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Basic Concepts for Petri Nets with Boolean Markings

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Summary. Contains basic concepts for Petri nets with Boolean markings and the firability/firing of single transitions as well as sequences of transitions [7]. The concept of a Boolean marking is introduced as a mapping of a Boolean TRUE/FALSE to each of the places in a place/transition net. This simplifies the conventional definitions of the firability and firing of a transition. One note of caution in this article - the definition of firing a transition does not require that the transition be firable. Therefore, it is advisable to check that transitions ARE firable before firing them.

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The papers [12], [1], [15], [17], [18], [4], [5], [13], [10], [11], [9], [2], [3], [14], [6], [16], and [8] provide the notation and terminology for this paper.

1. Preliminaries

The following four propositions are true:

(1) Let A, B be non empty set, and let f be a function from A into B, and let C be a subset of A, and let v be an element of B. Then $f +\cdot (C \longmapsto v)$ is a function from A into B.

(2) Let X, Y be non empty set, and let A, B be subsets of X, and let f be a function from X into Y. If $f\,^\circ A \cap f\,^\circ B = \emptyset$, then $A \cap B = \emptyset$.

(3) For all sets A, B and for every function f and for arbitrary x such that $A \cap B = \emptyset$ holds $(f +\cdot (A \longmapsto x))\,^\circ B = f\,^\circ B$.

(4) Let n be a natural number, and let D be a non empty set, and let d be an element of D, and let z be a finite sequence of elements of D. If $\operatorname{len} z = n$, then $\pi_{n+1}(z \frown \langle d \rangle) = d$.

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2. Boolean Marking and Firability/Firing of Transitions

Let P_1 be a place/transition net structure. The functor Bool_marks_of P_1 yielding a non empty set of functions from the places of P_1 to *Boolean* is defined by:

(Def.1) Bool_marks_of $P_1 = Boolean^{\text{the places of } P_1}$.

Let P_1 be a place/transition net structure. A Boolean marking of P_1 is an element of Bool_marks_of P_1.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. We say that t is firable on M_0 if and only if:

(Def.2) $M_0 \circ ({}^*\{t\}) \subseteq \{true\}$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. The functor Firing(t, M_0) yields a Boolean marking of P_1 and is defined by:

(Def.3) $\text{Firing}(t, M_0) = M_0 +\cdot ({}^*\{t\} \longmapsto false) +\cdot (\{t\}^* \longmapsto true)$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let Q be a finite sequence of elements of the transitions of P_1. We say that Q is firable on M_0 if and only if the conditions (Def.4) are satisfied.

(Def.4) (i) $Q = \varepsilon$, or

(ii) there exists a finite sequence M of elements of Bool_marks_of P_1 such that $\text{len}\, Q = \text{len}\, M$ and $\pi_1 Q$ is firable on M_0 and $\pi_1 M = \text{Firing}(\pi_1 Q, M_0)$ and for every natural number i such that $i < \text{len}\, Q$ and $i > 0$ holds $\pi_{i+1} Q$ is firable on $\pi_i M$ and $\pi_{i+1} M = \text{Firing}(\pi_{i+1} Q, \pi_i M)$.

Let P_1 be a place/transition net structure, let M_0 be a Boolean marking of P_1, and let Q be a finite sequence of elements of the transitions of P_1. The functor Firing(Q, M_0) yielding a Boolean marking of P_1 is defined as follows:

(Def.5) (i) $\text{Firing}(Q, M_0) = M_0$ if $Q = \varepsilon$,

(ii) there exists a finite sequence M of elements of Bool_marks_of P_1 such that $\text{len}\, Q = \text{len}\, M$ and $\text{Firing}(Q, M_0) = \pi_{\text{len}\, M} M$ and $\pi_1 M = \text{Firing}(\pi_1 Q, M_0)$ and for every natural number i such that $i < \text{len}\, Q$ and $i > 0$ holds $\pi_{i+1} M = \text{Firing}(\pi_{i+1} Q, \pi_i M)$, otherwise.

One can prove the following propositions:

(5) For every non empty set A and for arbitrary y and for every function f holds $(f +\cdot (A \longmapsto y)) \circ A = \{y\}$.

(6) Let P_1 be a place/transition net structure, and let M_0 be a Boolean marking of P_1, and let t be a transition of P_1, and let s be a place of P_1. If $s \in \{t\}^*$, then $(\text{Firing}(t, M_0))(s) = true$.

(7) Let P_1 be a place/transition net structure and let S_1 be a non empty set of places of P_1. Then S_1 is deadlock-like if and only if for every Boolean marking M_0 of P_1 such that $M_0 \circ S_1 = \{false\}$ and for every transition t of P_1 such that t is firable on M_0 holds $(\text{Firing}(t, M_0)) \circ S_1 = \{false\}$.

(8) Let D be a non empty set, and let Q_0, Q_1 be finite sequences of elements of D, and let i be a natural number. If $1 \leq i$ and $i \leq \operatorname{len} Q_0$, then $\pi_i(Q_0 \frown Q_1) = \pi_i Q_0$.

(9) Let D be a non empty set, and let Q_0, Q_1 be finite sequences of elements of D, and let i be a natural number. If $1 \leq i$ and $i \leq \operatorname{len} Q_1$, then $\pi_{\operatorname{len} Q_0 + i}(Q_0 \frown Q_1) = \pi_i Q_1$.

(10) Let P_1 be a place/transition net structure, and let M_0 be a Boolean marking of P_1, and let Q_0, Q_1 be finite sequences of elements of the transitions of P_1. Then $\mathrm{Firing}(Q_0 \frown Q_1, M_0) = \mathrm{Firing}(Q_1, \mathrm{Firing}(Q_0, M_0))$.

(11) Let P_1 be a place/transition net structure, and let M_0 be a Boolean marking of P_1, and let Q_0, Q_1 be finite sequences of elements of the transitions of P_1. If $Q_0 \frown Q_1$ is firable on M_0, then Q_1 is firable on $\mathrm{Firing}(Q_0, M_0)$ and Q_0 is firable on M_0.

(12) Let P_1 be a place/transition net structure, and let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. Then t is firable on M_0 if and only if $\langle t \rangle$ is firable on M_0.

(13) Let P_1 be a place/transition net structure, and let M_0 be a Boolean marking of P_1, and let t be a transition of P_1. Then $\mathrm{Firing}(t, M_0) = \mathrm{Firing}(\langle t \rangle, M_0)$.

(14) Let P_1 be a place/transition net structure and let S_1 be a non empty set of places of P_1. Then S_1 is deadlock-like if and only if for every Boolean marking M_0 of P_1 such that $M_0 \circ S_1 = \{\mathit{false}\}$ and for every finite sequence Q of elements of the transitions of P_1 such that Q is firable on M_0 holds $(\mathrm{Firing}(Q, M_0)) \circ S_1 = \{\mathit{false}\}$.

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