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Finite Sums of Vectors in Left Module over Associative Ring [1]

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Summary. Definition of a finite sequence of the vectors of Left Module over Associative Ring and some theorems concerning these sums. Written as a generalization of the article [11].

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The terminology and notation used here have been introduced in the following papers: [10], [3], [2], [4], [6], [12], [9], [5], [1], [7], and [8]. For simplicity we adopt the following convention: x is arbitrary, R is an associative ring, a is a scalar of R, V is a left module over R, and v, v_1, v_2, w, u are vectors of V. Let us consider R, V, x. The predicate $x \in V$ is defined by:

(Def.1) $x \in$ the carrier of the carrier of V.

The following two propositions are true:

(1) $x \in V$ if and only if $x \in$ the carrier of the carrier of V.

(2) $v \in V$.

We adopt the following convention: F, G, H will denote finite sequences of elements of the carrier of the carrier of V, f will denote a function from $\mathbb{N}$ into the carrier of the carrier of V, and i, j, k, n will denote natural numbers. Let us consider R, V, F. The functor $\sum F$ yielding a vector of V is defined by:

(Def.2) there exists f such that $\sum F = f(\operatorname{len} F)$ and $f(0) = \Theta_V$ and for all j, v such that $j < \operatorname{len} F$ and $v = F(j+1)$ holds $f(j+1) = f(j) + v$.

One can prove the following propositions:

(3) If there exists f such that $u = f(\operatorname{len} F)$ and $f(0) = \Theta_V$ and for all j, v such that $j < \operatorname{len} F$ and $v = F(j+1)$ holds $f(j+1) = f(j) + v$, then $u = \sum F$.

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(4) There exists f such that $\sum F = f(\operatorname{len} F)$ and $f(0) = \Theta_V$ and for all j, v such that $j < \operatorname{len} F$ and $v = F(j+1)$ holds $f(j+1) = f(j) + v$.

(5) If $k \in \operatorname{Seg} n$ and $\operatorname{len} F = n$, then $F(k)$ is a vector of V.

(6) If $\operatorname{len} F = \operatorname{len} G + 1$ and $G = F \restriction \operatorname{Seg} \operatorname{len} G$ and $v = F(\operatorname{len} F)$, then $\sum F = \sum G + v$.

(7) $\sum(F \frown G) = \sum F + \sum G$.

(8) If $\operatorname{len} F = \operatorname{len} G$ and $\operatorname{len} F = \operatorname{len} H$ and for every k such that $k \in \operatorname{Seg} \operatorname{len} F$ holds $H(k) = \pi_k F + \pi_k G$, then $\sum H = \sum F + \sum G$.

(9) If $\operatorname{len} F = \operatorname{len} G$ and for all k, v such that $k \in \operatorname{Seg} \operatorname{len} F$ and $v = G(k)$ holds $F(k) = a \cdot v$, then $\sum F = a \cdot \sum G$.

(10) If $\operatorname{len} F = \operatorname{len} G$ and for every k such that $k \in \operatorname{Seg} \operatorname{len} F$ holds $G(k) = a \cdot \pi_k F$, then $\sum G = a \cdot \sum F$.

(11) If $\operatorname{len} F = \operatorname{len} G$ and for all k, v such that $k \in \operatorname{Seg} \operatorname{len} F$ and $v = G(k)$ holds $F(k) = -v$, then $\sum F = -\sum G$.

(12) If $\operatorname{len} F = \operatorname{len} G$ and for every k such that $k \in \operatorname{Seg} \operatorname{len} F$ holds $G(k) = -\pi_k F$, then $\sum G = -\sum F$.

(13) If $\operatorname{len} F = \operatorname{len} G$ and $\operatorname{len} F = \operatorname{len} H$ and for every k such that $k \in \operatorname{Seg} \operatorname{len} F$ holds $H(k) = \pi_k F - \pi_k G$, then $\sum H = \sum F - \sum G$.

(14) If $\operatorname{rng} F = \operatorname{rng} G$ and F is one-to-one and G is one-to-one, then $\sum F = \sum G$.

(15) For all F, G and for every permutation f of $\operatorname{dom} F$ such that $\operatorname{len} F = \operatorname{len} G$ and for every i such that $i \in \operatorname{dom} G$ holds $G(i) = F(f(i))$ holds $\sum F = \sum G$.

(16) For every permutation f of $\operatorname{dom} F$ such that $G = F \cdot f$ holds $\sum F = \sum G$.

(17) $\sum \varepsilon_{\text{the carrier of the carrier of } V} = \Theta_V$.

(18) $\sum \langle v \rangle = v$.

(19) $\sum \langle v, u \rangle = v + u$.

(20) $\sum \langle v, u, w \rangle = v + u + w$.

(21) $a \cdot \sum \varepsilon_{\text{the carrier of the carrier of } V} = \Theta_V$.

(22) $a \cdot \sum \langle v \rangle = a \cdot v$.

(23) $a \cdot \sum \langle v, u \rangle = a \cdot v + a \cdot u$.

(24) $a \cdot \sum \langle v, u, w \rangle = a \cdot v + a \cdot u + a \cdot w$.

(25) $-\sum \varepsilon_{\text{the carrier of the carrier of } V} = \Theta_V$.

(26) $-\sum \langle v \rangle = -v$.

(27) $-\sum \langle v, u \rangle = (-v) - u$.

(28) $-\sum \langle v, u, w \rangle = (-v) - u - w$.

(29) $\sum \langle v, w \rangle = \sum \langle w, v \rangle$.

(30) $\sum \langle v, w \rangle = \sum \langle v \rangle + \sum \langle w \rangle$.

(31) $\sum \langle \Theta_V, \Theta_V \rangle = \Theta_V$.

(32) $\sum \langle \Theta_V, v \rangle = v$ and $\sum \langle v, \Theta_V \rangle = v$.

(33) $\sum\langle v, -v\rangle = \Theta_V$ and $\sum\langle -v, v\rangle = \Theta_V$.

We now state a number of propositions:

(34) $\sum\langle v, -w\rangle = v - w$ and $\sum\langle -w, v\rangle = v - w$.

(35) $\sum\langle -v, -w\rangle = -(v + w)$ and $\sum\langle -w, -v\rangle = -(v + w)$.

(36) $\sum\langle u, v, w\rangle = \sum\langle u\rangle + \sum\langle v\rangle + \sum\langle w\rangle$.

(37) $\sum\langle u, v, w\rangle = \sum\langle u, v\rangle + w$.

(38) $\sum\langle u, v, w\rangle = \sum\langle v, w\rangle + u$.

(39) $\sum\langle u, v, w\rangle = \sum\langle u, w\rangle + v$.

(40) $\sum\langle u, v, w\rangle = \sum\langle u, w, v\rangle$.

(41) $\sum\langle u, v, w\rangle = \sum\langle v, u, w\rangle$.

(42) $\sum\langle u, v, w\rangle = \sum\langle v, w, u\rangle$.

(43) $\sum\langle u, v, w\rangle = \sum\langle w, u, v\rangle$.

(44) $\sum\langle u, v, w\rangle = \sum\langle w, v, u\rangle$.

(45) $\sum\langle \Theta_V, \Theta_V, \Theta_V\rangle = \Theta_V$.

(46) $\sum\langle \Theta_V, \Theta_V, v\rangle = v$ and $\sum\langle \Theta_V, v, \Theta_V\rangle = v$ and $\sum\langle v, \Theta_V, \Theta_V\rangle = v$.

(47) $\sum\langle \Theta_V, u, v\rangle = u + v$ and $\sum\langle u, v, \Theta_V\rangle = u + v$ and $\sum\langle u, \Theta_V, v\rangle = u + v$.

(48) If $\operatorname{len} F = 0$, then $\sum F = \Theta_V$.

(49) If $\operatorname{len} F = 1$, then $\sum F = F(1)$.

(50) If $\operatorname{len} F = 2$ and $v_1 = F(1)$ and $v_2 = F(2)$, then $\sum F = v_1 + v_2$.

(51) If $\operatorname{len} F = 3$ and $v_1 = F(1)$ and $v_2 = F(2)$ and $v = F(3)$, then $\sum F = v_1 + v_2 + v$.

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