FORMALIZED MATHEMATICS
Vol.2, No.4, September–October 1991
Université Catholique de Louvain

Fix Point Theorem for Compact Spaces

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Summary. The Banach theorem in a compact metric spaces is proved.

MML Identifier: `ALI2`.

The terminology and notation used in this paper have been introduced in the following papers: [9], [15], [3], [4], [8], [11], [13], [9], [11], [5], [7], [18], [6], [17], [1], [2], [6], [4], and [5]. In the sequel M will be a metric space. Next we state the proposition

(1) For every set F such that F is finite and $F \neq \emptyset$ and for all sets B, C such that $B \in F$ and $C \in F$ holds $B \subseteq C$ or $C \subseteq B$ there exists a set m such that $m \in F$ and for every set C such that $C \in F$ holds $m \subseteq C$.

Let M be a metric space. A function from the carrier of M into the carrier of M is said to be a contraction of M if:

(Def.1) there exists a real number L such that $0 < L$ and $L < 1$ and for all points x, y of M holds $\rho(\mathrm{it}(x), \mathrm{it}(y)) \leq L \cdot \rho(x, y)$.

Next we state the proposition

(2) For every contraction f of M such that M_{top} is compact there exists a point c of M such that $f(c) = c$ and for every point x of M such that $f(x) = x$ holds $x = c$.

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ISSN 0777–4028

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Received July 17, 1990