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One-Dimensional Congruence of Segments, Basic Facts and Midpoint Relation [1]

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Summary. We study the theory of one-dimensional congruence of segments. The theory is characterized by a suitable formal axiom system; as a model of this system one can take the structure obtained from any weak directed geometrical bundle, with the congruence interpreted as in the case of "classical" vectors. Preliminary consequences of our axiom system are proved, basic relations of maximal distance and of midpoint are defined, and several fundamental properties of them are established.

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The papers [8], [2], [3], [10], [7], [4], [1], [5], [6], and [9] provide the terminology and notation for this paper. In the sequel A_1 will be a weak affine vector space. Let us consider A_1, and let a, b, c, d be elements of the points of A_1. The predicate $a, b \Leftrightarrow c, d$ is defined as follows:

(Def.1) $a, b \Rrightarrow c, d$ or $a, b \Rrightarrow d, c$.

An affine structure is called a weak segment-congruence space if:

(Def.2) (i) there exist elements a, b of the points of it such that $a \neq b$,

(ii) for all elements a, b of the points of it holds $a, b \Rrightarrow b, a$,

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(iii) for all elements a, b of the points of it such that $a, b \Rrightarrow a, a$ holds $a = b$,

(iv) for all elements a, b, c, d, p, q of the points of it such that $a, b \Rrightarrow p, q$ and $c, d \Rrightarrow p, q$ holds $a, b \Rrightarrow c, d$,

(v) for every elements a, c of the points of it there exists an element b of the points of it such that $a, b \Rrightarrow b, c$,

(vi) for all elements a, a', b, b', p of the points of it such that $a \neq a'$ and $b \neq b'$ and $p, a \Rrightarrow p, a'$ and $p, b \Rrightarrow p, b'$ holds $a, b \Rrightarrow a', b'$,

(vii) for all elements a, b of the points of it holds $a = b$ or there exists an element c of the points of it such that $a \neq c$ and $a, b \Rrightarrow b, c$ or there exist elements p, p' of the points of it such that $p \neq p'$ and $a, b \Rrightarrow p, p'$ and $a, p \Rrightarrow p, b$ and $a, p' \Rrightarrow p', b$,

(viii) for all elements a, b, b', p, p', c of the points of it such that $a, b \Rrightarrow b, c$ and $b, b' \Rrightarrow p, p'$ and $b, p \Rrightarrow p, b'$ and $b, p' \Rrightarrow p', b'$ holds $a, b' \Rrightarrow b', c$,

(ix) for all elements a, b, b', c of the points of it such that $a \neq c$ and $b \neq b'$ and $a, b \Rrightarrow b, c$ and $a, b' \Rrightarrow b', c$ there exist elements p, p' of the points of it such that $p \neq p'$ and $b, b' \Rrightarrow p, p'$ and $b, p \Rrightarrow p, b'$ and $b, p' \Rrightarrow p', b'$,

(x) for all elements a, b, c, p, p', q, q' of the points of it such that $a, b \Rrightarrow p, p'$ and $a, c \Rrightarrow q, q'$ and $a, p \Rrightarrow p, b$ and $a, q \Rrightarrow q, c$ and $a, p' \Rrightarrow p', b$ and $a, q' \Rrightarrow q', c$ there exist elements r, r' of the points of it such that $b, c \Rrightarrow r, r'$ and $b, r \Rrightarrow r, c$ and $b, r' \Rrightarrow r', c$.

We adopt the following rules: A_1 is a weak segment-congruence space and a, b, b', b'', c, d, p, p' are elements of the points of A_1. Let us consider A_1, and let a, b, c, d be elements of the points of A_1. The predicate $a, b \Leftrightarrow c, d$ is defined by:

(Def.3) $a, b \Rrightarrow c, d$.

We now state several propositions:

(1) $a, b \Leftrightarrow a, b$.

(2) If $a, b \Leftrightarrow c, d$, then $c, d \Leftrightarrow a, b$.

(3) If $a, b \Leftrightarrow c, d$, then $a, b \Leftrightarrow d, c$.

(4) If $a, b \Leftrightarrow c, d$, then $b, a \Leftrightarrow c, d$.

(5) For all a, b holds $a, a \Leftrightarrow b, b$.

(6) If $a, b \Leftrightarrow c, c$, then $a = b$.

(7) If $a, b \Leftrightarrow p, p'$ and $p, p' \Leftrightarrow b, c$ and $a, b \Leftrightarrow b, c$ and $a, p \Leftrightarrow p, b$ and $a, p' \Leftrightarrow p', b$, then $a = c$.

(8) If $a, b \Leftrightarrow a, b'$ and $a, b' \Leftrightarrow a, b''$ and $a, b \Leftrightarrow a, b''$, then $b = b'$ or $b = b''$ or $b' = b''$.

Let us consider A_1, a, b. We say that a, b are in a maximal distance if and only if:

(Def.4) there exist p, p' such that $p \neq p'$ and $a, b \Leftrightarrow p, p'$ and $a, p \Leftrightarrow p, b$ and $a, p' \Leftrightarrow p', b$.

Let us consider A_1, a, b, c. We say that b is a midpoint of a, c if and only if:

(Def.5) $a = b$ and $b = c$ and $a = c$ or $a = c$ and a, b are in a maximal distance or $a \neq c$ and $a, b \Leftrightarrow b, c$.

Next we state three propositions:

(11)[2] If $a \neq b$ and a, b are not in a maximal distance, then there exists c such that $a \neq c$ and $a, b \Leftrightarrow b, c$.

(12) If a, b are in a maximal distance and $a, b \Leftrightarrow b, c$, then $a = c$.

(13) If a, b are in a maximal distance, then $a \neq b$.

References

[1] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.

[2] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[3] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[4] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Formalized Mathematics*, 1(**2**):335–342, 1990.

[5] Grzegorz Lewandowski and Krzysztof Prażmowski. A construction of an abstract space of congruence of vectors. *Formalized Mathematics*, 1(**4**):685–688, 1990.

[6] Grzegorz Lewandowski, Krzysztof Prażmowski, and Bożena Lewandowska. Directed geometrical bundles and their analytical representation. *Formalized Mathematics*, 2(**1**):135–141, 1991.

[7] Henryk Oryszczyszyn and Krzysztof Prażmowski. Analytical ordered affine spaces. *Formalized Mathematics*, 1(**3**):601–605, 1990.

[8] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[9] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.

[10] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(**1**):181–186, 1990.

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[2]The propositions (9)–(10) were either repeated or obvious.