

Calculus of Propositions ¹

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Summary. Continues the analysis of the classical language of first order (see [6], [1], [3], [4], [2]). Three connectives : truth, negation and conjunction are primary (see [6]). The others (alternative, implication and equivalence) are defined with respect to them (see [1]). We prove some important tautologies of the calculus of propositions. Most of them are given as axioms of the classical logical calculus (see [5]). In the last part of our article we give some basic rules of inference.

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The notation and terminology used here have been introduced in the papers [3] and [4]. In the sequel p, q, r, s are elements of CQC–WFF. One can prove the following propositions:

- (1) $\neg(p \wedge \neg p) \in \text{Taut.}$
- (2) $p \vee \neg p \in \text{Taut.}$
- (3) $p \Rightarrow p \vee q \in \text{Taut.}$
- (4) $q \Rightarrow p \vee q \in \text{Taut.}$
- (5) $p \vee q \Rightarrow (\neg p \Rightarrow q) \in \text{Taut.}$
- (6) $\neg(p \vee q) \Rightarrow \neg p \wedge \neg q \in \text{Taut.}$
- (7) $\neg p \wedge \neg q \Rightarrow \neg(p \vee q) \in \text{Taut.}$
- (8) $p \vee q \Rightarrow q \vee p \in \text{Taut.}$
- (9) $\neg p \vee p \in \text{Taut.}$
- (10) $\neg(p \vee q) \Rightarrow \neg p \in \text{Taut.}$
- (11) $p \vee p \Rightarrow p \in \text{Taut.}$
- (12) $p \Rightarrow p \vee p \in \text{Taut.}$
- (13) $p \wedge \neg p \Rightarrow q \in \text{Taut.}$
- (14) $(p \Rightarrow q) \Rightarrow \neg p \vee q \in \text{Taut.}$

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- (15) $p \wedge q \Rightarrow \neg(p \Rightarrow \neg q) \in \text{Taut.}$
- (16) $\neg(p \Rightarrow \neg q) \Rightarrow p \wedge q \in \text{Taut.}$
- (17) $\neg(p \wedge q) \Rightarrow \neg p \vee \neg q \in \text{Taut.}$
- (18) $\neg p \vee \neg q \Rightarrow \neg(p \wedge q) \in \text{Taut.}$
- (19) $p \wedge q \Rightarrow p \in \text{Taut.}$
- (20) $p \wedge q \Rightarrow p \vee q \in \text{Taut.}$
- (21) $p \wedge q \Rightarrow q \in \text{Taut.}$
- (22) $p \Rightarrow p \wedge p \in \text{Taut.}$
- (23) $(p \Leftrightarrow q) \Rightarrow (p \Rightarrow q) \in \text{Taut.}$
- (24) $(p \Leftrightarrow q) \Rightarrow (q \Rightarrow p) \in \text{Taut.}$
- (25) $p \vee q \vee r \Rightarrow p \vee (q \vee r) \in \text{Taut.}$
- (26) $p \wedge q \wedge r \Rightarrow p \wedge (q \wedge r) \in \text{Taut.}$
- (27) $p \vee (q \vee r) \Rightarrow p \vee q \vee r \in \text{Taut.}$
- (28) $p \Rightarrow (q \Rightarrow p \wedge q) \in \text{Taut.}$
- (29) $(p \Rightarrow q) \Rightarrow ((q \Rightarrow p) \Rightarrow (p \Leftrightarrow q)) \in \text{Taut.}$
- (30) $p \vee q \Leftrightarrow q \vee p \in \text{Taut.}$
- (31) $(p \wedge q \Rightarrow r) \Rightarrow (p \Rightarrow (q \Rightarrow r)) \in \text{Taut.}$

The following propositions are true:

- (32) $(p \Rightarrow (q \Rightarrow r)) \Rightarrow (p \wedge q \Rightarrow r) \in \text{Taut.}$
- (33) $(r \Rightarrow p) \Rightarrow ((r \Rightarrow q) \Rightarrow (r \Rightarrow p \wedge q)) \in \text{Taut.}$
- (34) $(p \vee q \Rightarrow r) \Rightarrow (p \Rightarrow r) \vee (q \Rightarrow r) \in \text{Taut.}$
- (35) $(p \Rightarrow r) \Rightarrow ((q \Rightarrow r) \Rightarrow (p \vee q \Rightarrow r)) \in \text{Taut.}$
- (36) $(p \Rightarrow r) \wedge (q \Rightarrow r) \Rightarrow (p \vee q \Rightarrow r) \in \text{Taut.}$
- (37) $(p \Rightarrow q \wedge \neg q) \Rightarrow \neg p \in \text{Taut.}$
- (38) $(p \vee q) \wedge (p \vee r) \Rightarrow p \vee q \wedge r \in \text{Taut.}$
- (39) $p \wedge (q \vee r) \Rightarrow p \wedge q \vee p \wedge r \in \text{Taut.}$
- (40) $(p \vee r) \wedge (q \vee r) \Rightarrow p \wedge q \vee r \in \text{Taut.}$
- (41) $(p \vee q) \wedge r \Rightarrow p \wedge r \vee q \wedge r \in \text{Taut.}$
- (42) If $p \in \text{Taut}$, then $p \vee q \in \text{Taut}$.
- (43) If $q \in \text{Taut}$, then $p \vee q \in \text{Taut}$.
- (44) If $p \wedge q \in \text{Taut}$, then $p \in \text{Taut}$.
- (45) If $p \wedge q \in \text{Taut}$, then $q \in \text{Taut}$.
- (46) If $p \wedge q \in \text{Taut}$, then $p \vee q \in \text{Taut}$.
- (47) If $p \in \text{Taut}$ and $q \in \text{Taut}$, then $p \wedge q \in \text{Taut}$.
- (48) If $p \Rightarrow q \in \text{Taut}$, then $p \vee r \Rightarrow q \vee r \in \text{Taut}$.
- (49) If $p \Rightarrow q \in \text{Taut}$, then $r \vee p \Rightarrow r \vee q \in \text{Taut}$.
- (50) If $p \Rightarrow q \in \text{Taut}$, then $r \wedge p \Rightarrow r \wedge q \in \text{Taut}$.
- (51) If $p \Rightarrow q \in \text{Taut}$, then $p \wedge r \Rightarrow q \wedge r \in \text{Taut}$.
- (52) If $r \Rightarrow p \in \text{Taut}$ and $r \Rightarrow q \in \text{Taut}$, then $r \Rightarrow p \wedge q \in \text{Taut}$.

- (53) If $p \Rightarrow r \in \text{Taut}$ and $q \Rightarrow r \in \text{Taut}$, then $p \vee q \Rightarrow r \in \text{Taut}$.
- (54) If $p \vee q \in \text{Taut}$ and $\neg p \in \text{Taut}$, then $q \in \text{Taut}$.
- (55) If $p \vee q \in \text{Taut}$ and $\neg q \in \text{Taut}$, then $p \in \text{Taut}$.
- (56) If $p \Rightarrow q \in \text{Taut}$ and $r \Rightarrow s \in \text{Taut}$, then $p \wedge r \Rightarrow q \wedge s \in \text{Taut}$.
- (57) If $p \Rightarrow q \in \text{Taut}$ and $r \Rightarrow s \in \text{Taut}$, then $p \vee r \Rightarrow q \vee s \in \text{Taut}$.
- (58) If $p \wedge \neg q \Rightarrow \neg p \in \text{Taut}$, then $p \Rightarrow q \in \text{Taut}$.

References

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