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Groups, Rings, Left- and Right-Modules [1]

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Summary. The notion of group was defined as a group structure introduced in the article [6]. The article contains the basic properties of groups, rings, left- and right-modules of an associative ring.

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The articles [11], [10], [13], [3], [1], [12], [9], [4], [2], [5], [6], [7], and [8] provide the notation and terminology for this paper. A group structure is called a group if:

(Def.1) for all elements x, y, z of it holds $x+y+z = x+(y+z)$ and $x+0_{\rm it} = x$ and $x+-x = 0_{\rm it}$.

In the sequel G denotes a group structure and x, y denote elements of G. We see that the Abelian group is a group.

Let us consider G, x, y. The functor $x-'y$ yielding an element of G is defined by:

(Def.2) $x -' y = x + -y$.

In the sequel G denotes a group and u, v, w denote elements of G. One can prove the following propositions:

(1) $(-v) + v = 0_G$.

(2) $0_G + v = v$.

(3) $v + w = 0_G$ if and only if $-v = w$.

(4) $-0_G = 0_G$.

(5) (i) $-(v + w) = (-w) -' v$,

(ii) $- - v = v$,

(iii) $-((-v) + w) = (-w) + v$,

(iv) $-(v -' w) = w -' v$,

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(v) $-((-v) -' w) = w + v$,

(vi) $u -' (v + w) = u -' w -' v$.

(6) $0_G -' v = -v$ and $v -' 0_G = v$.

In the sequel G denotes an Abelian group and u, v, w denote elements of G. The following four propositions are true:

(7) (i) $-(v + w) = (-w) - v$,

(ii) $- - v = v$,

(iii) $-((-v) + w) = (-w) + v$,

(iv) $-(v - w) = w - v$,

(v) $-((-v) - w) = w + v$,

(vi) $u - (v + w) = u - w - v$.

(8) $0_G - v = -v$ and $v - 0_G = v$.

(9) $(-u) - v = (-v) - u$ and $(-u) + v = v - u$ and $u - v = (-v) + u$ and $u - v - w = u - w - v$.

(10) (i) $-(v + w) = (-v) - w$,

(ii) $-((-v) + w) = v - w$,

(iii) $-(v - w) = (-v) + w$,

(iv) $-((-v) - w) = v + w$,

(v) $u - (v + w) = u - v - w$.

For simplicity we adopt the following convention: R will denote an associative ring, a, b will denote scalars of R, V will denote a left module over R, and v, w will denote vectors of V. We now state several propositions:

(11) $-(a - b) = (-a) + b$.

(12) $a + 0_R = a$ and $0_R + a = a$.

(13) If $a = 0_R$ or $b = 0_R$, then $a \cdot b = 0_R$.

(14) $(-1_R) \cdot a = -a$ and $a \cdot -1_R = -a$.

(15) $a = 0_R$ if and only if $-a = 0_R$.

(16) $v + -v = \Theta_V$ and $(-v) + v = \Theta_V$.

(17) $-\Theta_V = \Theta_V$.

(18) $v + w = \Theta_V$ if and only if $-v = w$.

(19) $\Theta_V + v = v$ and $v + \Theta_V = v$ and $\Theta_V - v = -v$ and $v - \Theta_V = v$.

In the sequel x, y denote scalars of R. Next we state several propositions:

(20) $0_R \cdot v = \Theta_V$ and $(-1_R) \cdot v = -v$ and $x \cdot (\Theta_V) = \Theta_V$.

(21) $-x \cdot v = (-x) \cdot v$ and $w - x \cdot v = w + (-x) \cdot v$.

(22) $x \cdot -v = -x \cdot v$.

(23) $x \cdot (v - w) = x \cdot v - x \cdot w$.

(24) $v - x \cdot (y \cdot w) = v - x \cdot y \cdot w$.

In the sequel F will be a skew field, x will be a scalar of F, V will be a left module over F, and v will be a vector of V. The following two propositions are true:

(25) $x \cdot v = \Theta_V$ if and only if $x = 0_F$ or $v = \Theta_V$.

(26) If $x \neq 0_F$, then $x^{-1} \cdot (x \cdot v) = v$.

We adopt the following rules: V will denote a left module over R and v, v_1, v_2, u, w will denote vectors of V. The following propositions are true:

(27) $v - v = \Theta_V$.

(28) (i) $- - v = v$,

(ii) $-(v + w) = (-v) + -w$,

(iii) $-((-v) + w) = v + -w$,

(iv) $-(v + w) = (-v) - w$,

(v) $-(v - w) = (-v) + w$,

(vi) $-((-v) + w) = v - w$,

(vii) $-((-v) - w) = v + w$.

(29) $(u + v) - w = u + (v - w)$.

(30) $v = v_1 + v_2$ if and only if $v_1 = v - v_2$.

(31) $v - (u - w) = (v - u) + w$.

(32) If $v + u = v$ or $u + v = v$, then $u = \Theta_V$.

In the sequel R denotes an associative ring, V denotes a right module over R, and v, w denote vectors of V. We now state four propositions:

(33) $v + -v = \Theta_V$ and $(-v) + v = \Theta_V$.

(34) $-\Theta_V = \Theta_V$.

(35) $v + w = \Theta_V$ if and only if $-v = w$.

(36) $\Theta_V + v = v$ and $v + \Theta_V = v$ and $\Theta_V - v = -v$ and $v - \Theta_V = v$.

In the sequel x, y are scalars of R. We now state several propositions:

(37) $v \cdot 0_R = \Theta_V$ and $v \cdot -1_R = -v$ and $(\Theta_V) \cdot x = \Theta_V$.

(38) $-v \cdot x = v \cdot -x$ and $w - v \cdot x = w + v \cdot -x$.

(39) $(-v) \cdot x = -v \cdot x$.

(40) $(v - w) \cdot x = v \cdot x - w \cdot x$.

(41) $v - w \cdot y \cdot x = v - w \cdot (y \cdot x)$.

In the sequel F denotes a skew field, x denotes a scalar of F, V denotes a right module over F, and v denotes a vector of V. One can prove the following two propositions:

(42) $v \cdot x = \Theta_V$ if and only if $x = 0_F$ or $v = \Theta_V$.

(43) If $x \neq 0_F$, then $v \cdot x \cdot x^{-1} = v$.

We follow the rules: V will denote a right module over R and v, v_1, v_2, u, w will denote vectors of V. The following propositions are true:

(44) $v - v = \Theta_V$.

(45) (i) $- - v = v$,

(ii) $-(v + w) = (-v) + -w$,

(iii) $-((-v) + w) = v + -w$,

(iv) $-(v + w) = (-v) - w$,

(v) $-(v - w) = (-v) + w$,

(vi) $-((-v) + w) = v - w$,

(vii) $-((-v) - w) = v + w$.

(46) $(u + v) - w = u + (v - w)$.

(47) $v = v_1 + v_2$ if and only if $v_1 = v - v_2$.

(48) $v - (u - w) = (v - u) + w$.

(49) If $v + u = v$ or $u + v = v$, then $u = \Theta_V$.

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