three unary predicates $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$, and states that:

there exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ but if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$

provided the parameters satisfy the following conditions:

- for every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$ but if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ but if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$,
- for every element c of $\mathcal{A}$ holds $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$.

The scheme *ExFuncD4* concerns a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a unary functor $\mathcal{F}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{G}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{H}$ yielding an element of $\mathcal{B}$, a unary functor $\mathcal{I}$ yielding an element of $\mathcal{B}$, and four unary predicates $\mathcal{P}$, $\mathcal{Q}$, $\mathcal{R}$, and $\mathcal{S}$, and states that:

there exists a function f from $\mathcal{A}$ into $\mathcal{B}$ such that for every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then $f(c) = \mathcal{F}(c)$ but if $\mathcal{Q}[c]$, then $f(c) = \mathcal{G}(c)$ but if $\mathcal{R}[c]$, then $f(c) = \mathcal{H}(c)$ but if $\mathcal{S}[c]$, then $f(c) = \mathcal{I}(c)$

provided the following conditions are met:

- for every element c of $\mathcal{A}$ holds if $\mathcal{P}[c]$, then not $\mathcal{Q}[c]$ but if $\mathcal{P}[c]$, then not $\mathcal{R}[c]$ but if $\mathcal{P}[c]$, then not $\mathcal{S}[c]$ but if $\mathcal{Q}[c]$, then not $\mathcal{R}[c]$ but if $\mathcal{Q}[c]$, then not $\mathcal{S}[c]$ but if $\mathcal{R}[c]$, then not $\mathcal{S}[c]$,
- for every element c of $\mathcal{A}$ holds $\mathcal{P}[c]$ or $\mathcal{Q}[c]$ or $\mathcal{R}[c]$ or $\mathcal{S}[c]$.

The scheme *FuncExC_D2* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{C}$, and a binary predicate $\mathcal{P}$, and states that:

there exists a function f from $[\!:\mathcal{A}, \mathcal{B}:\!]$ into $\mathcal{C}$ such that for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\langle c, d \rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c, d]$, then $f(\langle c, d \rangle) = \mathcal{F}(c, d)$ but if not $\mathcal{P}[c, d]$, then $f(\langle c, d \rangle) = \mathcal{G}(c, d)$

for all values of the parameters.

The scheme *FuncExC_D3* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a non-empty set $\mathcal{C}$, a binary functor $\mathcal{F}$ yielding an element of $\mathcal{C}$, a binary functor $\mathcal{G}$ yielding an element of $\mathcal{C}$, a binary functor $\mathcal{H}$ yielding an element of $\mathcal{C}$, and three binary predicates $\mathcal{P}$, $\mathcal{Q}$, and $\mathcal{R}$, and states that:

there exists a function f from $[\!:\mathcal{A}, \mathcal{B}:\!]$ into $\mathcal{C}$ such that for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\langle c, d \rangle \in \operatorname{dom} f$ if and only if $\mathcal{P}[c, d]$ or $\mathcal{Q}[c, d]$ or $\mathcal{R}[c, d]$ and for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ such that $\langle c, d \rangle \in \operatorname{dom} f$ holds if $\mathcal{P}[c, d]$, then $f(\langle c, d \rangle) = \mathcal{F}(c, d)$ but if $\mathcal{Q}[c, d]$, then $f(\langle c, d \rangle) = \mathcal{G}(c, d)$ but if $\mathcal{R}[c, d]$, then $f(\langle c, d \rangle) = \mathcal{H}(c, d)$

provided the parameters have the following properties:

- for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds if $\mathcal{P}[c, d]$, then not $\mathcal{Q}[c, d]$ but if $\mathcal{P}[c, d]$, then not $\mathcal{R}[c, d]$ but if $\mathcal{Q}[c, d]$, then not $\mathcal{R}[c, d]$,
- for every element c of $\mathcal{A}$ and for every element d of $\mathcal{B}$ holds $\mathcal{P}[c, d]$ or $\mathcal{Q}[c, d]$ or $\mathcal{R}[c, d]$.

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Received September 21, 1990