

# One-Dimensional Congruence of Segments, Basic Facts and Midpoint Relation <sup>1</sup>

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**Summary.** We study the theory of one-dimensional congruence of segments. The theory is characterized by a suitable formal axiom system; as a model of this system one can take the structure obtained from any weak directed geometrical bundle, with the congruence interpreted as in the case of "classical" vectors. Preliminary consequences of our axiom system are proved, basic relations of maximal distance and of midpoint are defined, and several fundamental properties of them are established.

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The papers [8], [2], [3], [10], [7], [4], [1], [5], [6], and [9] provide the terminology and notation for this paper. In the sequel  $A_1$  will be a weak affine vector space. Let us consider  $A_1$ , and let  $a, b, c, d$  be elements of the points of  $A_1$ . The predicate  $a, b \Leftrightarrow c, d$  is defined as follows:

(Def.1)  $a, b \Rightarrow c, d$  or  $a, b \Rightarrow d, c$ .

An affine structure is called a weak segment-congruence space if:

(Def.2) (i) there exist elements  $a, b$  of the points of it such that  $a \neq b$ ,  
(ii) for all elements  $a, b$  of the points of it holds  $a, b \Rightarrow b, a$ ,

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- (iii) for all elements  $a, b$  of the points of it such that  $a, b \ni a$ ,  $a$  holds  $a = b$ ,
- (iv) for all elements  $a, b, c, d, p, q$  of the points of it such that  $a, b \ni p, q$  and  $c, d \ni p, q$  holds  $a, b \ni c, d$ ,
- (v) for every elements  $a, c$  of the points of it there exists an element  $b$  of the points of it such that  $a, b \ni b, c$ ,
- (vi) for all elements  $a, a', b, b', p$  of the points of it such that  $a \neq a'$  and  $b \neq b'$  and  $p, a \ni p, a'$  and  $p, b \ni p, b'$  holds  $a, b \ni a', b'$ ,
- (vii) for all elements  $a, b$  of the points of it holds  $a = b$  or there exists an element  $c$  of the points of it such that  $a \neq c$  and  $a, b \ni b, c$  or there exist elements  $p, p'$  of the points of it such that  $p \neq p'$  and  $a, b \ni p, p'$  and  $a, p \ni p, b$  and  $a, p' \ni p', b$ ,
- (viii) for all elements  $a, b, b', p, p', c$  of the points of it such that  $a, b \ni b, c$  and  $b, b' \ni p, p'$  and  $b, p \ni p, b'$  and  $b, p' \ni p', b'$  holds  $a, b' \ni b', c$ ,
- (ix) for all elements  $a, b, b', c$  of the points of it such that  $a \neq c$  and  $b \neq b'$  and  $a, b \ni b, c$  and  $a, b' \ni b', c$  there exist elements  $p, p'$  of the points of it such that  $p \neq p'$  and  $b, b' \ni p, p'$  and  $b, p \ni p, b'$  and  $b, p' \ni p', b'$ ,
- (x) for all elements  $a, b, c, p, p', q, q'$  of the points of it such that  $a, b \ni p, p'$  and  $a, c \ni q, q'$  and  $a, p \ni p, b$  and  $a, q \ni q, c$  and  $a, p' \ni p', b$  and  $a, q' \ni q', c$  there exist elements  $r, r'$  of the points of it such that  $b, c \ni r, r'$  and  $b, r \ni r, c$  and  $b, r' \ni r', c$ .

We adopt the following rules:  $A_1$  is a weak segment-congruence space and  $a, b, b', b'', c, d, p, p'$  are elements of the points of  $A_1$ . Let us consider  $A_1$ , and let  $a, b, c, d$  be elements of the points of  $A_1$ . The predicate  $a, b \ni c, d$  is defined by:

(Def.3)  $a, b \ni c, d$ .

We now state several propositions:

- (1)  $a, b \ni a, b$ .
- (2) If  $a, b \ni c, d$ , then  $c, d \ni a, b$ .
- (3) If  $a, b \ni c, d$ , then  $a, b \ni d, c$ .
- (4) If  $a, b \ni c, d$ , then  $b, a \ni c, d$ .
- (5) For all  $a, b$  holds  $a, a \ni b, b$ .
- (6) If  $a, b \ni c, c$ , then  $a = b$ .
- (7) If  $a, b \ni p, p'$  and  $p, p' \ni b, c$  and  $a, b \ni b, c$  and  $a, p \ni p, b$  and  $a, p' \ni p', b$ , then  $a = c$ .
- (8) If  $a, b \ni a, b'$  and  $a, b' \ni a, b''$  and  $a, b \ni a, b''$ , then  $b = b'$  or  $b = b''$  or  $b' = b''$ .

Let us consider  $A_1, a, b$ . We say that  $a, b$  are in a maximal distance if and only if:

(Def.4) there exist  $p, p'$  such that  $p \neq p'$  and  $a, b \ni p, p'$  and  $a, p \ni p, b$  and  $a, p' \ni p', b$ .

Let us consider  $A_1, a, b, c$ . We say that  $b$  is a midpoint of  $a, c$  if and only if:

(Def.5)  $a = b$  and  $b = c$  and  $a = c$  or  $a = c$  and  $a, b$  are in a maximal distance or  $a \neq c$  and  $a, b \ni b, c$ .

Next we state three propositions:

- (11)<sup>2</sup> If  $a \neq b$  and  $a, b$  are not in a maximal distance, then there exists  $c$  such that  $a \neq c$  and  $a, b \Leftrightarrow b, c$ .
- (12) If  $a, b$  are in a maximal distance and  $a, b \Leftrightarrow b, c$ , then  $a = c$ .
- (13) If  $a, b$  are in a maximal distance, then  $a \neq b$ .

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<sup>2</sup>The propositions (9)–(10) were either repeated or obvious.