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Maximal Anti-Discrete Subspaces of Topological Spaces

Zbigniew Karno
Warsaw University
Białystok

Summary. Let X be a topological space and let A be a subset of X. A is said to be *anti-discrete* provided for every open subset G of X either $A \cap G = \emptyset$ or $A \subseteq G$; equivalently, for every closed subset F of X either $A \cap F = \emptyset$ or $A \subseteq F$. An anti-discrete subset M of X is said to be *maximal anti-discrete* provided for every anti-discrete subset A of X if $M \subseteq A$ then $M = A$. A subspace of X is *maximal anti-discrete* iff its carrier is maximal anti-discrete in X. The purpose is to list a few properties of maximal anti-discrete sets and subspaces in Mizar formalism.

It is shown that every $x \in X$ is contained in a unique maximal anti-discrete subset $\mathrm{M}(x)$ of X, denoted in the text by MaxADSet(x). Such subset can be defined by

$$\mathrm{M}(x) = \bigcap \{S \subseteq X : x \in S, \text{ and } S \text{ is open or closed in } X\}.$$

It has the following remarkable properties: (1) $y \in \mathrm{M}(x)$ iff $\mathrm{M}(y) = \mathrm{M}(x)$, (2) either $\mathrm{M}(x) \cap \mathrm{M}(y) = \emptyset$ or $\mathrm{M}(x) = \mathrm{M}(y)$, (3) $\mathrm{M}(x) = \mathrm{M}(y)$ iff $\overline{\{x\}} = \overline{\{y\}}$, and (4) $\mathrm{M}(x) \cap \mathrm{M}(y) = \emptyset$ iff $\overline{\{x\}} \neq \overline{\{y\}}$. It follows from these properties that $\{\mathrm{M}(x) : x \in X\}$ is the T_0-partition of X defined by M.H. Stone in [7].

Moreover, it is shown that the operation M defined on all subsets of X by

$$\mathrm{M}(A) = \bigcup \{\mathrm{M}(x) : x \in A\},$$

denoted in the text by MaxADSet(A), satisfies the Kuratowski closure axioms (see e.g., [4]), i.e., (1) $\mathrm{M}(A \cup B) = \mathrm{M}(A) \cup \mathrm{M}(B)$, (2) $\mathrm{M}(A) = \mathrm{M}(\mathrm{M}(A))$, (3) $A \subseteq \mathrm{M}(A)$,and (4) $\mathrm{M}(\emptyset) = \emptyset$. Note that this operation commutes with the usual closure operation of X, and if A is an open (or a closed) subset of X, then $\mathrm{M}(A) = A$.

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The articles [11], [12], [8], [10], [5], [6], [13], [9], [3], [1], and [2] provide the terminology and notation for this paper.

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1. Properties of the Closure and the Interior Operations

Let X be a topological space and let A be a non empty subset of X. Observe that $\overline{A}$ is non empty.

Let X be a topological space and let A be an empty subset of X. One can check that $\overline{A}$ is empty.

Let X be a topological space and let A be a non proper subset of X. One can check that $\overline{A}$ is non proper.

Let X be a non trivial topological space and let A be a non trivial non empty subset of X. Observe that $\overline{A}$ is non trivial.

In the sequel X is a topological space.

We now state three propositions:

(1) For every subset A of X holds $\overline{A} = \bigcap\{F : F$ ranges over subsets of X, F is closed $\wedge\ A \subseteq F\}$.

(2) For every point x of X holds $\overline{\{x\}} = \bigcap\{F : F$ ranges over subsets of X, F is closed $\wedge\ x \in F\}$.

(3) For all subsets A, B of X such that $B \subseteq \overline{A}$ holds $\overline{B} \subseteq \overline{A}$.

Let X be a topological space and let A be a non proper subset of X. Note that $\operatorname{Int} A$ is non proper.

Let X be a topological space and let A be a proper subset of X. One can check that $\operatorname{Int} A$ is proper.

Let X be a topological space and let A be an empty subset of X. Note that $\operatorname{Int} A$ is empty.

Next we state two propositions:

(4) For every subset A of X holds $\operatorname{Int} A = \bigcup\{G : G$ ranges over subsets of X, G is open $\wedge\ G \subseteq A\}$.

(5) For all subsets A, B of X such that $\operatorname{Int} A \subseteq B$ holds $\operatorname{Int} A \subseteq \operatorname{Int} B$.

2. Anti-Discrete Subsets of Topological Structures

Let Y be a topological structure. A subset of Y is anti-discrete if:

(Def.1) For every point x of Y and for every subset G of Y such that G is open and $x \in G$ holds if $x \in$ it, then it $\subseteq G$.

Let Y be a non empty topological structure. Let us observe that a subset of Y is anti-discrete if:

(Def.2) For every point x of Y and for every subset F of Y such that F is closed and $x \in F$ holds if $x \in$ it, then it $\subseteq F$.

Let Y be a topological structure. Let us observe that a subset of Y is anti-discrete if:

(Def.3) For every subset G of Y such that G is open holds it $\cap G = \emptyset$ or it $\subseteq G$.

Let Y be a topological structure. Let us observe that a subset of Y is anti-discrete if:

(Def.4) For every subset F of Y such that F is closed holds it $\cap F = \emptyset$ or it $\subseteq F$.

Next we state the proposition

(6) Let Y_0, Y_1 be topological structures, and let D_0 be a subset of Y_0, and let D_1 be a subset of Y_1. Suppose the topological structure of Y_0 = the topological structure of Y_1 and $D_0 = D_1$. If D_0 is anti-discrete, then D_1 is anti-discrete.

In the sequel Y will denote a non empty topological structure.

Next we state three propositions:

(7) For all subsets A, B of Y such that $B \subseteq A$ holds if A is anti-discrete, then B is anti-discrete.

(8) For every point x of Y holds $\{x\}$ is anti-discrete.

(9) Every empty subset of Y is anti-discrete.

Let Y be a topological structure. A family of subsets of Y is anti-discrete-set-family if:

(Def.5) For every subset A of Y such that $A \in$ it holds A is anti-discrete.

One can prove the following propositions:

(10) Let F be a family of subsets of Y. Suppose F is anti-discrete-set-family. If $\bigcap F \neq \emptyset$, then $\bigcup F$ is anti-discrete.

(11) For every family F of subsets of Y such that F is anti-discrete-set-family holds $\bigcap F$ is anti-discrete.

Let Y be a non empty topological structure and let x be a point of Y. The functor MaxADSF(x) yields a non empty family of subsets of Y and is defined by:

(Def.6) MaxADSF$(x) = \{A : A$ ranges over subsets of Y, A is anti-discrete $\wedge\ x \in A\}$.

In the sequel x will denote a point of Y.

We now state four propositions:

(12) MaxADSF(x) is anti-discrete-set-family.

(13) $\{x\} = \bigcap$ MaxADSF(x).

(14) $\{x\} \subseteq \bigcup$ MaxADSF(x).

(15) $\bigcup$ MaxADSF(x) is anti-discrete.

3. Maximal Anti-Discrete Subsets of Topological Structures

Let Y be a topological structure. A subset of Y is maximal anti-discrete if:

(Def.7) It is anti-discrete and for every subset D of Y such that D is anti-discrete and it $\subseteq D$ holds it $= D$.

We now state the proposition

(16) Let Y_0, Y_1 be topological structures, and let D_0 be a subset of Y_0, and let D_1 be a subset of Y_1. Suppose the topological structure of Y_0 = the topological structure of Y_1 and $D_0 = D_1$. If D_0 is maximal anti-discrete, then D_1 is maximal anti-discrete.

In the sequel Y will denote a non empty topological structure.

One can prove the following propositions:

(17) Every empty subset of Y is not maximal anti-discrete.

(18) For every non empty subset A of Y such that A is anti-discrete and open holds A is maximal anti-discrete.

(19) For every non empty subset A of Y such that A is anti-discrete and closed holds A is maximal anti-discrete.

Let Y be a non empty topological structure and let x be a point of Y. The functor MaxADSet(x) yielding a non empty subset of Y is defined by:

(Def.8) $\mathrm{MaxADSet}(x) = \bigcup \mathrm{MaxADSF}(x)$.

We now state several propositions:

(20) For every point x of Y holds $\{x\} \subseteq \mathrm{MaxADSet}(x)$.

(21) For every subset D of Y and for every point x of Y such that D is anti-discrete and $x \in D$ holds $D \subseteq \mathrm{MaxADSet}(x)$.

(22) For every point x of Y holds MaxADSet(x) is maximal anti-discrete.

(23) For all points x, y of Y holds $y \in \mathrm{MaxADSet}(x)$ iff $\mathrm{MaxADSet}(y) = \mathrm{MaxADSet}(x)$.

(24) For all points x, y of Y holds $\mathrm{MaxADSet}(x) \cap \mathrm{MaxADSet}(y) = \emptyset$ or $\mathrm{MaxADSet}(x) = \mathrm{MaxADSet}(y)$.

(25) For every subset F of Y and for every point x of Y such that F is closed and $x \in F$ holds $\mathrm{MaxADSet}(x) \subseteq F$.

(26) For every subset G of Y and for every point x of Y such that G is open and $x \in G$ holds $\mathrm{MaxADSet}(x) \subseteq G$.

(27) Let x be a point of Y. Suppose $\{F : F$ ranges over subsets of Y, F is closed $\wedge\, x \in F\} \neq \emptyset$. Then $\mathrm{MaxADSet}(x) \subseteq \bigcap\{F : F$ ranges over subsets of Y, F is closed $\wedge\, x \in F\}$.

(28) Let x be a point of Y. Suppose $\{G : G$ ranges over subsets of Y, G is open $\wedge\, x \in G\} \neq \emptyset$. Then $\mathrm{MaxADSet}(x) \subseteq \bigcap\{G : G$ ranges over subsets of Y, G is open $\wedge\, x \in G\}$.

Let Y be a non empty topological structure. Let us observe that a subset of Y is maximal anti-discrete if:

(Def.9) There exists a point x of Y such that $x \in$ it and it $= \mathrm{MaxADSet}(x)$.

The following proposition is true

(29) For every subset A of Y and for every point x of Y such that $x \in A$ holds if A is maximal anti-discrete, then $A = \mathrm{MaxADSet}(x)$.

Let Y be a non empty topological structure. Let us observe that a non empty subset of Y is maximal anti-discrete if:

(Def.10) For every point x of Y such that $x \in$ it holds it $=$ MaxADSet(x).

Let Y be a non empty topological structure and let A be a subset of Y. The functor MaxADSet(A) yielding a subset of Y is defined as follows:

(Def.11) MaxADSet$(A) = \bigcup\{$MaxADSet$(a) : a$ ranges over points of Y, $a \in A\}$.

Next we state a number of propositions:

(30) For every point x of Y holds MaxADSet$(x) =$ MaxADSet$(\{x\})$.

(31) For every subset A of Y and for every point x of Y such that MaxADSet$(x) \cap$ MaxADSet$(A) \neq \emptyset$ holds MaxADSet$(x) \cap A \neq \emptyset$.

(32) For every subset A of Y and for every point x of Y such that MaxADSet$(x) \cap$ MaxADSet$(A) \neq \emptyset$ holds MaxADSet$(x) \subseteq$ MaxADSet(A).

(33) For all subsets A, B of Y such that $A \subseteq B$ holds MaxADSet$(A) \subseteq$ MaxADSet(B).

(34) For every subset A of Y holds $A \subseteq$ MaxADSet(A).

(35) For every subset A of Y holds MaxADSet$(A) =$ MaxADSet(MaxADSet(A)).

(36) For all subsets A, B of Y such that $A \subseteq$ MaxADSet(B) holds MaxADSet$(A) \subseteq$ MaxADSet(B).

(37) For all subsets A, B of Y holds $B \subseteq$ MaxADSet(A) and $A \subseteq$ MaxADSet(B) iff MaxADSet$(A) =$ MaxADSet(B).

(38) For all subsets A, B of Y holds MaxADSet$(A \cup B) =$ MaxADSet$(A) \cup$ MaxADSet(B).

(39) For all subsets A, B of Y holds MaxADSet$(A \cap B) \subseteq$ MaxADSet$(A) \cap$ MaxADSet(B).

Let Y be a non empty topological structure and let A be a non empty subset of Y. One can verify that MaxADSet(A) is non empty.

Let Y be a non empty topological structure and let A be an empty subset of Y. One can verify that MaxADSet(A) is empty.

Let Y be a non empty topological structure and let A be a non proper subset of Y. Observe that MaxADSet(A) is non proper.

Let Y be a non trivial non empty topological structure and let A be a non trivial non empty subset of Y. Note that MaxADSet(A) is non trivial.

The following four propositions are true:

(40) For every subset G of Y and for every subset A of Y such that G is open and $A \subseteq G$ holds MaxADSet$(A) \subseteq G$.

(41) Let A be a subset of Y. Suppose $\{G : G$ ranges over subsets of Y, G is open $\wedge\ A \subseteq G\} \neq \emptyset$. Then MaxADSet$(A) \subseteq \bigcap\{G : G$ ranges over subsets of Y, G is open $\wedge\ A \subseteq G\}$.

(42) For every subset F of Y and for every subset A of Y such that F is closed and $A \subseteq F$ holds MaxADSet$(A) \subseteq F$.

(43) Let A be a subset of Y. Suppose $\{F : F$ ranges over subsets of Y, F is closed $\wedge$ $A \subseteq F\} \neq \emptyset$. Then $\mathrm{MaxADSet}(A) \subseteq \bigcap\{F : F$ ranges over subsets of Y, F is closed $\wedge$ $A \subseteq F\}$.

4. Anti-Discrete and Maximal Anti-discrete Subsets of Topological Spaces

Let X be a topological space. Let us observe that a subset of X is anti-discrete if:

(Def.12) For every point x of X such that $x \in$ it holds it $\subseteq \overline{\{x\}}$.

Let X be a topological space. Let us observe that a subset of X is anti-discrete if:

(Def.13) For every point x of X such that $x \in$ it holds $\overline{\text{it}} = \overline{\{x\}}$.

Let X be a topological space. Let us observe that a subset of X is anti-discrete if:

(Def.14) For all points x, y of X such that $x \in$ it and $y \in$ it holds $\overline{\{x\}} = \overline{\{y\}}$.

In the sequel X will be a topological space.

The following four propositions are true:

(44) For every point x of X and for every subset D of X such that D is anti-discrete and $\overline{\{x\}} \subseteq D$ holds $D = \overline{\{x\}}$.

(45) Let A be a subset of X. Then A is anti-discrete and closed if and only if for every point x of X such that $x \in A$ holds $A = \overline{\{x\}}$.

(46) For every subset A of X such that A is anti-discrete and A is not open holds A is boundary.

(47) For every point x of X such that $\overline{\{x\}} = \{x\}$ holds $\{x\}$ is maximal anti-discrete.

In the sequel x, y will be points of X.

The following propositions are true:

(48) $\mathrm{MaxADSet}(x) \subseteq \bigcap\{G : G$ ranges over subsets of X, G is open $\wedge$ $x \in G\}$.

(49) $\mathrm{MaxADSet}(x) \subseteq \bigcap\{F : F$ ranges over subsets of X, F is closed $\wedge$ $x \in F\}$.

(50) $\mathrm{MaxADSet}(x) \subseteq \overline{\{x\}}$.

(51) $\mathrm{MaxADSet}(x) = \mathrm{MaxADSet}(y)$ iff $\overline{\{x\}} = \overline{\{y\}}$.

(52) $\mathrm{MaxADSet}(x) \cap \mathrm{MaxADSet}(y) = \emptyset$ iff $\overline{\{x\}} \neq \overline{\{y\}}$.

Let X be a topological space and let x be a point of X. Then $\mathrm{MaxADSet}(x)$ is a non empty subset of X and it can be characterized by the condition:

(Def.15) $\mathrm{MaxADSet}(x) = \overline{\{x\}} \cap \bigcap\{G : G$ ranges over subsets of X, G is open $\wedge$ $x \in G\}$.

The following propositions are true:

(53) Let x, y be points of X. Then $\overline{\{x\}} \subseteq \overline{\{y\}}$ if and only if $\bigcap\{G : G$ ranges over subsets of X, G is open $\wedge\ y \in G\} \subseteq \bigcap\{G : G$ ranges over subsets of X, G is open $\wedge\ x \in G\}$.

(54) For all points x, y of X holds $\overline{\{x\}} \subseteq \overline{\{y\}}$ iff $\operatorname{MaxADSet}(y) \subseteq \bigcap\{G : G$ ranges over subsets of X, G is open $\wedge\ x \in G\}$.

(55) Let x, y be points of X. Then $\operatorname{MaxADSet}(x) \cap \operatorname{MaxADSet}(y) = \emptyset$ if and only if one of the following conditions is satisfied:

(i) there exists a subset V of X such that V is open and $\operatorname{MaxADSet}(x) \subseteq V$ and $V \cap \operatorname{MaxADSet}(y) = \emptyset$, or

(ii) there exists a subset W of X such that W is open and $W \cap \operatorname{MaxADSet}(x) = \emptyset$ and $\operatorname{MaxADSet}(y) \subseteq W$.

(56) Let x, y be points of X. Then $\operatorname{MaxADSet}(x) \cap \operatorname{MaxADSet}(y) = \emptyset$ if and only if one of the following conditions is satisfied:

(i) there exists a subset E of X such that E is closed and $\operatorname{MaxADSet}(x) \subseteq E$ and $E \cap \operatorname{MaxADSet}(y) = \emptyset$, or

(ii) there exists a subset F of X such that F is closed and $F \cap \operatorname{MaxADSet}(x) = \emptyset$ and $\operatorname{MaxADSet}(y) \subseteq F$.

In the sequel A, B denote subsets of X.

The following propositions are true:

(57) $\operatorname{MaxADSet}(A) \subseteq \bigcap\{G : G$ ranges over subsets of X, G is open $\wedge\ A \subseteq G\}$.

(58) If A is open, then $\operatorname{MaxADSet}(A) = A$.

(59) $\operatorname{MaxADSet}(\operatorname{Int} A) = \operatorname{Int} A$.

(60) $\operatorname{MaxADSet}(A) \subseteq \bigcap\{F : F$ ranges over subsets of X, F is closed $\wedge\ A \subseteq F\}$.

(61) $\operatorname{MaxADSet}(A) \subseteq \overline{A}$.

(62) If A is closed, then $\operatorname{MaxADSet}(A) = A$.

(63) $\operatorname{MaxADSet}(\overline{A}) = \overline{A}$.

(64) $\overline{\operatorname{MaxADSet}(A)} = \overline{A}$.

(65) If $\operatorname{MaxADSet}(A) = \operatorname{MaxADSet}(B)$, then $\overline{A} = \overline{B}$.

(66) If A is closed or B is closed, then $\operatorname{MaxADSet}(A \cap B) = \operatorname{MaxADSet}(A) \cap \operatorname{MaxADSet}(B)$.

(67) If A is open or B is open, then $\operatorname{MaxADSet}(A \cap B) = \operatorname{MaxADSet}(A) \cap \operatorname{MaxADSet}(B)$.

5. Maximal Anti-Discrete Subspaces

In the sequel Y is a non empty topological structure.

One can prove the following two propositions:

(68) Let Y_0 be a subspace of Y and let A be a subset of Y. Suppose $A =$ the carrier of Y_0. If Y_0 is anti-discrete, then A is anti-discrete.

(69) Let Y_0 be a subspace of Y. Suppose Y_0 is topological space-like. Let A be a subset of Y. Suppose $A =$ the carrier of Y_0. If A is anti-discrete, then Y_0 is anti-discrete.

In the sequel X will be a topological space and Y_0 will be a subspace of X.

One can prove the following four propositions:

(70) If for every open subspace X_0 of X holds Y_0 misses X_0 or Y_0 is a subspace of X_0, then Y_0 is anti-discrete.

(71) If for every closed subspace X_0 of X holds Y_0 misses X_0 or Y_0 is a subspace of X_0, then Y_0 is anti-discrete.

(72) Let Y_0 be an anti-discrete subspace of X and let X_0 be an open subspace of X. Then Y_0 misses X_0 or Y_0 is a subspace of X_0.

(73) Let Y_0 be an anti-discrete subspace of X and let X_0 be a closed subspace of X. Then Y_0 misses X_0 or Y_0 is a subspace of X_0.

Let Y be a non empty topological structure. A subspace of Y is maximal anti-discrete if it satisfies the conditions (Def.16).

(Def.16) (i) It is anti-discrete, and

(ii) for every subspace Y_0 of Y such that Y_0 is anti-discrete holds if the carrier of it $\subseteq$ the carrier of Y_0, then the carrier of it $=$ the carrier of Y_0.

Let Y be a non empty topological structure. Note that every subspace of Y which is maximal anti-discrete is also anti-discrete and every subspace of Y which is non anti-discrete is also non maximal anti-discrete.

Next we state the proposition

(74) Let Y_0 be a subspace of X and let A be a subset of X. Suppose $A =$ the carrier of Y_0. Then Y_0 is maximal anti-discrete if and only if A is maximal anti-discrete.

Let X be a topological space. One can check the following observations:

* every subspace of X which is open and anti-discrete is also maximal anti-discrete,
* every subspace of X which is open and non maximal anti-discrete is also non anti-discrete,
* every subspace of X which is anti-discrete and non maximal anti-discrete is also non open,
* every subspace of X which is closed and anti-discrete is also maximal anti-discrete,
* every subspace of X which is closed and non maximal anti-discrete is also non anti-discrete, and
* every subspace of X which is anti-discrete and non maximal anti-discrete is also non closed.

Let Y be a non empty topological structure and let x be a point of Y. The functor MaxADSspace(x) yielding a strict subspace of Y is defined by:

(Def.17) The carrier of MaxADSspace(x) $=$ MaxADSet(x).

We now state three propositions:

(75) For every point x of Y holds Sspace(x) is a subspace of MaxADSspace(x).

(76) Let x, y be points of Y. Then y is a point of MaxADSspace(x) if and only if the topological structure of MaxADSspace(y) = the topological structure of MaxADSspace(x).

(77) Let x, y be points of Y. Then

(i) the carrier of MaxADSspace(x) misses the carrier of MaxADSspace(y), or

(ii) the topological structure of MaxADSspace(x) = the topological structure of MaxADSspace(y).

Let X be a topological space. One can check that there exists a subspace of X which is maximal anti-discrete and strict.

Let X be a topological space and let x be a point of X. One can check that MaxADSspace(x) is maximal anti-discrete.

One can prove the following propositions:

(78) Let X_0 be a closed subspace of X and let x be a point of X. If x is a point of X_0, then MaxADSspace(x) is a subspace of X_0.

(79) Let X_0 be an open subspace of X and let x be a point of X. If x is a point of X_0, then MaxADSspace(x) is a subspace of X_0.

(80) For every point x of X such that $\overline{\{x\}} = \{x\}$ holds Sspace(x) is maximal anti-discrete.

Let Y be a non empty topological structure and let A be a non empty subset of Y. The functor Sspace(A) yielding a strict subspace of Y is defined by:

(Def.18) The carrier of Sspace(A) = A.

One can prove the following propositions:

(81) Every non empty subset of Y is a subset of Sspace(A).

(82) Let Y_0 be a subspace of Y and let A be a non empty subset of Y. If A is a subset of Y_0, then Sspace(A) is a subspace of Y_0.

Let Y be a non trivial non empty topological structure. Note that there exists a subspace of Y which is non proper and strict.

Let Y be a non trivial non empty topological structure and let A be a non trivial non empty subset of Y. Observe that Sspace(A) is non trivial.

Let Y be a non empty topological structure and let A be a non proper non empty subset of Y. One can verify that Sspace(A) is non proper.

Let Y be a non empty topological structure and let A be a non empty subset of Y. The functor MaxADSspace(A) yields a strict subspace of Y and is defined by:

(Def.19) The carrier of MaxADSspace(A) = MaxADSet(A).

We now state several propositions:

(83) Every non empty subset of Y is a subset of MaxADSspace(A).

(84) For every non empty subset A of Y holds Sspace(A) is a subspace of MaxADSspace(A).

(85) For every point x of Y holds the topological structure of MaxADSspace(x) = the topological structure of MaxADSspace($\{x\}$).

(86) For all non empty subsets A, B of Y such that $A \subseteq B$ holds MaxADSspace(A) is a subspace of MaxADSspace(B).

(87) For every non empty subset A of Y holds the topological structure of MaxADSspace(A) = the topological structure of MaxADSspace(MaxADSet(A)).

(88) For all non empty subsets A, B of Y such that A is a subset of MaxADSspace(B) holds MaxADSspace(A) is a subspace of MaxADSspace(B).

(89) Let A, B be non empty subsets of Y. Then B is a subset of MaxADSspace(A) and A is a subset of MaxADSspace(B) if and only if the topological structure of MaxADSspace(A) = the topological structure of MaxADSspace(B).

Let Y be a non trivial non empty topological structure and let A be a non trivial non empty subset of Y. One can verify that MaxADSspace(A) is non trivial.

Let Y be a non empty topological structure and let A be a non proper non empty subset of Y. One can verify that MaxADSspace(A) is non proper.

The following two propositions are true:

(90) Let X_0 be an open subspace of X and let A be a non empty subset of X. If A is a subset of X_0, then MaxADSspace(A) is a subspace of X_0.

(91) Let X_0 be a closed subspace of X and let A be a non empty subset of X. If A is a subset of X_0, then MaxADSspace(A) is a subspace of X_0.

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