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Ternary Fields [1]

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Summary. The article contains part 3 of the set of papers concerning the theory of algebraic structures, based on the book [11] pp. 13-15 (pages 6-8 for English edition).

First the basic structure (F, 0, 1, T) is defined, where T is a ternary operation on F (three-argument operations have been introduced in the article [9]). Following it, the basic axioms of a Ternary Field are displayed, the mode is defined and its existence proved. The basic properties of a Ternary Field are also contemplated there.

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The articles [13], [12], [3], [4], [1], [2], [6], [5], [7], [8], [10], and [9] provide the notation and terminology for this paper. We consider ternary field structures which are systems

⟨a carrier, a zero, a unity, a operation⟩,

where the carrier is a non-empty set, the zero is an element of the carrier, the unity is an element of the carrier, and the operation is a ternary operation on the carrier.

In the sequel F denotes a ternary field structure. Let us consider F. A scalar of F is an element of the carrier of F.

In the sequel a, b, c are scalars of F. Let us consider F, a, b, c. The functor $\mathrm{T}(a,b,c)$ yields a scalar of F and is defined by:

(Def.1) $\mathrm{T}(a,b,c) =$ (the operation of F)(a, b, c).

Let us consider F. The functor 0_F yielding a scalar of F is defined as follows:

(Def.2) $0_F =$ the zero of F.

Let us consider F. The functor 1_F yields a scalar of F and is defined by:

(Def.3) $1_F =$ the unity of F.

The ternary operation $\mathrm{T}_{\mathbb{R}}$ on $\mathbb{R}$ is defined as follows:

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(Def.4) for all real numbers a, b, c holds $\mathrm{T}_{\mathbb{R}}(a, b, c) = a \cdot b + c$.

The ternary field structure $\mathbb{R}_{\mathrm{t}}$ is defined by:

(Def.5) $\mathbb{R}_{\mathrm{t}} = \langle \mathbb{R}, 0, 1, \mathrm{T}_{\mathbb{R}} \rangle$.

Let a, b, c be scalars of $\mathbb{R}_{\mathrm{t}}$. The functor $\mathrm{T}^e(a, b, c)$ yields a scalar of $\mathbb{R}_{\mathrm{t}}$ and is defined by:

(Def.6) $\mathrm{T}^e(a, b, c) = (\text{the operation of } \mathbb{R}_{\mathrm{t}})(a, b, c)$.

We now state several propositions:

(1) For every scalar a of $\mathbb{R}_{\mathrm{t}}$ holds a is a real number.

(2) For every real number a holds a is a scalar of $\mathbb{R}_{\mathrm{t}}$.

(3) For all real numbers u, u', v, v' such that $u \neq u'$ there exists a real number x such that $u \cdot x + v = u' \cdot x + v'$.

(5)[2] For all scalars u, a, v of $\mathbb{R}_{\mathrm{t}}$ and for all real numbers z, x, y such that $u = z$ and $a = x$ and $v = y$ holds $\mathrm{T}(u, a, v) = z \cdot x + y$.

(6) $0 = 0_{\mathbb{R}_{\mathrm{t}}}$.

(7) $1 = 1_{\mathbb{R}_{\mathrm{t}}}$.

A ternary field structure is called a ternary field if:

(Def.7) (i) $0_{\mathrm{it}} \neq 1_{\mathrm{it}}$,

(ii) for every scalar a of it holds $\mathrm{T}(a, 1_{\mathrm{it}}, 0_{\mathrm{it}}) = a$,

(iii) for every scalar a of it holds $\mathrm{T}(1_{\mathrm{it}}, a, 0_{\mathrm{it}}) = a$,

(iv) for all scalars a, b of it holds $\mathrm{T}(a, 0_{\mathrm{it}}, b) = b$,

(v) for all scalars a, b of it holds $\mathrm{T}(0_{\mathrm{it}}, a, b) = b$,

(vi) for every scalars u, a, b of it there exists a scalar v of it such that $\mathrm{T}(u, a, v) = b$,

(vii) for all scalars u, a, v, v' of it such that $\mathrm{T}(u, a, v) = \mathrm{T}(u, a, v')$ holds $v = v'$,

(viii) for all scalars a, a' of it such that $a \neq a'$ for every scalars b, b' of it there exist scalars u, v of it such that $\mathrm{T}(u, a, v) = b$ and $\mathrm{T}(u, a', v) = b'$,

(ix) for all scalars u, u' of it such that $u \neq u'$ for every scalars v, v' of it there exists a scalar a of it such that $\mathrm{T}(u, a, v) = \mathrm{T}(u', a, v')$,

(x) for all scalars a, a', u, u', v, v' of it such that $\mathrm{T}(u, a, v) = \mathrm{T}(u', a, v')$ and $\mathrm{T}(u, a', v) = \mathrm{T}(u', a', v')$ holds $a = a'$ or $u = u'$.

We adopt the following convention: F is a ternary field and a, a', b, c, x, x', u, u', v, v' are scalars of F. We now state several propositions:

(8) If $a \neq a'$ and $\mathrm{T}(u, a, v) = \mathrm{T}(u', a, v')$ and $\mathrm{T}(u, a', v) = \mathrm{T}(u', a', v')$, then $u = u'$ and $v = v'$.

(9) For every a, b, c there exists x such that $\mathrm{T}(a, b, x) = c$.

(10) If $\mathrm{T}(a, b, x) = \mathrm{T}(a, b, x')$, then $x = x'$.

(11) If $a \neq 0_F$, then for every b, c there exists x such that $\mathrm{T}(a, x, b) = c$.

(12) If $a \neq 0_F$ and $\mathrm{T}(a, x, b) = \mathrm{T}(a, x', b)$, then $x = x'$.

(13) If $a \neq 0_F$, then for every b, c there exists x such that $\mathrm{T}(x, a, b) = c$.

[2]The proposition (4) was either repeated or obvious.

(14) If $a \neq 0_F$ and $\mathrm{T}(x, a, b) = \mathrm{T}(x', a, b)$, then $x = x'$.

References

[1] Czesław Byliński. Basic functions and operations on functions. *Formalized Mathematics*, 1(**1**):245–254, 1990.

[2] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.

[3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[4] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[5] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[6] Eugeniusz Kusak, Wojciech Leończuk, and Michał Muzalewski. Abelian groups, fields and vector spaces. *Formalized Mathematics*, 1(**2**):335–342, 1990.

[7] Michał Muzalewski. Midpoint algebras. *Formalized Mathematics*, 1(**3**):483–488, 1990.

[8] Michał Muzalewski and Wojciech Skaba. From loops to abelian multiplicative groups with zero. *Formalized Mathematics*, 1(**5**):833–840, 1990.

[9] Michal Muzalewski and Wojciech Skaba. Three-argument operations and four-argument operations. *Formalized Mathematics*, 2(**2**):221–224, 1991.

[10] Wojciech Skaba and Michał Muzalewski. From double loops to fields. *Formalized Mathematics*, 2(**1**):185–191, 1991.

[11] Wanda Szmielew. *From Affine to Euclidean Geometry*. Volume 27, PWN – D.Reidel Publ. Co., Warszawa – Dordrecht, 1983.

[12] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(**1**):25–34, 1990.

[13] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

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