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# Linear Independence in Left Module over Domain [1]

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**Summary.** Notion of submodule generated by a set of vectors and linear independence of a set of vectors. A few theorems originated as a generalization of the theorems from the article [18].

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The articles [22], [5], [3], [2], [4], [6], [21], [16], [14], [15], [1], [17], [19], [20], [7], [8], [9], [12], [11], [10], and [13] provide the terminology and notation for this paper. For simplicity we adopt the following rules: $x$ is arbitrary, $R$ is an associative ring, $V$ is a left module over $R$, $v$, $v_1$, $v_2$ are vectors of $V$, $A$, $B$ are subsets of $V$, and $l$ is a linear combination of $A$. We now define two new predicates. Let us consider $R$, $V$, $A$. We say that $A$ is linearly independent if and only if:

(Def.1) for every $l$ such that $\sum l = \Theta_V$ holds support $l = \emptyset$.

$A$ is linearly dependent stands for $A$ is not linearly independent.

One can prove the following propositions:

(2)[2] If $A \subseteq B$ and $B$ is linearly independent, then $A$ is linearly independent.

(3) If $0_R \neq 1_R$ and $A$ is linearly independent, then $\Theta_V \notin A$.

(4) $\emptyset_{\text{the carrier of the carrier of } V}$ is linearly independent.

(5) If $0_R \neq 1_R$ and $\{v_1, v_2\}$ is linearly independent, then $v_1 \neq \Theta_V$ and $v_2 \neq \Theta_V$.

(6) If $0_R \neq 1_R$, then $\{v, \Theta_V\}$ is linearly dependent and $\{\Theta_V, v\}$ is linearly dependent.

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[2]The proposition (1) was either repeated or obvious.




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For simplicity we follow the rules: $R$ will be an integral domain, $V$ will be a left module over $R$, $W$ will be a submodule of $V$, $A$, $B$ will be subsets of $V$, and $l$ will be a linear combination of $A$. Let us consider $R$, $V$, $A$. The functor $\mathrm{Lin}(A)$ yields a submodule of $V$ and is defined as follows:

(Def.2) the carrier of the carrier of $\mathrm{Lin}(A) = \{\sum l\}$.

One can prove the following propositions:

(7) If the carrier of the carrier of $W = \{\sum l\}$, then $W = \mathrm{Lin}(A)$.

(8) The carrier of the carrier of $\mathrm{Lin}(A) = \{\sum l\}$.

(9) $x \in \mathrm{Lin}(A)$ if and only if there exists $l$ such that $x = \sum l$.

(10) If $x \in A$, then $x \in \mathrm{Lin}(A)$.

We now state several propositions:

(11) $\mathrm{Lin}(\emptyset_{\text{the carrier of the carrier of } V}) = \mathbf{0}_V$.

(12) If $\mathrm{Lin}(A) = \mathbf{0}_V$, then $A = \emptyset$ or $A = \{\Theta_V\}$.

(13) If $0_R \neq 1_R$ and $A =$ the carrier of the carrier of $W$, then $\mathrm{Lin}(A) = W$.

(14) If $0_R \neq 1_R$ and $A =$ the carrier of the carrier of $V$, then $\mathrm{Lin}(A) = V$.

(15) If $A \subseteq B$, then $\mathrm{Lin}(A)$ is a submodule of $\mathrm{Lin}(B)$.

(16) If $\mathrm{Lin}(A) = V$ and $A \subseteq B$, then $\mathrm{Lin}(B) = V$.

(17) $\mathrm{Lin}(A \cup B) = \mathrm{Lin}(A) + \mathrm{Lin}(B)$.

(18) $\mathrm{Lin}(A \cap B)$ is a submodule of $\mathrm{Lin}(A) \cap \mathrm{Lin}(B)$.

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