FORMALIZED MATHEMATICS
Volume 5, Number 4, 1996
Warsaw University - Białystok

Left and Right Component of the Complement of a Special Closed Curve

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Summary. In the article the concept of the left and right component are introduced. These are the auxiliary notions needed in the proof of Jordan Curve Theorem.

MML Identifier: GOBOARD9.

The articles [23], [26], [7], [25], [11], [2], [21], [18], [27], [6], [5], [3], [24], [12], [1], [13], [20], [28], [19], [4], [9], [10], [14], [15], [16], [8], [22], and [17] provide the notation and terminology for this paper.

For simplicity we adopt the following rules: f will denote a non constant standard special circular sequence, i, j, k will denote natural numbers, p, q will denote points of $\mathcal{E}^2_{\rm T}$, and G will denote a Go-board.

The following propositions are true:

(1) $i -' i = 0$.

(2) $i -' j \leq i$.

(3) Let G_1 be a non empty topological space and let A_1, A_2, B be subsets of the carrier of G_1. Suppose A_1 is a component of B and A_2 is a component of B. Then $A_1 = A_2$ or A_1 misses A_2.

(4) Let G_1 be a non empty topological space, and let A, B be non empty subsets of the carrier of G_1, and let A_3 be a subset of the carrier of $G_1 \restriction B$.If $A = A_3$, then $G_1 \restriction A = G_1 \restriction B \restriction A_3$.

(5) Let G_1 be a non empty topological space and let A, B be non empty subsets of the carrier of G_1. Suppose $A \subseteq B$ and A is connected. Then there exists a subset C of the carrier of G_1 such that C is a component of B and $A \subseteq C$.

(6) Let G_1 be a non empty topological space and let A, B, C, D be subsets of the carrier of G_1. Suppose B is connected and C is a component of D and $A \subseteq C$ and A meets B and $B \subseteq D$. Then $B \subseteq C$.

ISSN 1426–2630

(7) $\mathcal{L}(p, q)$ is convex.

(8) $\mathcal{L}(p, q)$ is connected.

One can check that there exists a subset of the carrier of $\mathcal{E}^2_{\rm T}$ which is convex.

One can prove the following three propositions:

(9) For all convex subsets P, Q of the carrier of $\mathcal{E}^2_{\rm T}$ holds $P \cap Q$ is convex.

(10) For every finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds Rev(**X**-coordinate(f)) = **X**-coordinate(Rev(f)).

(11) For every finite sequence f of elements of $\mathcal{E}^2_{\rm T}$ holds Rev(**Y**-coordinate(f)) = **Y**-coordinate(Rev(f)).

Let us mention that there exists a finite sequence which is non constant.

Let f be a non constant finite sequence. Note that Rev(f) is non constant.

Let f be a standard special circular sequence. Then Rev(f) is a standard special circular sequence.

We now state a number of propositions:

(12) If $i \geq 1$ and $j \geq 1$ and $i + j = \operatorname{len} f$, then leftcell($f, i$) = rightcell(Rev($f$), j).

(13) If $i \geq 1$ and $j \geq 1$ and $i + j = \operatorname{len} f$, then leftcell(Rev($f$), i) = rightcell(f, j).

(14) Suppose $1 \leq k$ and $k + 1 \leq \operatorname{len} f$. Then there exist i, j such that $i \leq \operatorname{len}$ the Go-board of f and $j \leq \operatorname{width}$ the Go-board of f and cell(the Go-board of f, i, j) = leftcell(f, k).

(15) If $j \leq \operatorname{width} G$, then Int hstrip($G, j$) is convex.

(16) If $i \leq \operatorname{len} G$, then Int vstrip($G, i$) is convex.

(17) If $i \leq \operatorname{len} G$ and $j \leq \operatorname{width} G$, then Int cell($G, i, j$) $\neq \emptyset$.

(18) If $1 \leq k$ and $k + 1 \leq \operatorname{len} f$, then Int leftcell($f, k$) $\neq \emptyset$.

(19) If $1 \leq k$ and $k + 1 \leq \operatorname{len} f$, then Int rightcell($f, k$) $\neq \emptyset$.

(20) If $i \leq \operatorname{len} G$ and $j \leq \operatorname{width} G$, then Int cell($G, i, j$) is convex.

(21) If $i \leq \operatorname{len} G$ and $j \leq \operatorname{width} G$, then Int cell($G, i, j$) is connected.

(22) If $1 \leq k$ and $k + 1 \leq \operatorname{len} f$, then Int leftcell($f, k$) is connected.

(23) If $1 \leq k$ and $k + 1 \leq \operatorname{len} f$, then Int rightcell($f, k$) is connected.

Let us consider f. The functor LeftComp(f) yields a subset of the carrier of $\mathcal{E}^2_{\rm T}$ and is defined as follows:

(Def. 1) LeftComp(f) is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and Int leftcell(f, 1) $\subseteq$ LeftComp(f).

The functor RightComp(f) yields a subset of the carrier of $\mathcal{E}^2_{\rm T}$ and is defined by:

(Def. 2) RightComp(f) is a component of $(\widetilde{\mathcal{L}}(f))^{\rm c}$ and Int rightcell(f, 1) $\subseteq$ RightComp(f).

One can prove the following propositions:

(24) For every k such that $1 \leq k$ and $k + 1 \leq \operatorname{len} f$ holds Int leftcell(f, k) $\subseteq$ LeftComp(f).

(25) The Go-board of $\mathrm{Rev}(f) =$ the Go-board of f.

(26) $\mathrm{RightComp}(f) = \mathrm{LeftComp}(\mathrm{Rev}(f))$.

(27) $\mathrm{RightComp}(\mathrm{Rev}(f)) = \mathrm{LeftComp}(f)$.

(28) For every k such that $1 \leq k$ and $k+1 \leq \mathrm{len}\, f$ holds $\mathrm{Int}\,\mathrm{rightcell}(f,k) \subseteq \mathrm{RightComp}(f)$.

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Received October 29, 1995