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On the Decomposition of the Continuity

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Summary. This article is devoted to functions of general topological spaces. A function from X to Y is A-continuous if the counterimage of every open set V of Y belongs to A, where A is a collection of subsets of X. We give the following characteristics of the continuity, called decomposition of continuity: A function f is continuous if and only if it is both A-continuous and B-continuous.

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The articles [14], [12], [2], [1], [3], [10], [6], [8], [11], [5], [13], [9], [15], [7], and [4] provide the notation and terminology for this paper.

Let T be a topological space. A subset of the carrier of T is called an α-set of T if:

(Def.1) $\mathrm{It} \subseteq \mathrm{Int}\,\overline{\mathrm{Int}\,\mathrm{it}}$.

A subset of the carrier of T is semi-open if:

(Def.2) $\mathrm{It} \subseteq \overline{\mathrm{Int}\,\mathrm{it}}$.

A subset of the carrier of T is pre-open if:

(Def.3) $\mathrm{It} \subseteq \mathrm{Int}\,\overline{\mathrm{it}}$.

A subset of the carrier of T is pre-semi-open if:

(Def.4) $\mathrm{It} \subseteq \overline{\mathrm{Int}\,\overline{\mathrm{it}}}$.

A subset of the carrier of T is semi-pre-open if:

(Def.5) $\mathrm{It} \subseteq \overline{\mathrm{Int}\,\mathrm{it}} \cup \mathrm{Int}\,\overline{\mathrm{it}}$.

Let T be a topological space and let B be a subset of the carrier of T. The functor $\mathrm{sInt}(B)$ yielding a subset of the carrier of T is defined as follows:

(Def.6) $\mathrm{sInt}(B) = B \cap \overline{\mathrm{Int}\,B}$.

The functor $\mathrm{pInt}(B)$ yielding a subset of the carrier of T is defined as follows:

(Def.7) $\mathrm{pInt}(B) = B \cap \mathrm{Int}\,\overline{B}$.

The functor $\alpha\mathrm{Int}(B)$ yielding a subset of the carrier of T is defined as follows:

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(Def.8) $\alpha\mathrm{Int}(B) = B \cap \mathrm{Int}\,\overline{\mathrm{Int}\,B}$.

The functor psInt(B) yields a subset of the carrier of T and is defined as follows:

(Def.9) $\mathrm{psInt}(B) = B \cap \overline{\mathrm{Int}\,\overline{B}}$.

Let T be a topological space and let B be a subset of the carrier of T. The functor spInt(B) yields a subset of the carrier of T and is defined by:

(Def.10) $\mathrm{spInt}(B) = \mathrm{sInt}(B) \cup \mathrm{pInt}(B)$.

Let T be a topological space. The functor T^{α} yields a family of subsets of the carrier of T and is defined as follows:

(Def.11) $T^{\alpha} = \{B : B$ ranges over subsets of the carrier of T, B is anα-set of $T\}$.

The functor SO(T) yielding a family of subsets of the carrier of T is defined by:

(Def.12) $\mathrm{SO}(T) = \{B : B$ ranges over subsets of the carrier of T, B is semi-open$\}$.

The functor PO(T) yielding a family of subsets of the carrier of T is defined as follows:

(Def.13) $\mathrm{PO}(T) = \{B : B$ ranges over subsets of the carrier of T, B is pre-open$\}$.

The functor SPO(T) yielding a family of subsets of the carrier of T is defined as follows:

(Def.14) $\mathrm{SPO}(T) = \{B : B$ ranges over subsets of the carrier of T, B is semi-pre-open$\}$.

The functor PSO(T) yields a family of subsets of the carrier of T and is defined by:

(Def.15) $\mathrm{PSO}(T) = \{B : B$ ranges over subsets of the carrier of T, B is pre-semi-open$\}$.

The functor $D(c, \alpha)(T)$ yielding a family of subsets of the carrier of T is defined as follows:

(Def.16) $D(c, \alpha)(T) = \{B : B$ ranges over subsets of the carrier of T, $\mathrm{Int}\,B = \alpha\mathrm{Int}(B)\}$.

The functor $D(c, p)(T)$ yielding a family of subsets of the carrier of T is defined by:

(Def.17) $D(c, p)(T) = \{B : B$ ranges over subsets of the carrier of T, $\mathrm{Int}\,B = \mathrm{pInt}(B)\}$.

The functor $D(c, s)(T)$ yielding a family of subsets of the carrier of T is defined by:

(Def.18) $D(c, s)(T) = \{B : B$ ranges over subsets of the carrier of T, $\mathrm{Int}\,B = \mathrm{sInt}(B)\}$.

The functor $D(c, ps)(T)$ yielding a family of subsets of the carrier of T is defined as follows:

(Def.19) $D(c, ps)(T) = \{B : B$ ranges over subsets of the carrier of T, $\mathrm{Int}\,B = \mathrm{psInt}(B)\}$.

The functor $D(\alpha, p)(T)$ yields a family of subsets of the carrier of T and is defined as follows:

(Def.20) $D(\alpha, p)(T) = \{B : B$ ranges over subsets of the carrier of T, $\alpha\mathrm{Int}(B) = \mathrm{pInt}(B)\}$.

The functor $D(\alpha, s)(T)$ yielding a family of subsets of the carrier of T is defined as follows:

(Def.21) $D(\alpha, s)(T) = \{B : B$ ranges over subsets of the carrier of T, $\alpha\mathrm{Int}(B) = \mathrm{sInt}(B)\}$.

The functor $D(\alpha, ps)(T)$ yields a family of subsets of the carrier of T and is defined as follows:

(Def.22) $D(\alpha, ps)(T) = \{B : B$ ranges over subsets of the carrier of T, $\alpha\mathrm{Int}(B) = \mathrm{psInt}(B)\}$.

The functor $D(p, sp)(T)$ yielding a family of subsets of the carrier of T is defined by:

(Def.23) $D(p, sp)(T) = \{B : B$ ranges over subsets of the carrier of T, $\mathrm{pInt}(B) = \mathrm{spInt}(B)\}$.

The functor $D(p, ps)(T)$ yielding a family of subsets of the carrier of T is defined by:

(Def.24) $D(p, ps)(T) = \{B : B$ ranges over subsets of the carrier of T, $\mathrm{pInt}(B) = \mathrm{psInt}(B)\}$.

The functor $D(sp, ps)(T)$ yields a family of subsets of the carrier of T and is defined as follows:

(Def.25) $D(sp, ps)(T) = \{B : B$ ranges over subsets of the carrier of T, $\mathrm{spInt}(B) = \mathrm{psInt}(B)\}$.

In the sequel T will be a topological space and B will be a subset of the carrier of T.

One can prove the following propositions:

(1) $\alpha\mathrm{Int}(B) = \mathrm{pInt}(B)$ iff $\mathrm{sInt}(B) = \mathrm{psInt}(B)$.

(2) B is an α-set of T iff $B = \alpha\mathrm{Int}(B)$.

(3) B is semi-open iff $B = \mathrm{sInt}(B)$.

(4) B is pre-open iff $B = \mathrm{pInt}(B)$.

(5) B is pre-semi-open iff $B = \mathrm{psInt}(B)$.

(6) B is semi-pre-open iff $B = \mathrm{spInt}(B)$.

(7) $T^{\alpha} \cap D(c, \alpha)(T) =$ the topology of T.

(8) $\mathrm{SO}(T) \cap D(c, s)(T) =$ the topology of T.

(9) $\mathrm{PO}(T) \cap D(c, p)(T) =$ the topology of T.

(10) $\mathrm{PSO}(T) \cap D(c, ps)(T) =$ the topology of T.

(11) $\mathrm{PO}(T) \cap D(\alpha, p)(T) = T^{\alpha}$.

(12) $\mathrm{SO}(T) \cap D(\alpha, s)(T) = T^{\alpha}$.

(13) $\mathrm{PSO}(T) \cap D(\alpha, ps)(T) = T^{\alpha}$.

(14) $\mathrm{SPO}(T) \cap D(p, sp)(T) = \mathrm{PO}(T)$.

(15) $\mathrm{PSO}(T) \cap D(p, ps)(T) = \mathrm{PO}(T)$.

(16) $\mathrm{PSO}(T) \cap D(\alpha, p)(T) = \mathrm{SO}(T)$.

(17) $\mathrm{PSO}(T) \cap D(sp, ps)(T) = \mathrm{SPO}(T)$.

Let X, Y be topological spaces and let f be a mapping from X into Y. We say that f is s -continuous if and only if:

(Def.26) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in \mathrm{SO}(X)$.

We say that f is p -continuous if and only if:

(Def.27) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in \mathrm{PO}(X)$.

We say that f is α -continuous if and only if:

(Def.28) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in X^{\alpha}$.

We say that f is ps -continuous if and only if:

(Def.29) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in \mathrm{PSO}(X)$.

We say that f is sp -continuous if and only if:

(Def.30) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in \mathrm{SPO}(X)$.

We say that f is (c, α) -continuous if and only if:

(Def.31) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(c, \alpha)(X)$.

We say that f is (c, s) -continuous if and only if:

(Def.32) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(c, s)(X)$.

We say that f is (c, p) -continuous if and only if:

(Def.33) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(c, p)(X)$.

We say that f is (c, ps) -continuous if and only if:

(Def.34) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(c, ps)(X)$.

We say that f is (α, p) -continuous if and only if:

(Def.35) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(\alpha, p)(X)$.

We say that f is (α, s) -continuous if and only if:

(Def.36) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(\alpha, s)(X)$.

We say that f is (α, ps) -continuous if and only if:

(Def.37) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(\alpha, ps)(X)$.

We say that f is (p, ps) -continuous if and only if:

(Def.38) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(p, ps)(X)$.

We say that f is (p, sp) -continuous if and only if:

(Def.39) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(p, sp)(X)$.

We say that f is (sp, ps) -continuous if and only if:

(Def.40) For every subset G of the carrier of Y such that G is open holds $f^{-1}G \in D(sp, ps)(X)$.

In the sequel X, Y will denote topological spaces and f will denote a mapping from X into Y.

The following propositions are true:

(18) f is α -continuous iff f is p -continuous and (α, p) -continuous.

(19) f is α -continuous iff f is s -continuous and (α, s) -continuous.

(20) f is α -continuous iff f is ps -continuous and (α, ps) -continuous.

(21) f is p -continuous iff f is sp -continuous and (p, sp) -continuous.

(22) f is p -continuous iff f is ps -continuous and (p, ps) -continuous.

(23) f is s -continuous iff f is ps -continuous and (α, p) -continuous.

(24) f is sp -continuous iff f is ps -continuous and (sp, ps) -continuous.

(25) f is continuous iff f is α -continuous and (c, α) -continuous.

(26) f is continuous iff f is s -continuous and (c, s) -continuous.

(27) f is continuous iff f is p -continuous and (c, p) -continuous.

(28) f is continuous iff f is ps -continuous and (c, ps) -continuous.

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