FORMALIZED MATHEMATICS
Volume 3, Number **2**, 1992
Université Catholique de Louvain

Basic Notation of Universal Algebra

Jarosław Kotowicz
Warsaw University
Białystok

Beata Madras
Warsaw University
Białystok

Małgorzata Korolkiewicz
Warsaw University
Białystok

MML Identifier: `UNIALG_1`.

The papers [6], [3], [1], [5], [4], and [2] provide the terminology and notation for this paper. For simplicity we adopt the following convention: A denotes a non-empty set, a denotes an element of A, x, y denote finite sequences of elements of A, h denotes a partial function from A^* to A, and n denotes a natural number. We now define two new attributes. Let us consider A. A partial function from A^* to A is homogeneous if:

(Def.1) for all x, y such that $x \in \mathrm{dom}\,\mathrm{it}$ and $y \in \mathrm{dom}\,\mathrm{it}$ holds $\mathrm{len}\,x = \mathrm{len}\,y$.

Let us consider A. A partial function from A^* to A is quasi total if:

(Def.2) for all x, y such that $\mathrm{len}\,x = \mathrm{len}\,y$ and $x \in \mathrm{dom}\,\mathrm{it}$ holds $y \in \mathrm{dom}\,\mathrm{it}$.

Let us consider A. Note that there exists a homogeneous quasi total non-empty partial function from A^* to A.

We now state three propositions:

(1) h is a non-empty partial function from A^* to A if and only if $\mathrm{dom}\,h \neq \emptyset$.

(2) $\{\varepsilon_A\} \longmapsto a$ is a homogeneous quasi total non-empty partial function from A^* to A.

(3) $\{\varepsilon_A\} \longmapsto a$ is an element of $A^* \dot{\rightarrow} A$.

We now define four new constructions. We consider universal algebra structures which are extension of a 1-sorted structure and are systems

⟨a carrier, a characteristic⟩,

where the carrier is a non-empty set and the characteristic is a finite sequence of elements of $(\text{the carrier})^* \dot{\rightarrow}$the carrier. Let us consider A. A finite sequence of elements of $A^* \dot{\rightarrow} A$ is homogeneous if:

ISSN 0777–4028

(Def.3) for all n, h such that $n \in \operatorname{dom} \mathrm{it}$ and $h = \mathrm{it}(n)$ holds h is homogeneous.

Let us consider A. A finite sequence of elements of $A^* \dot{\rightarrow} A$ is quasi total if:

(Def.4) for all n, h such that $n \in \operatorname{dom} \mathrm{it}$ and $h = \mathrm{it}(n)$ holds h is quasi total.

Let us consider A. A finite sequence of elements of $A^* \dot{\rightarrow} A$ is non-empty if:

(Def.5) for all n, h such that $n \in \operatorname{dom} \mathrm{it}$ and $h = \mathrm{it}(n)$ holds h is non-empty.

In the sequel U will be a universal algebra structure. We now define four new constructions. Let us consider U. The functor $\mathrm{Opers}\, U$ yielding a finite sequence of elements of (the carrier of $U)^* \dot{\rightarrow}$the carrier of U is defined as follows:

(Def.6) $\mathrm{Opers}\, U =$ the characteristic of U.

A universal algebra structure is partial if:

(Def.7) $\mathrm{Opers}\, \mathrm{it}$ is homogeneous.

A universal algebra structure is quasi total if:

(Def.8) $\mathrm{Opers}\, \mathrm{it}$ is quasi total.

A universal algebra structure is non-empty if:

(Def.9) $\mathrm{Opers}\, \mathrm{it} \neq \varepsilon$ and $\mathrm{Opers}\, \mathrm{it}$ is non-empty.

We now state the proposition

(4) For every element x of $A^* \dot{\rightarrow} A$ such that $x = \{\varepsilon_A\} \longmapsto a$ holds $\langle x \rangle$ is homogeneous, quasi total and non-empty.

Let us note that there exists a quasi total partial non-empty strict universal algebra structure.

A universal algebra is a quasi total partial non-empty universal algebra structure.

In the sequel U will be a universal algebra. Let us consider A, and let f be a homogeneous quasi total non-empty partial function from A^* to A. The functor $\mathrm{arity}\, f$ yielding a natural number is defined as follows:

(Def.10) if $x \in \operatorname{dom} f$, then $\mathrm{arity}\, f = \operatorname{len} x$.

The following proposition is true

(5) For every U and for every n such that $n \in \operatorname{dom} \mathrm{Opers}\, U$ holds $(\mathrm{Opers}\, U)(n)$ is a homogeneous quasi total non-empty partial function from (the carrier of $U)^*$ to the carrier of U.

Let U be a universal algebra. The functor $\mathrm{signature}\, U$ yields a finite sequence of elements of $\mathbb{N}$ and is defined as follows:

(Def.11) $\operatorname{len} \mathrm{signature}\, U = \operatorname{len} \mathrm{Opers}\, U$
and for every n such that $n \in \operatorname{dom} \mathrm{signature}\, U$ and for every homogeneous quasi total non-empty partial function h from (the carrier of $U)^*$ to the carrier of U such that $h = (\mathrm{Opers}\, U)(n)$ holds $(\mathrm{signature}\, U)(n) = \mathrm{arity}\, h$.

References

[1] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[2] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Formalized Mathematics*, 1(**3**):529–536, 1990.

[3] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[4] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.

[5] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.

[6] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

Received December 29, 1992