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T_0 Topological Spaces

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The papers [7], [10], [9], [1], [2], [4], [3], [6], [5], and [8] provide the terminology and notation for this paper.

The following two propositions are true:

(1) Let A, B be non empty sets and let R_1, R_2 be relations between A and B. Suppose that for every element x of A and for every element y of B holds $\langle x, y\rangle \in R_1$ iff $\langle x, y\rangle \in R_2$. Then $R_1 = R_2$.

(2) Let X, Y be non empty sets, and let f be a function from X into Y, and let A be a subset of X. Suppose that for all elements x_1, x_2 of X such that $x_1 \in A$ and $f(x_1) = f(x_2)$ holds $x_2 \in A$. Then $f^{-1} f^{\circ} A = A$.

Let T, S be topological spaces. We say that T and S are homeomorphic if and only if:

(Def.1) There exists map from T into S which is a homeomorphism.

Let T, S be topological spaces and let f be a map from T into S. We say that f is open if and only if:

(Def.2) For every subset A of T such that A is open holds $f^{\circ} A$ is open.

Let T be a topological space. The functor Indiscernibility(T) yielding an equivalence relation of the carrier of T is defined by the condition (Def.3).

(Def.3) Let p, q be points of T. Then $\langle p, q\rangle \in$ Indiscernibility(T) if and only if for every subset A of T such that A is open holds $p \in A$ iff $q \in A$.

Let T be a topological space. The functor $T_{/\,\mathrm{Indiscernibility}\,T}$ yields a non empty partition of the carrier of T and is defined as follows:

(Def.4) $T_{/\,\mathrm{Indiscernibility}\,T} = \mathrm{Classes}\,\mathrm{Indiscernibility}(T)$.

Let T be a topological space. The functor T_0-reflex(T) yields a topological space and is defined as follows:

(Def.5) T_0-reflex(T) = the decomposition space of $T_{/\,\mathrm{Indiscernibility}\,T}$.

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Let T be a topological space. The functor T_0-map(T) yielding a continuous map from T into T_0-reflex(T) is defined as follows:

(Def.6) T_0-map(T) = the projection onto $T_{/\,\mathrm{Indiscernibility}\,T}$.

One can prove the following propositions:

(3) For every topological space T and for every point p of T holds $p \in (T_0\text{-map}(T))(p)$.

(4) For every topological space T holds dom T_0-map(T) = the carrier of T and rng T_0-map(T) $\subseteq$ the carrier of T_0-reflex(T).

(5) Let T be a topological space. Then the carrier of T_0-reflex(T) = $T_{/\,\mathrm{Indiscernibility}\,T}$ and the topology of T_0-reflex(T) = $\{A : A$ ranges over subsets of $T_{/\,\mathrm{Indiscernibility}\,T}$, $\bigcup A \in$ the topology of $T\}$.

(6) For every topological space T and for every subset V of T_0-reflex(T) holds V is open iff $\bigcup V \in$ the topology of T.

(7) Let T be a topological space and let C be arbitrary. Then C is a point of T_0-reflex(T) if and only if there exists a point p of T such that $C = [p]_{\mathrm{Indiscernibility}(T)}$.

(8) For every topological space T and for every point p of T holds $(T_0\text{-map}(T))(p) = [p]_{\mathrm{Indiscernibility}(T)}$.

(9) For every topological space T and for all points p, q of T holds $(T_0\text{-map}(T))(q) = (T_0\text{-map}(T))(p)$ iff $\langle q, p\rangle \in \mathrm{Indiscernibility}(T)$.

(10) Let T be a topological space and let A be a subset of T. Suppose A is open. Let p, q be points of T. If $p \in A$ and $(T_0\text{-map}(T))(p) = (T_0\text{-map}(T))(q)$, then $q \in A$.

(11) Let T be a topological space and let A be a subset of T. Suppose A is open. Let C be a subset of T. If $C \in T_{/\,\mathrm{Indiscernibility}\,T}$ and C meets A, then $C \subseteq A$.

(12) For every topological space T holds T_0-map(T) is open.

A topological structure is discernible if it satisfies the condition (Def.7).

(Def.7) Let x, y be points of it. Suppose $x \neq y$. Then there exists a subset V of it such that V is open but $x \in V$ and $y \notin V$ or $y \in V$ and $x \notin V$.

Let us note that there exists a topological space which is discernible.

A T_0-space is a discernible topological space.

One can prove the following propositions:

(13) For every topological space T holds T_0-reflex(T) is a T_0-space.

(14) Let T, S be topological spaces. Given a map h from T_0-reflex(S) into T_0-reflex(T) such that h is a homeomorphism and T_0-map(T) and $h \cdot T_0$-map(S) are fiberwise equipotent. Then T and S are homeomorphic.

(15) Let T be a topological space, and let T_0 be a T_0-space, and let f be a continuous map from T into T_0, and let p, q be points of T. If $\langle p, q\rangle \in \mathrm{Indiscernibility}(T)$, then $f(p) = f(q)$.

(16) Let T be a topological space, and let T_0 be a T_0-space, and let f be a continuous map from T into T_0, and let p be a point of T. Then $f^\circ([p]_{\mathrm{Indiscernibility}(T)}) = \{f(p)\}$.

(17) Let T be a topological space, and let T_0 be a T_0-space, and let f be a continuous map from T into T_0. Then there exists a continuous map h from T_0-reflex(T) into T_0 such that $f = h \cdot T_0$-map(T).

References

[1] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[2] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[3] Agata Darmochwał. Families of subsets, subspaces and mappings in topological spaces. *Formalized Mathematics*, 1(**2**):257–261, 1990.

[4] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.

[5] Konrad Raczkowski and Paweł Sadowski. Equivalence relations and classes of abstraction. *Formalized Mathematics*, 1(**3**):441–444, 1990.

[6] Andrzej Trybulec. A Borsuk theorem on homotopy types. *Formalized Mathematics*, 2(**4**):535–545, 1991.

[7] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[8] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

[9] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

[10] Edmund Woronowicz. Relations defined on sets. *Formalized Mathematics*, 1(**1**):181–186, 1990.

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