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Categories of Groups

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Summary. We define the category of groups and its subcategories: category of Abelian groups and category of groups with the operator of $\frac{1}{2}$. The carriers of the groups are included in a universum. The universum is a parameter of the categories.

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The articles [13], [2], [14], [3], [1], [11], [7], [5], [4], [12], [10], [6], [9], and [8] provide the notation and terminology for this paper. For simplicity we follow the rules: x, y will be arbitrary, D will be a non-empty set, U_1 will be a universal class, and G, H will be group structures. Let us consider x. Then $\{x\}$ is a non-empty set.

The following propositions are true:

(1) For all sets X, Y, A and for all x, y such that $\langle x, y\rangle \in A$ and $A \subseteq [: X, Y :]$ holds x is an element of X and y is an element of Y.

(2) For all sets X, Y, A and for an arbitrary z such that $z \in A$ and $A \subseteq [: X, Y :]$ there exists an element x of X and there exists an element y of Y such that $z = \langle x, y\rangle$.

(3) For all elements u_1, u_2, u_3, u_4 of U_1 holds $\langle u_1, u_2, u_3\rangle$ is an element of U_1 and $\langle u_1, u_2, u_3, u_4\rangle$ is an element of U_1.

(4) For all x, y such that $x \in y$ and $y \in U_1$ holds $x \in U_1$.

In this article we present several logical schemes. The scheme *PartLambda2* deals with a set $\mathcal{A}$, a set $\mathcal{B}$, a set $\mathcal{C}$, a binary functor $\mathcal{F}$, and a binary predicate $\mathcal{P}$, and states that:

there exists a partial function f from $[: \mathcal{A}, \mathcal{B} :]$ to $\mathcal{C}$ such that for all x, y holds $\langle x, y\rangle \in \mathrm{dom}\, f$ if and only if $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x, y]$ and for all x, y such that $\langle x, y\rangle \in \mathrm{dom}\, f$ holds $f(\langle x, y\rangle) = \mathcal{F}(x, y)$

provided the following requirement is met:

- for all x, y such that $x \in \mathcal{A}$ and $y \in \mathcal{B}$ and $\mathcal{P}[x, y]$ holds $\mathcal{F}(x, y) \in \mathcal{C}$.

The scheme *PartLambda2D* deals with a non-empty set $\mathcal{A}$, a non-empty set $\mathcal{B}$, a set $\mathcal{C}$, a binary functor $\mathcal{F}$, and a binary predicate $\mathcal{P}$, and states that:

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there exists a partial function f from $[\!:\mathcal{A}, \mathcal{B}:\!]$ to $\mathcal{C}$ such that for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ holds $\langle x, y\rangle \in \operatorname{dom} f$ if and only if $\mathcal{P}[x, y]$ and for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ such that $\langle x, y\rangle \in \operatorname{dom} f$ holds $f(\langle x, y\rangle) = \mathcal{F}(x, y)$
provided the parameters satisfy the following condition:

- for every element x of $\mathcal{A}$ and for every element y of $\mathcal{B}$ such that $\mathcal{P}[x, y]$ holds $\mathcal{F}(x, y) \in \mathcal{C}$.

We now define three new functors. op_2 is a binary operation on $\{\emptyset\}$.
op_1 is a unary operation on $\{\emptyset\}$.
op_0 is an element of $\{\emptyset\}$.

We now state three propositions:

(5) $\mathrm{op}_2(\emptyset, \emptyset) = \emptyset$ and $\mathrm{op}_1(\emptyset) = \emptyset$ and $\mathrm{op}_0 = \emptyset$.

(6) $\{\emptyset\} \in U_1$ and $\langle \{\emptyset\}, \{\emptyset\}\rangle \in U_1$ and $[\!:\{\emptyset\}, \{\emptyset\}:\!] \in U_1$ and $\mathrm{op}_2 \in U_1$ and $\mathrm{op}_1 \in U_1$.

(7) $\langle \{\emptyset\}, \mathrm{op}_2, \mathrm{op}_1, \mathrm{op}_0\rangle$ is a group with the operator $\frac{1}{2}$.

The trivial group being a group with the operator $\frac{1}{2}$ is defined as follows:

(Def.1) the trivial group$= \langle \{\emptyset\}, \mathrm{op}_2, \mathrm{op}_1, \mathrm{op}_0\rangle$.

We now state the proposition

(8) If $G =$ the trivial group , then for every element x of G holds $x = \emptyset$ and for all elements x, y of G holds $x + y = \emptyset$ and for every element x of G holds $-x = \emptyset$ and $0_G = \emptyset$.

In the sequel C denotes a category and O denotes a non-empty subset of the objects of C. Let us consider C, O. The functor $\mathrm{Morphs}\, O$ yields a non-empty subset of the morphisms of C and is defined by:

(Def.2) $\mathrm{Morphs}\, O = \bigcup\{\hom(a, b) : a \in O \wedge b \in O\}$, where a ranges over objects of C, and b ranges over objects of C.

We now define four new functors. Let us consider C, O. The functor $\operatorname{dom} O$ yielding a function from $\mathrm{Morphs}\, O$ into O is defined by:

(Def.3) $\operatorname{dom} O =$ (the dom-map of C) $\restriction \mathrm{Morphs}\, O$.

The functor $\operatorname{cod} O$ yields a function from $\mathrm{Morphs}\, O$ into O and is defined by:

(Def.4) $\operatorname{cod} O =$ (the cod-map of C) $\restriction \mathrm{Morphs}\, O$.

The functor $\operatorname{comp} O$ yielding a partial function from $[\!:\mathrm{Morphs}\, O, \mathrm{Morphs}\, O$ **qua** a non-empty set $:\!]$ to $\mathrm{Morphs}\, O$ is defined as follows:

(Def.5) $\operatorname{comp} O =$ (the composition of C) $\restriction [\!:\mathrm{Morphs}\, O, \mathrm{Morphs}\, O:\!]$.

The functor I_O yielding a function from O into $\mathrm{Morphs}\, O$ is defined by:

(Def.6) $\mathrm{I}_O =$ (the id-map of C) $\restriction O$.

Next we state the proposition

(9) $\langle O, \mathrm{Morphs}\, O, \operatorname{dom} O, \operatorname{cod} O, \operatorname{comp} O, \mathrm{I}_O\rangle$ is full subcategory of C.

Let us consider C, O. The functor $\operatorname{cat} O$ yielding a subcategory of C is defined as follows:

(Def.7) $\operatorname{cat} O = \langle O, \mathrm{Morphs}\, O, \operatorname{dom} O, \operatorname{cod} O, \operatorname{comp} O, \mathrm{I}_O\rangle$.

Next we state the proposition

(10) The objects of $\mathrm{cat}\,O = O$.

Let us consider G, H. A map from G into H is a function from the carrier of G into the carrier of H.

Let G_1, G_2, G_3 be group structures, and let f be a map from G_1 into G_2, and let g be a map from G_2 into G_3. Then $g \cdot f$ is a map from G_1 into G_3.

Let us consider G. The functor id_G yields a map from G into G and is defined by:

(Def.8) $\mathrm{id}_G = \mathrm{id}_{(\text{the carrier of } G)}$.

One can prove the following two propositions:

(11) For every element x of G holds $\mathrm{id}_G(x) = x$.

(12) For every map f from G into H holds $f \cdot \mathrm{id}_G = f$ and $\mathrm{id}_H \cdot f = f$.

Let us consider G, H. The functor $\mathrm{zero}(G, H)$ yielding a map from G into H is defined by:

(Def.9) $\mathrm{zero}(G, H) = (\text{the carrier of } G) \longmapsto 0_H$.

Let us consider G, H, and let f be a map from G into H. We say that f is additive if and only if:

(Def.10) for all elements x, y of G holds $f(x + y) = f(x) + f(y)$.

One can prove the following propositions:

(13) For all G_1, G_2, G_3 being group structures and for every map f from G_1 into G_2 and for every map g from G_2 into G_3 and for every element x of G_1 holds $(g \cdot f)(x) = g(f(x))$.

(14) For all G_1, G_2, G_3 being group structures and for every map f from G_1 into G_2 and for every map g from G_2 into G_3 such that f is additive and g is additive holds $g \cdot f$ is additive.

(15) For every element x of G holds $(\mathrm{zero}(G, H))(x) = 0_H$.

(16) For every group H holds $\mathrm{zero}(G, H)$ is additive.

In the sequel G, H are groups. We consider group morphism structures which are systems

⟨a dom-map, a cod-map, a `Fun`⟩,

where the dom-map, the cod-map are a group and the `Fun` is a map from the dom-map into the cod-map.

We now define two new functors. Let f be a group morphism structure. The functor $\mathrm{dom}\,f$ yielding a group is defined as follows:

(Def.11) $\mathrm{dom}\,f =$ the dom-map of f.

The functor $\mathrm{cod}\,f$ yields a group and is defined by:

(Def.12) $\mathrm{cod}\,f =$ the cod-map of f.

Let f be a group morphism structure. The functor $\mathrm{fun}\,f$ yields a map from $\mathrm{dom}\,f$ into $\mathrm{cod}\,f$ and is defined by:

(Def.13) $\mathrm{fun}\,f =$ the `Fun` of f.

Next we state the proposition

(17) For every f being a group morphism structure and for all groups G_1, G_2 and for every map f_0 from G_1 into G_2 such that $f = \langle G_1, G_2, f_0 \rangle$ holds $\mathrm{dom}\, f = G_1$ and $\mathrm{cod}\, f = G_2$ and $\mathrm{fun}\, f = f_0$.

Let us consider G, H. The functor $\mathrm{ZERO}\, G$ yielding a group morphism structure is defined as follows:

(Def.14) $\mathrm{ZERO}\, G = \langle G, H, \mathrm{zero}(G, H) \rangle$.

A group morphism structure is said to be a morphism of groups if:

(Def.15) fun it is additive.

One can prove the following proposition

(18) For every morphism F of groups holds the `Fun` of F is additive.

Let us consider G, H. Then $\mathrm{ZERO}\, G$ is a morphism of groups.

Let us consider G, H. A morphism of groups is said to be a morphism from G to H if:

(Def.16) $\mathrm{dom}\,\mathrm{it} = G$ and $\mathrm{cod}\,\mathrm{it} = H$.

We now state three propositions:

(19) For every f being a group morphism structure such that $\mathrm{dom}\, f = G$ and $\mathrm{cod}\, f = H$ and $\mathrm{fun}\, f$ is additive holds f is a morphism from G to H.

(20) For every map f from G into H such that f is additive holds $\langle G, H, f \rangle$ is a morphism from G to H.

(21) id_G is additive.

Let us consider G. The functor I_G yields a morphism from G to G and is defined by:

(Def.17) $\mathrm{I}_G = \langle G, G, \mathrm{id}_G \rangle$.

Let us consider G, H. Then $\mathrm{ZERO}\, G$ is a morphism from G to H.

We now state several propositions:

(22) For every morphism F from G to H there exists a map f from G into H such that $F = \langle G, H, f \rangle$ and f is additive.

(23) For every morphism F from G to H there exists a map f from G into H such that $F = \langle G, H, f \rangle$.

(24) For every morphism F of groups there exist G, H such that F is a morphism from G to H.

(25) For every morphism F of groups there exist groups G, H and there exists a map f from G into H such that F is a morphism from G to H and $F = \langle G, H, f \rangle$ and f is additive.

(26) For all morphisms g, f of groups such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ there exist groups G_1, G_2, G_3 such that g is a morphism from G_2 to G_3 and f is a morphism from G_1 to G_2.

(27) For every morphism F of groups holds F is a morphism from $\mathrm{dom}\, F$ to $\mathrm{cod}\, F$.

Let G, F be morphisms of groups. Let us assume that $\mathrm{dom}\, G = \mathrm{cod}\, F$. The functor $G \cdot F$ yielding a morphism of groups is defined by:

(Def.18) for all groups G_1, G_2, G_3 and for every map g from G_2 into G_3 and for every map f from G_1 into G_2 such that $G = \langle G_2, G_3, g\rangle$ and $F = \langle G_1, G_2, f\rangle$ holds $G \cdot F = \langle G_1, G_3, g \cdot f\rangle$.

Next we state the proposition

(28) For all groups G_1, G_2, G_3 and for every morphism G from G_2 to G_3 and for every morphism F from G_1 to G_2 holds $G \cdot F$ is a morphism from G_1 to G_3.

Let G_1, G_2, G_3 be groups, and let G be a morphism from G_2 to G_3, and let F be a morphism from G_1 to G_2. Then $G \cdot F$ is a morphism from G_1 to G_3.

The following propositions are true:

(29) For all groups G_1, G_2, G_3 and for every morphism G from G_2 to G_3 and for every morphism F from G_1 to G_2 and for every map g from G_2 into G_3 and for every map f from G_1 into G_2 such that $G = \langle G_2, G_3, g\rangle$ and $F = \langle G_1, G_2, f\rangle$ holds $G \cdot F = \langle G_1, G_3, g \cdot f\rangle$.

(30) For all morphisms f, g of groups such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ there exist groups G_1, G_2, G_3 and there exists a map f_0 from G_1 into G_2 and there exists a map g_0 from G_2 into G_3 such that $f = \langle G_1, G_2, f_0\rangle$ and $g = \langle G_2, G_3, g_0\rangle$ and $g \cdot f = \langle G_1, G_3, g_0 \cdot f_0\rangle$.

(31) For all morphisms f, g of groups such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $\mathrm{dom}(g \cdot f) = \mathrm{dom}\, f$ and $\mathrm{cod}(g \cdot f) = \mathrm{cod}\, g$.

(32) For all groups G_1, G_2, G_3, G_4 and for every morphism f from G_1 to G_2 and for every morphism g from G_2 to G_3 and for every morphism h from G_3 to G_4 holds $h \cdot (g \cdot f) = h \cdot g \cdot f$.

(33) For all morphisms f, g, h of groups such that $\mathrm{dom}\, h = \mathrm{cod}\, g$ and $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $h \cdot (g \cdot f) = h \cdot g \cdot f$.

(34) $\mathrm{dom}(\mathrm{I}_G) = G$ and $\mathrm{cod}(\mathrm{I}_G) = G$ and for every morphism f of groups such that $\mathrm{cod}\, f = G$ holds $\mathrm{I}_G \cdot f = f$ and for every morphism g of groups such that $\mathrm{dom}\, g = G$ holds $g \cdot \mathrm{I}_G = g$.

A non-empty set is called a non-empty set of groups if:

(Def.19) for every element x of it holds x is a group.

In the sequel V will be a non-empty set of groups. Let us consider V. We see that the element of V is a group.

We now state two propositions:

(35) For every morphism f of groups and for every element x of $\{f\}$ holds x is a morphism of groups.

(36) For every morphism f from G to H and for every element x of $\{f\}$ holds x is a morphism from G to H.

A non-empty set is called a non-empty set of morphisms of groups if:

(Def.20) for every element x of it holds x is a morphism of groups.

Let M be a non-empty set of morphisms of groups. We see that the element of M is a morphism of groups.

We now state the proposition

(37) For every morphism f of groups holds $\{f\}$ is a non-empty set of morphisms of groups.

Let us consider G, H. A non-empty set of morphisms of groups is called a non-empty set of morphisms from G into H if:

(Def.21) for every element x of it holds x is a morphism from G to H.

The following two propositions are true:

(38) D is a non-empty set of morphisms from G into H if and only if for every element x of D holds x is a morphism from G to H.

(39) For every morphism f from G to H holds $\{f\}$ is a non-empty set of morphisms from G into H.

Let us consider G, H. The functor $\mathrm{Morphs}(G, H)$ yields a non-empty set of morphisms from G into H and is defined by:

(Def.22) $x \in \mathrm{Morphs}(G, H)$ if and only if x is a morphism from G to H.

Let us consider G, H, and let M be a non-empty set of morphisms from G into H. We see that the element of M is a morphism from G to H.

Let us consider x, y. The predicate $\mathrm{P_{ob}}\, x, y$ is defined by:

(Def.23) there exist arbitrary x_1, x_2, x_3, x_4 such that $x = \langle x_1, x_2, x_3, x_4 \rangle$ and there exists G such that $y = G$ and $x_1 =$ the carrier of G and $x_2 =$ the addition of G and $x_3 =$ the reverse-map of G and $x_4 =$ the zero of G.

One can prove the following two propositions:

(40) For arbitrary x, y_1, y_2 such that $\mathrm{P_{ob}}\, x, y_1$ and $\mathrm{P_{ob}}\, x, y_2$ holds $y_1 = y_2$.

(41) There exists x such that $x \in U_1$ and $\mathrm{P_{ob}}\, x$,the trivial group .

Let us consider U_1. The functor $\mathrm{GroupObj}(U_1)$ yields a non-empty set and is defined as follows:

(Def.24) for every y holds $y \in \mathrm{GroupObj}(U_1)$ if and only if there exists x such that $x \in U_1$ and $\mathrm{P_{ob}}\, x, y$.

The following propositions are true:

(42) The trivial group$\in \mathrm{GroupObj}(U_1)$.

(43) For every element x of $\mathrm{GroupObj}(U_1)$ holds x is a group.

Let us consider U_1. Then $\mathrm{GroupObj}(U_1)$ is a non-empty set of groups.

Let us consider V. The functor $\mathrm{Morphs}\, V$ yielding a non-empty set of morphisms of groups is defined by:

(Def.25) for every x holds $x \in \mathrm{Morphs}\, V$ if and only if there exist elements G, H of V such that x is a morphism from G to H.

Let us consider V, and let F be an element of $\mathrm{Morphs}\, V$. Then $\mathrm{dom}\, F$ is an element of V. Then $\mathrm{cod}\, F$ is an element of V.

Let us consider V, and let G be an element of V. The functor I_G yields an element of $\mathrm{Morphs}\, V$ and is defined by:

(Def.26) $\mathrm{I}_G = \mathrm{I}_G$.

We now define three new functors. Let us consider V. The functor $\mathrm{dom}\, V$ yields a function from $\mathrm{Morphs}\, V$ into V and is defined as follows:

(Def.27) for every element f of $\mathrm{Morphs}\, V$ holds $(\mathrm{dom}\, V)(f) = \mathrm{dom}\, f$.

The functor $\mathrm{cod}\, V$ yields a function from $\mathrm{Morphs}\, V$ into V and is defined as follows:

(Def.28) for every element f of $\mathrm{Morphs}\, V$ holds $(\mathrm{cod}\, V)(f) = \mathrm{cod}\, f$.

The functor I_V yielding a function from V into $\mathrm{Morphs}\, V$ is defined as follows:

(Def.29) for every element G of V holds $\mathrm{I}_V(G) = \mathrm{I}_G$.

One can prove the following two propositions:

(44) For all elements g, f of $\mathrm{Morphs}\, V$ such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ there exist elements G_1, G_2, G_3 of V such that g is a morphism from G_2 to G_3 and f is a morphism from G_1 to G_2.

(45) For all elements g, f of $\mathrm{Morphs}\, V$ such that $\mathrm{dom}\, g = \mathrm{cod}\, f$ holds $g \cdot f \in \mathrm{Morphs}\, V$.

Let us consider V. The functor $\mathrm{comp}\, V$ yields a partial function from $[\!:\mathrm{Morphs}\, V, \mathrm{Morphs}\, V:\!]$ to $\mathrm{Morphs}\, V$ and is defined by:

(Def.30) for all elements g, f of $\mathrm{Morphs}\, V$ holds $\langle g, f\rangle \in \mathrm{dom}\,\mathrm{comp}\, V$ if and only if $\mathrm{dom}\, g = \mathrm{cod}\, f$ and for all elements g, f of $\mathrm{Morphs}\, V$ such that $\langle g, f\rangle \in \mathrm{dom}\,\mathrm{comp}\, V$ holds $(\mathrm{comp}\, V)(\langle g, f\rangle) = g \cdot f$.

Let us consider U_1. The functor $\mathrm{GroupCat}(U_1)$ yielding a category structure is defined by:

(Def.31) $\mathrm{GroupCat}(U_1) = \langle \mathrm{GroupObj}(U_1), \mathrm{Morphs}\, \mathrm{GroupObj}(U_1), \mathrm{dom}\, \mathrm{GroupObj}(U_1), \mathrm{cod}\, \mathrm{GroupObj}(U_1), \mathrm{comp}\, \mathrm{GroupObj}(U_1), \mathrm{I}_{\mathrm{GroupObj}(U_1)}\rangle$.

Next we state several propositions:

(46) For all morphisms f, g of $\mathrm{GroupCat}(U_1)$ holds $\langle g, f\rangle \in \mathrm{dom}$ (the composition of $\mathrm{GroupCat}(U_1)$) if and only if $\mathrm{dom}\, g = \mathrm{cod}\, f$.

(47) For every morphism f of $\mathrm{GroupCat}(U_1)$ and for every element f' of $\mathrm{Morphs}\, \mathrm{GroupObj}(U_1)$ and for every object b of $\mathrm{GroupCat}(U_1)$ and for every element b' of $\mathrm{GroupObj}(U_1)$ holds f is an element of $\mathrm{Morphs}\, \mathrm{GroupObj}(U_1)$ and f' is a morphism of $\mathrm{GroupCat}(U_1)$ and b is an element of $\mathrm{GroupObj}(U_1)$ and b' is an object of $\mathrm{GroupCat}(U_1)$.

(48) For every object b of $\mathrm{GroupCat}(U_1)$ and for every element b' of $\mathrm{GroupObj}(U_1)$ such that $b = b'$ holds $\mathrm{id}_b = \mathrm{I}_{b'}$.

(49) For every morphism f of $\mathrm{GroupCat}(U_1)$ and for every element f' of $\mathrm{Morphs}\, \mathrm{GroupObj}(U_1)$ such that $f = f'$ holds $\mathrm{dom}\, f = \mathrm{dom}\, f'$ and $\mathrm{cod}\, f = \mathrm{cod}\, f'$.

(50) Let f, g be morphisms of GroupCat(U_1). Let f', g' be elements of Morphs GroupObj(U_1). Suppose $f = f'$ and $g = g'$. Then

(i) $\mathrm{dom}\, g = \mathrm{cod}\, f$ if and only if $\mathrm{dom}\, g' = \mathrm{cod}\, f'$,

(ii) $\mathrm{dom}\, g = \mathrm{cod}\, f$ if and only if $\langle g', f' \rangle \in \mathrm{dom}\,\mathrm{comp}\,\mathrm{GroupObj}(U_1)$,

(iii) if $\mathrm{dom}\, g = \mathrm{cod}\, f$, then $g \cdot f = g' \cdot f'$,

(iv) $\mathrm{dom}\, f = \mathrm{dom}\, g$ if and only if $\mathrm{dom}\, f' = \mathrm{dom}\, g'$,

(v) $\mathrm{cod}\, f = \mathrm{cod}\, g$ if and only if $\mathrm{cod}\, f' = \mathrm{cod}\, g'$.

Let us consider U_1. Then GroupCat(U_1) is a category.

Let us consider U_1. The functor AbGroupObj(U_1) yielding a non-empty subset of the objects of GroupCat(U_1) is defined as follows:

(Def.32) $\mathrm{AbGroupObj}(U_1) = \{G : \bigvee_H G = H\}$, where G ranges over elements of the objects of GroupCat(U_1), and H ranges over Abelian groups.

One can prove the following proposition

(51) The trivial group$\in$ AbGroupObj(U_1).

Let us consider U_1. The functor AbGroupCat(U_1) yielding a subcategory of GroupCat(U_1) is defined as follows:

(Def.33) $\mathrm{AbGroupCat}(U_1) = \mathrm{cat}\,\mathrm{AbGroupObj}(U_1)$.

We now state the proposition

(52) The objects of AbGroupCat(U_1) = AbGroupObj(U_1).

Let us consider U_1. The functor $\frac{1}{2}$ GroupObj(U_1) yields a non-empty subset of the objects of AbGroupCat(U_1) and is defined as follows:

(Def.34) $\frac{1}{2}\,\mathrm{GroupObj}(U_1) = \{G : \bigvee_H G = H\}$, where G ranges over elements of the objects of AbGroupCat(U_1), and H ranges over groups with the operator $\frac{1}{2}$.

Let us consider U_1. The functor $\frac{1}{2}$ GroupCat(U_1) yields a subcategory of AbGroupCat(U_1) and is defined by:

(Def.35) $\frac{1}{2}\,\mathrm{GroupCat}(U_1) = \mathrm{cat}\,\frac{1}{2}\,\mathrm{GroupObj}(U_1)$.

Next we state two propositions:

(53) The objects of $\frac{1}{2}$ GroupCat(U_1) = $\frac{1}{2}$ GroupObj(U_1).

(54) The trivial group$\in$ $\frac{1}{2}$ GroupObj(U_1).

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