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Two Programs for SCM. Part I - Preliminaries [1]

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Summary. In two articles (this one and [3]) we discuss correctness of two short programs for the **SCM** machine: one computes Fibonacci numbers and the other computes the *fusc* function of Dijkstra [7]. The limitations of current Mizar implementation rendered it impossible to present the correctness proofs for the programs in one article. This part is purely technical and contains a number of very specific lemmas about integer division, floor, exponentiation and logarithms. The formal definitions of the Fibonacci sequence and the *fusc* function may be of general interest.

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The terminology and notation used in this paper are introduced in the following papers: [12], [1], [14], [9], [13], [11], [10], [8], [5], [6], [2], [4], and [15].

Let X_1, X_2 be non empty set, let Y_1 be a non empty subset of X_1, and let Y_2 be a non empty subset of X_2. Then $[: Y_1, Y_2 :]$ is a non empty subset of $[: X_1, X_2 :]$.

Let X_1, X_2 be non empty set, let Y_1 be a non empty subset of X_1, let Y_2 be a non empty subset of X_2, and let x be an element of $[: Y_1, Y_2 :]$. Then x_1 is an element of Y_1. Then x_2 is an element of Y_2.

In the sequel n will denote a natural number.

Let us consider n. The functor $\mathrm{Fib}(n)$ yielding a natural number is defined by the condition (Def.1).

(Def.1) There exists a function f_1 from $\mathbb{N}$ into $[: \mathbb{N}, \mathbb{N} :]$ such that
- (i) $\mathrm{Fib}(n) = f_1(n)_1$,
- (ii) $f_1(0) = \langle 0, 1 \rangle$, and

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(iii) for every natural number n and for every element x of $[: \mathbb{N}, \mathbb{N} :]$ such that $x = f_1(n)$ holds $f_1(n+1) = \langle x_2, x_1 + x_2 \rangle$.

We now state a number of propositions:

(1) $\mathrm{Fib}(0) = 0$ and $\mathrm{Fib}(1) = 1$ and for every natural number n holds $\mathrm{Fib}(n + 1 + 1) = \mathrm{Fib}(n) + \mathrm{Fib}(n+1)$.

(2) For every integer i holds $i \div +1 = i$.

(3) For all integers i, j such that $j > 0$ and $i \div j = 0$ holds $i < j$.

(4) For all integers i, j such that $0 \leq i$ and $i < j$ holds $i \div j = 0$.

(5) For all integers i, j, k such that $j > 0$ and $k > 0$ holds $i \div j \div k = i \div j \cdot k$.

(6) For every integer i holds $i \bmod +2 = 0$ or $i \bmod +2 = 1$.

(7) For every integer i such that i is a natural number holds $i \div +2$ is a natural number.

(8) For every natural number k such that $k > 0$ and for every natural number n holds $k^n > 0$.

(9)[2] For every natural number n holds $2^n = 2^n$.

(10) For all real numbers a, b, c such that $a \leq b$ and $c > 1$ holds $c^a \leq c^b$.

Let a, n be natural numbers. Then a^n is a natural number.

Next we state several propositions:

(11) For all real numbers r, s such that $r \geq s$ holds $\lfloor r \rfloor \geq \lfloor s \rfloor$.

(12) For all real numbers a, b, c such that $a > 1$ and $b > 0$ and $c \geq b$ holds $\log_a c \geq \log_a b$.

(13) For every natural number n such that $n > 0$ holds $\lfloor \log_2(2 \cdot n) \rfloor + 1 \neq \lfloor \log_2(2 \cdot n + 1) \rfloor$.

(14) For every natural number n such that $n > 0$ holds $\lfloor \log_2(2 \cdot n) \rfloor + 1 \geq \lfloor \log_2(2 \cdot n + 1) \rfloor$.

(15) For every natural number n such that $n > 0$ holds $\lfloor \log_2(2 \cdot n) \rfloor = \lfloor \log_2(2 \cdot n + 1) \rfloor$.

(16) For every natural number n such that $n > 0$ holds $\lfloor \log_2 n \rfloor + 1 = \lfloor \log_2(2 \cdot n + 1) \rfloor$.

Let f be a function from $\mathbb{N}$ into $\mathbb{N}^*$ and let n be a natural number. Then $f(n)$ is a finite sequence of elements of $\mathbb{N}$.

Let n be a natural number. The functor $\mathrm{Fusc}(n)$ yields a natural number and is defined by:

(Def.2) (i) $\mathrm{Fusc}(n) = 0$ if $n = 0$,

(ii) there exists a natural number l and there exists a function f_2 from $\mathbb{N}$ into $\mathbb{N}^*$ such that $l+1 = n$ and $\mathrm{Fusc}(n) = \pi_n f_2(l)$ and $f_2(0) = \langle 1 \rangle$ and for every natural number n holds for every natural number k such that $n+2 = 2 \cdot k$ holds $f_2(n+1) = f_2(n) \frown \langle \pi_k f_2(n) \rangle$ and for every natural number k such that $n+2 = 2 \cdot k + 1$ holds $f_2(n+1) = f_2(n) \frown \langle \pi_k f_2(n) + \pi_{k+1} f_2(n) \rangle$, otherwise.

[2]Both power functions in this theorem are different. The first is defined in [10] and the second in [8].

The following propositions are true:

(17) $\mathrm{Fusc}(0) = 0$ and $\mathrm{Fusc}(1) = 1$ and for every natural number n holds $\mathrm{Fusc}(2 \cdot n) = \mathrm{Fusc}(n)$ and $\mathrm{Fusc}(2 \cdot n + 1) = \mathrm{Fusc}(n) + \mathrm{Fusc}(n + 1)$.

(18) For all natural numbers n_1, n'_1 such that $n_1 \neq 0$ and $n_1 = 2 \cdot n'_1$ holds $n'_1 < n_1$.

(19) For all natural numbers n_1, n'_1 such that $n_1 = 2 \cdot n'_1 + 1$ holds $n'_1 < n_1$.

(20) For all natural numbers A, B holds $B = A \cdot \mathrm{Fusc}(0) + B \cdot \mathrm{Fusc}(0 + 1)$.

(21) For all natural numbers n_1, n'_1, A, B, N such that $n_1 = 2 \cdot n'_1 + 1$ and $\mathrm{Fusc}(N) = A \cdot \mathrm{Fusc}(n_1) + B \cdot \mathrm{Fusc}(n_1 + 1)$ holds $\mathrm{Fusc}(N) = A \cdot \mathrm{Fusc}(n'_1) + (B + A) \cdot \mathrm{Fusc}(n'_1 + 1)$.

(22) For all natural numbers n_1, n'_1, A, B, N such that $n_1 = 2 \cdot n'_1$ and $\mathrm{Fusc}(N) = A \cdot \mathrm{Fusc}(n_1) + B \cdot \mathrm{Fusc}(n_1 + 1)$ holds $\mathrm{Fusc}(N) = (A + B) \cdot \mathrm{Fusc}(n'_1) + B \cdot \mathrm{Fusc}(n'_1 + 1)$.

(23) $6 + 1 = 6 \cdot (\lfloor \log_2 1 \rfloor + 1) + 1$.

(24) For every natural number n'_1 such that $n'_1 > 0$ holds $\lfloor \log_2 n'_1 \rfloor$ is a natural number and $6 \cdot (\lfloor \log_2 n'_1 \rfloor + 1) + 1 > 0$.

(25) For all natural numbers n_1, n'_1 such that $n_1 = 2 \cdot n'_1 + 1$ and $n'_1 > 0$ holds $6 + (6 \cdot (\lfloor \log_2 n'_1 \rfloor + 1) + 1) = 6 \cdot (\lfloor \log_2 n_1 \rfloor + 1) + 1$.

(26) For all natural numbers n_1, n'_1 such that $n_1 = 2 \cdot n'_1$ and $n'_1 > 0$ holds $6 + (6 \cdot (\lfloor \log_2 n'_1 \rfloor + 1) + 1) = 6 \cdot (\lfloor \log_2 n_1 \rfloor + 1) + 1$.

(27) For every natural number N such that $N \neq 0$ holds $6 \cdot N - 4 > 0$.

(28) For every natural number N holds $6 + (6 \cdot N - 4) = 6 \cdot (N + 1) - 4$.

(29) For all natural numbers m, k, N such that $m = (k + 1 + N) - 1$ holds $m = (k + (N + 1)) - 1$.

(30) For every natural number N holds $2 + (6 \cdot N - 4) = 6 \cdot N - 2$.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[3] Grzegorz Bancerek and Piotr Rudnicki. Two programs for **SCM**. Part II - programs. *Formalized Mathematics*, 4(**1**):73–75, 1993.

[4] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Formalized Mathematics*, 1(**3**):529–536, 1990.

[5] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[6] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[7] Edsger W. Dijkstra. *Selected Writings on Computing, a Personal Perspective.*

[8] Rafał Kwiatek. Factorial and Newton coefficients. *Formalized Mathematics*, 1(**5**):887–890, 1990.

[9] Rafał Kwiatek and Grzegorz Zwara. The divisibility of integers and integer relative primes. *Formalized Mathematics*, 1(**5**):829–832, 1990.

[10] Konrad Raczkowski and Andrzej Nędzusiak. Real exponents and logarithms. *Formalized Mathematics*, 2(**2**):213–216, 1991.

[11] Andrzej Trybulec. Domains and their Cartesian products. *Formalized Mathematics*, 1(**1**):115–122, 1990.
[12] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[13] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.
[14] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[15] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.

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