FORMALIZED MATHEMATICS
Volume 5, Number 3, 1996
Warsaw University - Białystok

On the Go-Board of a Standard Special Circular Sequence

Andrzej Trybulec
Warsaw University
Białystok

MML Identifier: GOBOARD7.

The articles [21], [24], [5], [23], [9], [2], [19], [17], [1], [4], [3], [7], [22], [10], [11], [18], [25], [6], [8], [12], [13], [15], [20], [16], and [14] provide the terminology and notation for this paper.

1. Preliminaries

For simplicity we adopt the following convention: f will denote a standard special circular sequence, i, j, k, n, i_1, i_2, j_1, j_2 will denote natural numbers, r, s, r_1, r_2 will denote real numbers, p, q, p_1 will denote points of $\mathcal{E}^2_{\rm T}$, and G will denote a Go-board.

The following propositions are true:

(1) If $|r_1 - r_2| > s$, then $r_1 + s < r_2$ or $r_2 + s < r_1$.

(2) $|r - s| = 0$ iff $r = s$.

(3) For all points p, p_1, q of $\mathcal{E}^n_{\rm T}$ such that $p + p_1 = q + p_1$ holds $p = q$.

(4) For all points p, p_1, q of $\mathcal{E}^n_{\rm T}$ such that $p_1 + p = p_1 + q$ holds $p = q$.

(5) If $p_1 \in \mathcal{L}(p, q)$ and $p_{\bf 1} = q_{\bf 1}$, then $(p_1)_{\bf 1} = q_{\bf 1}$.

(6) If $p_1 \in \mathcal{L}(p, q)$ and $p_{\bf 2} = q_{\bf 2}$, then $(p_1)_{\bf 2} = q_{\bf 2}$.

(7) $\frac{1}{2} \cdot (p + q) \in \mathcal{L}(p, q)$.

(8) If $p_{\bf 1} = q_{\bf 1}$ and $q_{\bf 1} = (p_1)_{\bf 1}$ and $p_{\bf 2} \leq q_{\bf 2}$ and $q_{\bf 2} \leq (p_1)_{\bf 2}$, then $q \in \mathcal{L}(p, p_1)$.

(9) If $p_{\bf 1} \leq q_{\bf 1}$ and $q_{\bf 1} \leq (p_1)_{\bf 1}$ and $p_{\bf 2} = q_{\bf 2}$ and $q_{\bf 2} = (p_1)_{\bf 2}$, then $q \in \mathcal{L}(p, p_1)$.

(10) Let D be a non empty set, and given i, j, and let M be a matrix over D. If $1 \leq i$ and $i \leq \operatorname{len} M$ and $1 \leq j$ and $j \leq \operatorname{width} M$, then $\langle i, j \rangle \in$ the indices of M.

ISSN 1426–2630

(11) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\frac{1}{2} \cdot (G_{i,j} + G_{i+1,j+1}) = \frac{1}{2} \cdot (G_{i,j+1} + G_{i+1,j})$.

(12) Suppose $\mathcal{L}(f,k)$ is horizontal. Then there exists j such that $1 \leq j$ and $j \leq \operatorname{width}$ the Go-board of f and for every p such that $p \in \mathcal{L}(f,k)$ holds $p_{\mathbf{2}} = ((\text{the Go-board of } f)_{1,j})_{\mathbf{2}}$.

(13) Suppose $\mathcal{L}(f,k)$ is vertical. Then there exists i such that $1 \leq i$ and $i \leq \operatorname{len}$ the Go-board of f and for every p such that $p \in \mathcal{L}(f,k)$ holds $p_{\mathbf{1}} = ((\text{the Go-board of } f)_{i,1})_{\mathbf{1}}$.

(14) If $i \leq \operatorname{len}$ the Go-board of f and $j \leq \operatorname{width}$ the Go-board of f, then Int cell(the Go-board of f, i, j) misses $\widetilde{\mathcal{L}}(f)$.

2. Segments on a Go-Board

Next we state a number of propositions:

(15) If $1 \leq i$ and $i \leq \operatorname{len} G$ and $1 \leq j$ and $j+2 \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i,j+1}) \cap \mathcal{L}(G_{i,j+1}, G_{i,j+2}) = \{G_{i,j+1}\}$.

(16) If $1 \leq i$ and $i+2 \leq \operatorname{len} G$ and $1 \leq j$ and $j \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i+1,j}) \cap \mathcal{L}(G_{i+1,j}, G_{i+2,j}) = \{G_{i+1,j}\}$.

(17) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i,j+1}) \cap \mathcal{L}(G_{i,j+1}, G_{i+1,j+1}) = \{G_{i,j+1}\}$.

(18) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j+1}, G_{i+1,j+1}) \cap \mathcal{L}(G_{i+1,j}, G_{i+1,j+1}) = \{G_{i+1,j+1}\}$.

(19) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i+1,j}) \cap \mathcal{L}(G_{i,j}, G_{i,j+1}) = \{G_{i,j}\}$.

(20) If $1 \leq i$ and $i+1 \leq \operatorname{len} G$ and $1 \leq j$ and $j+1 \leq \operatorname{width} G$, then $\mathcal{L}(G_{i,j}, G_{i+1,j}) \cap \mathcal{L}(G_{i+1,j}, G_{i+1,j+1}) = \{G_{i+1,j}\}$.

(21) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2+1 \leq \operatorname{width} G$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1})$ meets $\mathcal{L}(G_{i_2,j_2}, G_{i_2,j_2+1})$. Then $i_1 = i_2$ and $|j_1 - j_2| \leq 1$.

(22) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1+1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1+1,j_1})$ meets $\mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2})$. Then $j_1 = j_2$ and $|i_1 - i_2| \leq 1$.

(23) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2+1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1})$ meets $\mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2})$. Then $i_1 = i_2$ or $i_1 = i_2+1$ but $j_1 = j_2$ or $j_1+1 = j_2$.

(24) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1+1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2+1 \leq \operatorname{width} G$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1})$ meets $\mathcal{L}(G_{i_2,j_2}, G_{i_2,j_2+1})$. Then

(i) $j_1 = j_2$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1}) = \mathcal{L}(G_{i_2,j_2}, G_{i_2,j_2+1})$, or
(ii) $j_1 = j_2 + 1$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1}) \cap \mathcal{L}(G_{i_2,j_2}, G_{i_2,j_2+1}) = \{G_{i_1,j_1}\}$, or
(iii) $j_1 + 1 = j_2$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1}) \cap \mathcal{L}(G_{i_2,j_2}, G_{i_2,j_2+1}) = \{G_{i_2,j_2}\}$.

(25) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 + 1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 + 1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1+1,j_1})$ meets $\mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2})$. Then
(i) $i_1 = i_2$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1+1,j_1}) = \mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2})$, or
(ii) $i_1 = i_2 + 1$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1+1,j_1}) \cap \mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2}) = \{G_{i_1,j_1}\}$, or
(iii) $i_1 + 1 = i_2$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1+1,j_1}) \cap \mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2}) = \{G_{i_2,j_2}\}$.

(26) Let i_1, j_1, i_2, j_2 be natural numbers. Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 + 1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 + 1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1})$ meets $\mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2})$. Then $j_1 = j_2$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1}) \cap \mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2}) = \{G_{i_1,j_1}\}$ or $j_1 + 1 = j_2$ and $\mathcal{L}(G_{i_1,j_1}, G_{i_1,j_1+1}) \cap \mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2}) = \{G_{i_1,j_1+1}\}$.

(27) Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 + 1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 + 1 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot (G_{i_1,j_1} + G_{i_1,j_1+1}) \in \mathcal{L}(G_{i_2,j_2}, G_{i_2,j_2+1})$. Then $i_1 = i_2$ and $j_1 = j_2$.

(28) Suppose $1 \leq i_1$ and $i_1 + 1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$ and $1 \leq i_2$ and $i_2 + 1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot (G_{i_1,j_1} + G_{i_1+1,j_1}) \in \mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2})$. Then $i_1 = i_2$ and $j_1 = j_2$.

(29) Suppose $1 \leq i_1$ and $i_1 + 1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 \leq \operatorname{width} G$. Then it is not true that there exist i_2, j_2 such that $1 \leq i_2$ and $i_2 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 + 1 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot (G_{i_1,j_1} + G_{i_1+1,j_1}) \in \mathcal{L}(G_{i_2,j_2}, G_{i_2,j_2+1})$.

(30) Suppose $1 \leq i_1$ and $i_1 \leq \operatorname{len} G$ and $1 \leq j_1$ and $j_1 + 1 \leq \operatorname{width} G$. Then it is not true that there exist i_2, j_2 such that $1 \leq i_2$ and $i_2 + 1 \leq \operatorname{len} G$ and $1 \leq j_2$ and $j_2 \leq \operatorname{width} G$ and $\frac{1}{2} \cdot (G_{i_1,j_1} + G_{i_1,j_1+1}) \in \mathcal{L}(G_{i_2,j_2}, G_{i_2+1,j_2})$.

3. Standard Special Circular Sequences

In the sequel f will be a non constant standard special circular sequence.

The following propositions are true:

(31) For every standard non empty finite sequence f of elements of $\mathcal{E}^2_{\mathrm{T}}$ such that $i \in \operatorname{dom} f$ and $i + 1 \in \operatorname{dom} f$ holds $\pi_i f \neq \pi_{i+1} f$.

(32) There exists i such that $i \in \operatorname{dom} f$ and $(\pi_i f)_{\mathbf{1}} \neq (\pi_1 f)_{\mathbf{1}}$.

(33) There exists i such that $i \in \operatorname{dom} f$ and $(\pi_i f)_{\mathbf{2}} \neq (\pi_1 f)_{\mathbf{2}}$.

(34) len the Go-board of $f > 1$.

(35) width the Go-board of $f > 1$.

(36) $\operatorname{len} f > 4$.

(37) Let f be a circular s.c.c. finite sequence of elements of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $\operatorname{len} f > 4$. Let i, j be natural numbers. If $1 \leq i$ and $i < j$ and $j < \operatorname{len} f$, then $\pi_i f \neq \pi_j f$.

(38) For all natural numbers i, j such that $1 \leq i$ and $i < j$ and $j < \operatorname{len} f$ holds $\pi_i f \neq \pi_j f$.

(39) For all natural numbers i, j such that $1 < i$ and $i < j$ and $j \leq \operatorname{len} f$ holds $\pi_i f \neq \pi_j f$.

(40) For every natural number i such that $1 < i$ and $i \leq \operatorname{len} f$ and $\pi_i f = \pi_1 f$ holds $i = \operatorname{len} f$.

(41) Suppose that
(i) $1 \leq i$,
(ii) $i \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+1 \leq \operatorname{width}$ the Go-board of f, and
(v) $\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i,j+1}) \in \widetilde{\mathcal{L}}(f)$.
Then there exists k such that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i,j+1}) = \mathcal{L}(f, k)$.

(42) Suppose that
(i) $1 \leq i$,
(ii) $i+1 \leq \operatorname{len}$ the Go-board of f
(iii) $1 \leq j$,
(iv) $j \leq \operatorname{width}$ the Go-board of f and
(v) $\frac{1}{2} \cdot (($the Go-board of $f)_{i,j} + ($the Go-board of $f)_{i+1,j}) \in \widetilde{\mathcal{L}}(f)$.
Then there exists k such that $1 \leq k$ and $k+1 \leq \operatorname{len} f$ and $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i+1,j}) = \mathcal{L}(f, k)$.

(43) Suppose that
(i) $1 \leq i$,
(ii) $i+1 \leq \operatorname{len}$ the Go-board of f
(iii) $1 \leq j$,
(iv) $j+1 \leq \operatorname{width}$ the Go-board of f
(v) $1 \leq k$,
(vi) $k+1 < \operatorname{len} f$,
(vii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k)$, and
(viii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k+1)$.
Then $\pi_k f = ($the Go-board of $f)_{i,j+1}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i+1,j+1}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i+1,j}$.

(44) Suppose that
(i) $1 \leq i$,
(ii) $i \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+1 < \operatorname{width}$ the Go-board of f,
(v) $1 \leq k$,
(vi) $k+1 < \operatorname{len} f$,
(vii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i,j+2}) = \mathcal{L}(f, k)$, and
(viii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i,j+1}) = \mathcal{L}(f, k+1)$.
Then $\pi_k f = ($the Go-board of $f)_{i,j+2}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i,j+1}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i,j}$.

(45) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq$ len the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq$ width the Go-board of f,

(v) $1 \leq k$,

(vi) $k+1 < \text{len}\, f$,

(vii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f,k)$, and

(viii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i,j+1}) = \mathcal{L}(f,k+1)$.

Then $\pi_k f =$ (the Go-board of $f)_{i+1,j+1}$ and $\pi_{k+1} f =$ (the Go-board of $f)_{i,j+1}$ and $\pi_{k+2} f =$ (the Go-board of $f)_{i,j}$.

(46) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq$ len the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq$ width the Go-board of f,

(v) $1 \leq k$,

(vi) $k+1 < \text{len}\, f$,

(vii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f,k)$, and

(viii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f,k+1)$.

Then $\pi_k f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j+1}$ and $\pi_{k+2} f =$ (the Go-board of $f)_{i,j+1}$.

(47) Suppose that

(i) $1 \leq i$,

(ii) $i+1 <$ len the Go-board of f,

(iii) $1 \leq j$,

(iv) $j \leq$ width the Go-board of f,

(v) $1 \leq k$,

(vi) $k+1 < \text{len}\, f$,

(vii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+2,j}) = \mathcal{L}(f,k)$, and

(viii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i+1,j}) = \mathcal{L}(f,k+1)$.

Then $\pi_k f =$ (the Go-board of $f)_{i+2,j}$ and $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_{k+2} f =$ (the Go-board of $f)_{i,j}$.

(48) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq$ len the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq$ width the Go-board of f,

(v) $1 \leq k$,

(vi) $k+1 < \text{len}\, f$,

(vii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f,k)$, and

(viii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i+1,j}) = \mathcal{L}(f,k+1)$.

Then $\pi_k f =$ (the Go-board of $f)_{i+1,j+1}$ and $\pi_{k+1} f =$ (the Go-board of $f)_{i+1,j}$ and $\pi_{k+2} f =$ (the Go-board of $f)_{i,j}$.

(49) Suppose that
(i) $1 \leq i$,
(ii) $i + 1 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j + 1 \leq \operatorname{width}$ the Go-board of f,
(v) $1 \leq k$,
(vi) $k + 1 < \operatorname{len} f$,
(vii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k)$, and
(viii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k+1)$.

Then $\pi_k f = ($the Go-board of $f)_{i+1,j}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i+1,j+1}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i,j+1}$.

(50) Suppose that
(i) $1 \leq i$,
(ii) $i \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j + 1 < \operatorname{width}$ the Go-board of f,
(v) $1 \leq k$,
(vi) $k + 1 < \operatorname{len} f$,
(vii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i,j+1}) = \mathcal{L}(f, k)$, and
(viii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i,j+2}) = \mathcal{L}(f, k+1)$.

Then $\pi_k f = ($the Go-board of $f)_{i,j}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i,j+1}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i,j+2}$.

(51) Suppose that
(i) $1 \leq i$,
(ii) $i + 1 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j + 1 \leq \operatorname{width}$ the Go-board of f,
(v) $1 \leq k$,
(vi) $k + 1 < \operatorname{len} f$,
(vii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i,j+1}) = \mathcal{L}(f, k)$, and
(viii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k+1)$.

Then $\pi_k f = ($the Go-board of $f)_{i,j}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i,j+1}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i+1,j+1}$.

(52) Suppose that
(i) $1 \leq i$,
(ii) $i + 1 \leq \operatorname{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j + 1 \leq \operatorname{width}$ the Go-board of f,
(v) $1 \leq k$,
(vi) $k + 1 < \operatorname{len} f$,
(vii) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k)$, and
(viii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k+1)$.

Then $\pi_k f = ($the Go-board of $f)_{i,j+1}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i+1,j+1}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i+1,j}$.

(53) Suppose that

(i) $1 \leq i$,

(ii) $i + 1 < \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j \leq \operatorname{width}$ the Go-board of f,

(v) $1 \leq k$,

(vi) $k + 1 < \operatorname{len} f$,

(vii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i+1,j}) = \mathcal{L}(f, k)$, and

(viii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+2,j}) = \mathcal{L}(f, k+1)$.

Then $\pi_k f = ($the Go-board of $f)_{i,j}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i+1,j}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i+2,j}$.

(54) Suppose that

(i) $1 \leq i$,

(ii) $i + 1 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j + 1 \leq \operatorname{width}$ the Go-board of f,

(v) $1 \leq k$,

(vi) $k + 1 < \operatorname{len} f$,

(vii) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i+1,j}) = \mathcal{L}(f, k)$, and

(viii) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+1,j+1}) = \mathcal{L}(f, k+1)$.

Then $\pi_k f = ($the Go-board of $f)_{i,j}$ and $\pi_{k+1} f = ($the Go-board of $f)_{i+1,j}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i+1,j+1}$.

(55) Suppose that

(i) $1 \leq i$,

(ii) $i \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j + 1 < \operatorname{width}$ the Go-board of f,

(v) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i,j+1}) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i,j+2}) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $\pi_1 f = ($the Go-board of $f)_{i,j+1}$ but $\pi_2 f = ($the Go-board of $f)_{i,j}$ and $\pi_{\operatorname{len} f -' 1} f = ($the Go-board of $f)_{i,j+2}$ or $\pi_2 f = ($the Go-board of $f)_{i,j+2}$ and $\pi_{\operatorname{len} f -' 1} f = ($the Go-board of $f)_{i,j}$, or

(viii) there exists k such that $1 \leq k$ and $k + 1 < \operatorname{len} f$ and $\pi_{k+1} f = ($the Go-board of $f)_{i,j+1}$ and $\pi_k f = ($the Go-board of $f)_{i,j}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i,j+2}$ or $\pi_k f = ($the Go-board of $f)_{i,j+2}$ and $\pi_{k+2} f = ($the Go-board of $f)_{i,j}$.

(56) Suppose that

(i) $1 \leq i$,

(ii) $i + 1 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j + 1 \leq \operatorname{width}$ the Go-board of f,

(v) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i,j+1}) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $\pi_1 f$ = (the Go-board of $f)_{i,j+1}$ but $\pi_2 f$ = (the Go-board of $f)_{i,j}$ and $\pi_{\operatorname{len} f -' 1} f$ = (the Go-board of $f)_{i+1,j+1}$ or $\pi_2 f$ = (the Go-board of $f)_{i+1,j+1}$ and $\pi_{\operatorname{len} f -' 1} f$ = (the Go-board of $f)_{i,j}$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\pi_{k+1} f$ = (the Go-board of $f)_{i,j+1}$ and $\pi_k f$ = (the Go-board of $f)_{i,j}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i+1,j+1}$ or $\pi_k f$ = (the Go-board of $f)_{i+1,j+1}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i,j}$.

(57) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq \operatorname{width}$ the Go-board of f,

(v) $\mathcal{L}(($the Go-board of $f)_{i,j+1}, ($the Go-board of $f)_{i+1,j+1}) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}(($the Go-board of $f)_{i+1,j+1}, ($the Go-board of $f)_{i+1,j}) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $\pi_1 f$ = (the Go-board of $f)_{i+1,j+1}$ but $\pi_2 f$ = (the Go-board of $f)_{i,j+1}$ and $\pi_{\operatorname{len} f -' 1} f$ = (the Go-board of $f)_{i+1,j}$ or $\pi_2 f$ = (the Go-board of $f)_{i+1,j}$ and $\pi_{\operatorname{len} f -' 1} f$ = (the Go-board of $f)_{i,j+1}$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\pi_{k+1} f$ = (the Go-board of $f)_{i+1,j+1}$ and $\pi_k f$ = (the Go-board of $f)_{i,j+1}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i+1,j}$ or $\pi_k f$ = (the Go-board of $f)_{i+1,j}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i,j+1}$.

(58) Suppose that

(i) $1 \leq i$,

(ii) $i+1 < \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j \leq \operatorname{width}$ the Go-board of f,

(v) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i+1,j}) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}(($the Go-board of $f)_{i+1,j}, ($the Go-board of $f)_{i+2,j}) \subseteq \widetilde{\mathcal{L}}(f)$.

Then

(vii) $\pi_1 f$ = (the Go-board of $f)_{i+1,j}$ but $\pi_2 f$ = (the Go-board of $f)_{i,j}$ and $\pi_{\operatorname{len} f -' 1} f$ = (the Go-board of $f)_{i+2,j}$ or $\pi_2 f$ = (the Go-board of $f)_{i+2,j}$ and $\pi_{\operatorname{len} f -' 1} f$ = (the Go-board of $f)_{i,j}$, or

(viii) there exists k such that $1 \leq k$ and $k+1 < \operatorname{len} f$ and $\pi_{k+1} f$ = (the Go-board of $f)_{i+1,j}$ and $\pi_k f$ = (the Go-board of $f)_{i,j}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i+2,j}$ or $\pi_k f$ = (the Go-board of $f)_{i+2,j}$ and $\pi_{k+2} f$ = (the Go-board of $f)_{i,j}$.

(59) Suppose that

(i) $1 \leq i$,

(ii) $i+1 \leq \operatorname{len}$ the Go-board of f,

(iii) $1 \leq j$,

(iv) $j+1 \leq \operatorname{width}$ the Go-board of f,

(v) $\mathcal{L}(($the Go-board of $f)_{i,j}, ($the Go-board of $f)_{i+1,j}) \subseteq \widetilde{\mathcal{L}}(f)$, and

(vi) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j}, (\text{the Go-board of } f)_{i+1,j+1}) \subseteq \widetilde{\mathcal{L}}(f)$.
Then
(vii) $\pi_1 f = (\text{the Go-board of } f)_{i+1,j}$ but $\pi_2 f = (\text{the Go-board of } f)_{i,j}$ and $\pi_{\text{len} f -' 1} f = (\text{the Go-board of } f)_{i+1,j+1}$ or $\pi_2 f = (\text{the Go-board of } f)_{i+1,j+1}$ and $\pi_{\text{len} f -' 1} f = (\text{the Go-board of } f)_{i,j}$, or
(viii) there exists k such that $1 \leq k$ and $k+1 < \text{len} f$ and $\pi_{k+1} f = (\text{the Go-board of } f)_{i+1,j}$ and $\pi_k f = (\text{the Go-board of } f)_{i,j}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i+1,j+1}$ or $\pi_k f = (\text{the Go-board of } f)_{i+1,j+1}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i,j}$.

(60) Suppose that
(i) $1 \leq i$,
(ii) $i+1 \leq \text{len}$ the Go-board of f,
(iii) $1 \leq j$,
(iv) $j+1 \leq \text{width}$ the Go-board of f,
(v) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j}, (\text{the Go-board of } f)_{i+1,j+1}) \subseteq \widetilde{\mathcal{L}}(f)$, and
(vi) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j+1}, (\text{the Go-board of } f)_{i,j+1}) \subseteq \widetilde{\mathcal{L}}(f)$.
Then
(vii) $\pi_1 f = (\text{the Go-board of } f)_{i+1,j+1}$ but $\pi_2 f = (\text{the Go-board of } f)_{i+1,j}$ and $\pi_{\text{len} f -' 1} f = (\text{the Go-board of } f)_{i,j+1}$ or $\pi_2 f = (\text{the Go-board of } f)_{i,j+1}$ and $\pi_{\text{len} f -' 1} f = (\text{the Go-board of } f)_{i+1,j}$, or
(viii) there exists k such that $1 \leq k$ and $k+1 < \text{len} f$ and $\pi_{k+1} f = (\text{the Go-board of } f)_{i+1,j+1}$ and $\pi_k f = (\text{the Go-board of } f)_{i+1,j}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i,j+1}$ or $\pi_k f = (\text{the Go-board of } f)_{i,j+1}$ and $\pi_{k+2} f = (\text{the Go-board of } f)_{i+1,j}$.

(61) Suppose $1 \leq i$ and $i < \text{len}$ the Go-board of f and $1 \leq j$ and $j+1 <$ width the Go-board of f. Then
(i) $\mathcal{L}((\text{the Go-board of } f)_{i,j}, (\text{the Go-board of } f)_{i,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or
(ii) $\mathcal{L}((\text{the Go-board of } f)_{i,j+1}, (\text{the Go-board of } f)_{i,j+2}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or
(iii) $\mathcal{L}((\text{the Go-board of } f)_{i,j+1}, (\text{the Go-board of } f)_{i+1,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$.

(62) Suppose $1 \leq i$ and $i < \text{len}$ the Go-board of f and $1 \leq j$ and $j+1 <$ width the Go-board of f. Then
(i) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j}, (\text{the Go-board of } f)_{i+1,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or
(ii) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j+1}, (\text{the Go-board of } f)_{i+1,j+2}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or
(iii) $\mathcal{L}((\text{the Go-board of } f)_{i,j+1}, (\text{the Go-board of } f)_{i+1,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$.

(63) Suppose $1 \leq j$ and $j < \text{width}$ the Go-board of f and $1 \leq i$ and $i+1 <$ len the Go-board of f. Then
(i) $\mathcal{L}((\text{the Go-board of } f)_{i,j}, (\text{the Go-board of } f)_{i+1,j}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or
(ii) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j}, (\text{the Go-board of } f)_{i+2,j}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or
(iii) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j}, (\text{the Go-board of } f)_{i+1,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$.

(64) Suppose $1 \leq j$ and $j < \text{width}$ the Go-board of f and $1 \leq i$ and $i+1 <$ len the Go-board of f. Then
(i) $\mathcal{L}((\text{the Go-board of } f)_{i,j+1}, (\text{the Go-board of } f)_{i+1,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or
(ii) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j+1}, (\text{the Go-board of } f)_{i+2,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$, or

(iii) $\mathcal{L}((\text{the Go-board of } f)_{i+1,j}, (\text{the Go-board of } f)_{i+1,j+1}) \not\subseteq \widetilde{\mathcal{L}}(f)$.

References

[1] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.

[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.

[4] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[5] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.

[6] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.

[7] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.

[8] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\rm T}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.

[9] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.

[10] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.

[11] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Formalized Mathematics*, 1(**3**):607–610, 1990.

[12] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part I. *Formalized Mathematics*, 3(**1**):107–115, 1992.

[13] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part II. *Formalized Mathematics*, 3(**1**):117–121, 1992.

[14] Rafał Kwiatek and Grzegorz Zwara. The divisibility of integers and integer relative primes. *Formalized Mathematics*, 1(**5**):829–832, 1990.

[15] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons, part I. *Formalized Mathematics*, 5(**1**):97–102, 1996.

[16] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-Board into cells. *Formalized Mathematics*, 5(**3**):323–328, 1996.

[17] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.

[18] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.

[19] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.

[20] Andrzej Trybulec. On the decomposition of finite sequences. *Formalized Mathematics*, 5(**3**):317–322, 1996.

[21] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.

[22] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.

[23] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[24] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.

[25] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.

Received October 15, 1995