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On the Concept of the Triangulation

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The terminology and notation used in this paper have been introduced in the following articles: [22], [28], [11], [29], [31], [30], [27], [14], [2], [9], [10], [6], [19], [13], [5], [8], [25], [23], [3], [4], [12], [26], [15], [17], [18], [1], [21], [20], [24], [16], and [7].

1. Introduction

In this paper A will be a set and k, m, n will be natural numbers.

The scheme *Regr1* concerns a natural number $\mathcal{A}$ and a unary predicate $\mathcal{P}$, and states that:

For every k such that $k \leq \mathcal{A}$ holds $\mathcal{P}[k]$

provided the parameters meet the following conditions:

- $\mathcal{P}[\mathcal{A}]$,
- For every k such that $k < \mathcal{A}$ and $\mathcal{P}[k+1]$ holds $\mathcal{P}[k]$.

Let n be a natural number. Observe that $\mathrm{Seg}(n+1)$ is non empty.

Let X be a non empty set and let R be an order in X. Note that $\langle X, R \rangle$ is non empty.

One can prove the following proposition

(1) $\emptyset \mid^2 A = \emptyset$.

Let X be a set. Note that there exists a subset of $\mathrm{Fin}\, X$ which is non empty.

Let X be a non empty set. Note that there exists a subset of $\mathrm{Fin}\, X$ which is non empty and has non empty elements.

Let X be a non empty set and let F be a non empty subset of $\mathrm{Fin}\, X$ with non empty elements. Observe that there exists an element of F which is non empty.

A set has a non-empty element if:

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(Def.1) There exists a non empty set X such that $X \in$ it.

Let us mention that there exists a set which has a non-empty element.

Let X be a set with a non-empty element. Note that there exists an element of X which is non empty.

One can check that every set which has a non-empty element is non empty.

Let X be a non empty set. Note that there exists a subset of $\mathrm{Fin}\, X$ which has a non-empty element.

Let X be a non empty set, let F be a subset of $\mathrm{Fin}\, X$ with a non-empty element, let R be an order in X, and let A be an element of F. Then $R\,|^2\, A$ is an order in A.

The scheme *SubFinite* concerns a set $\mathcal{A}$, a subset $\mathcal{B}$ of $\mathcal{A}$, and a unary predicate $\mathcal{P}$, and states that:

$\mathcal{P}[\mathcal{B}]$

provided the following conditions are satisfied:

- $\mathcal{B}$ is finite,
- $\mathcal{P}[\emptyset_{\mathcal{A}}]$,
- For every element x of $\mathcal{A}$ and for every subset B of $\mathcal{A}$ such that $x \in \mathcal{B}$ and $B \subseteq \mathcal{B}$ and $\mathcal{P}[B]$ holds $\mathcal{P}[B \cup \{x\}]$.

We now state the proposition

(2) Let F be a non empty poset and let A be a subset of F. Suppose A is finite and $A \neq \emptyset$ and for all elements B, C of F such that $B \in A$ and $C \in A$ holds $B \leq C$ or $C \leq B$. Then there exists an element m of F such that $m \in A$ and for every element C of F such that $C \in A$ holds $m \leq C$.

Let X be a non empty set and let F be a subset of $\mathrm{Fin}\, X$ with a non-empty element. Observe that there exists an element of F which is finite and non empty.

Let A be a non empty poset and let a_1, a_2 be elements of A. We introduce $a_2 \geq a_1$ as a synonym of $a_1 \leq a_2$ We introduce $a_2 > a_1$ as a synonym of $a_1 < a_2$.

Let P be a non empty poset. Note that there exists a subset of P which is non empty and finite.

Let P be a non empty poset, let A be a non empty finite subset of P, and let x be an element of P. One can check that $\mathrm{InitSegm}(A, x)$ is finite.

The following proposition is true

(3) For all finite sets A, B such that $A \subseteq B$ and $\mathrm{card}\, A = \mathrm{card}\, B$ holds $A = B$.

Let A, B be non empty sets, let f be a function from A into B, and let x be an element of A. Then $f(x)$ is an element of B.

Let F be a non empty poset and let A be a non empty subset of F. We see that the element of A is an element of F.

Let X be a non empty set, let F be a subset of $\mathrm{Fin}\, X$ with a non-empty element, let A be a non empty element of F, and let R be an order in X. Let us assume that R linearly orders A. The functor $\mathrm{SgmX}(R, A)$ yields a finite sequence of elements of the carrier of $\langle A, R\,|^2\, A\rangle$ and is defined by the conditions (Def.2).

(Def.2) (i) $\operatorname{rng} \operatorname{SgmX}(R, A) = A$, and

(ii) for all natural numbers n, m and for all elements p, q of $\langle A, R |^2 A\rangle$ such that $n \in \operatorname{dom} \operatorname{SgmX}(R, A)$ and $m \in \operatorname{dom} \operatorname{SgmX}(R, A)$ and $n < m$ and $p = \pi_n \operatorname{SgmX}(R, A)$ and $q = \pi_m \operatorname{SgmX}(R, A)$ holds $p > q$.

Next we state the proposition

(4) Let X be a non empty set, and let F be a subset of $\operatorname{Fin} X$ with a non-empty element, and let A be a non empty element of F, and let R be an order in X, and let f be a finite sequence of elements of the carrier of $\langle X, R\rangle$. Suppose that

(i) $\operatorname{rng} f = A$, and

(ii) for all natural numbers n, m and for all elements p, q of $\langle X, R\rangle$ such that $n \in \operatorname{dom} f$ and $m \in \operatorname{dom} f$ and $n < m$ and $p = \pi_n f$ and $q = \pi_m f$ holds $p > q$.

Then $f = \operatorname{SgmX}(R, A)$.

2. Abstract Complexes

Let C be a non empty poset. The functor symplexes(C) yields a subset of Fin (the carrier of C) and is defined by:

(Def.3) symplexes(C) $= \{A : A$ ranges over elements of Fin (the carrier of C), the internal relation of C linearly orders $A\}$.

Let C be a non empty poset. Note that symplexes(C) has a non-empty element.

In the sequel C denotes a non empty poset.

Next we state three propositions:

(5) For every element x of C holds $\{x\} \in$ symplexes(C).

(6) $\emptyset \in$ symplexes(C).

(7) For arbitrary x, s such that $x \subseteq s$ and $s \in$ symplexes(C) holds $x \in$ symplexes(C).

Let us consider C. Observe that every element of symplexes(C) is finite.

One can prove the following propositions:

(8) For every non empty poset C and for every non empty element A of symplexes(C) holds SgmX(the internal relation of C, A) is one-to-one.

(9) Let C be a non empty poset and let A be a non empty element of symplexes(C). If $\overline{\overline{A}} = n$, then len SgmX(the internal relation of C, A) $= n$.

(10) Let C be a non empty poset and let A be a non empty element of symplexes(C). If $\overline{\overline{A}} = n$, then dom SgmX(the internal relation of C, A) $= \operatorname{Seg} n$.

Let C be a non empty poset. One can verify that there exists an element of symplexes(C) which is non empty.

3. Triangulations

A set sequence is a many sorted set indexed by $\mathbb{N}$.

A set sequence is lower non-empty if:

(Def.4) For every n such that it(n) is non empty and for every m such that $m < n$ holds it(m) is non empty.

Let us observe that there exists a set sequence which is lower non-empty.

Let X be a set sequence. The functor FuncsSeq(X) yields a set sequence and is defined by:

(Def.5) For every natural number n holds $(\mathrm{FuncsSeq}(X))(n) = X(n)^{X(n+1)}$.

Let X be a lower non-empty set sequence and let n be a natural number. Observe that $(\mathrm{FuncsSeq}(X))(n)$ is non empty.

Let us consider n and let f be an element of $(\mathrm{Seg}(n+1))^{\mathrm{Seg}\,n}$. The functor ${}^{@}f$ yields a finite sequence of elements of $\mathbb{R}$ and is defined as follows:

(Def.6) ${}^{@}f = f$.

The set sequence NatEmbSeq is defined by:

(Def.7) For every natural number n holds $(\mathrm{NatEmbSeq})(n) = \{f : f$ ranges over elements of $(\mathrm{Seg}(n+1))^{\mathrm{Seg}\,n}$, ${}^{@}f$ is increasing$\}$.

Let us consider n. Observe that $(\mathrm{NatEmbSeq})(n)$ is non empty.

Let n be a natural number. Note that every element of NatEmbSeq(n) is function-like and relation-like.

Let X be a set sequence.

(Def.8) A many sorted function from NatEmbSeq into FuncsSeq(X) is called a triangulation of X.

We consider triangulation structures as systems

$\langle$ a skeleton sequence, a faces assignment $\rangle$,

where the skeleton sequence is a set sequence and the faces assignment is a many sorted function from NatEmbSeq into FuncsSeq(the skeleton sequence).

Let T be a triangulation structure. We say that T is lower non-empty if and only if:

(Def.9) The skeleton sequence of T is lower non-empty.

Let us note that there exists a triangulation structure which is lower non-empty and strict.

Let T be a lower non-empty triangulation structure. Note that the skeleton sequence of T is lower non-empty.

Let S be a lower non-empty set sequence and let F be a many sorted function from NatEmbSeq into FuncsSeq(S). Note that $\langle S, F\rangle$ is lower non-empty.

4. Relationship Between Abstract Complexes and Triangulations

Let T be a triangulation structure and let n be a natural number. A symplex of T and n is an element of (the skeleton sequence of T)(n).

Let n be a natural number. A face of n is an element of (NatEmbSeq)(n).

Let T be a lower non-empty triangulation structure, let n be a natural number, let x be a symplex of T and $n+1$, and let f be a face of n. Let us assume that (the skeleton sequence of T)($n+1$) $\neq \emptyset$. The functor face(x, f) yields a symplex of T and n and is defined by:

(Def.10) For all functions F, G such that $F =$ (the faces assignment of T)(n) and $G = F(f)$ holds face(x, f) $= G(x)$.

Let C be a non empty poset. The functor Triang(C) yielding a lower non-empty strict triangulation structure is defined by the conditions (Def.11).

(Def.11) (i) (The skeleton sequence of Triang(C))(0) $= \{\emptyset\}$,

(ii) for every natural number n such that $n > 0$ holds (the skeleton sequence of Triang(C))(n) $=$ {SgmX(the internal relation of C, A) : A ranges over non empty elements of symplexes(C), $\overline{\overline{A}} = n$}, and

(iii) for every natural number n and for every face f of n and for every element s of (the skeleton sequence of Triang(C))($n+1$) such that $s \in$ (the skeleton sequence of Triang(C))($n+1$) and for every non empty element A of symplexes(C) such that SgmX(the internal relation of C, A) $= s$ holds face(s, f) $=$ SgmX(the internal relation of C, A) $\cdot f$.

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