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An Extension of SCM

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The articles [19], [25], [9], [20], [11], [14], [2], [18], [26], [6], [7], [17], [16], [22], [3], [8], [10], [23], [1], [15], [5], [24], [12], [13], [21], and [4] provide the notation and terminology for this paper.

In this paper x will be arbitrary and k will denote a natural number.

The subset $\text{Data-Loc}_{\text{SCM}_{\text{FSA}}}$ of $\mathbb{Z}$ is defined as follows:

(Def. 1) $\text{Data-Loc}_{\text{SCM}_{\text{FSA}}} = \text{Data-Loc}_{\text{SCM}}$.

The subset $\text{Data}^*\text{-Loc}_{\text{SCM}_{\text{FSA}}}$ of $\mathbb{Z}$ is defined as follows:

(Def. 2) $\text{Data}^*\text{-Loc}_{\text{SCM}_{\text{FSA}}} = \mathbb{Z} \setminus \mathbb{N}$.

The subset $\text{Instr-Loc}_{\text{SCM}_{\text{FSA}}}$ of $\mathbb{Z}$ is defined as follows:

(Def. 3) $\text{Instr-Loc}_{\text{SCM}_{\text{FSA}}} = \text{Instr-Loc}_{\text{SCM}}$.

One can check the following observations:

* $\text{Data}^*\text{-Loc}_{\text{SCM}_{\text{FSA}}}$ is non empty,
* $\text{Data-Loc}_{\text{SCM}_{\text{FSA}}}$ is non empty, and
* $\text{Instr-Loc}_{\text{SCM}_{\text{FSA}}}$ is non empty.

For simplicity we adopt the following convention: J, K are elements of $\mathbb{Z}_{13}$, a is an element of $\text{Instr-Loc}_{\text{SCM}_{\text{FSA}}}$, b, c, c_1 are elements of $\text{Data-Loc}_{\text{SCM}_{\text{FSA}}}$, and f, f_1 are elements of $\text{Data}^*\text{-Loc}_{\text{SCM}_{\text{FSA}}}$.

The subset $\text{Instr}_{\text{SCM}_{\text{FSA}}}$ of $[\!:\mathbb{Z}_{13}, (\bigcup\{\mathbb{Z}, \mathbb{Z}^*\} \cup \mathbb{Z})^*:\!]$ is defined by:

(Def. 4) $\text{Instr}_{\text{SCM}_{\text{FSA}}} = \text{Instr}_{\text{SCM}} \cup \{\langle J, \langle c, f, b\rangle\rangle : J \in \{9, 10\}\} \cup \{\langle K, \langle c_1, f_1\rangle\rangle : K \in \{11, 12\}\}$.

The following two propositions are true:

(1) $\text{Instr}_{\text{SCM}_{\text{FSA}}} = \text{Instr}_{\text{SCM}} \cup \{\langle J, \langle c, f, b\rangle\rangle : J \in \{9, 10\}\} \cup \{\langle K, \langle c_1, f_1\rangle\rangle : K \in \{11, 12\}\}$.

(2) $\text{Instr}_{\text{SCM}} \subseteq \text{Instr}_{\text{SCM}_{\text{FSA}}}$.

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Let us observe that $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is non empty.

Let I be an element of $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$. The functor $\mathrm{InsCode}(I)$ yielding a natural number is defined by:

(Def. 5) $\mathrm{InsCode}(I) = I_{\mathbf{1}}$.

The following two propositions are true:

(3) For every element I of $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ such that $\mathrm{InsCode}(I) \leq 8$ holds $I \in \mathrm{Instr}_{\mathrm{SCM}}$.

(4) $\langle 0, \varepsilon \rangle \in \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

The function $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}$ from $\mathbb{Z}$ into $\{\mathbb{Z}, \mathbb{Z}^*\} \cup \{\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}, \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}\}$ is defined by:

(Def. 6) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}} = (\mathbb{Z} \longmapsto \mathbb{Z}^*) {+\cdot} \mathrm{OK}_{\mathrm{SCM}} {+\cdot} (\mathrm{Instr}_{\mathrm{SCM}} \dot{\longmapsto} \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}) \cdot (\mathrm{OK}_{\mathrm{SCM}} \restriction \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}})$.

One can prove the following propositions:

(5) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}} = (\mathbb{Z} \longmapsto \mathbb{Z}^*) {+\cdot} \mathrm{OK}_{\mathrm{SCM}} {+\cdot} (\mathrm{Instr}_{\mathrm{SCM}} \dot{\longmapsto} \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}) \cdot (\mathrm{OK}_{\mathrm{SCM}} \restriction \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}})$.

(6) If $x \in \{9, 10\}$, then $\langle x, \langle c, f, b \rangle \rangle \in \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(7) If $x \in \{11, 12\}$, then $\langle x, \langle c, f \rangle \rangle \in \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(8) $\mathbb{Z} = \{0\} \cup \mathrm{Data\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \cup \mathrm{Data}^*\text{-}\mathrm{Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \cup \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(9) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(0) = \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(10) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(b) = \mathbb{Z}$.

(11) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(a) = \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(12) $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(f) = \mathbb{Z}^*$.

(13) $\mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}$ and $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}$ and $\mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ and $\mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}^*$ and $\mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}} \neq \mathbb{Z}^*$.

(14) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ holds $i = 0$.

(15) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \mathbb{Z}$ holds $i \in \mathrm{Data\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(16) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \mathrm{Instr}_{\mathrm{SCM}_{\mathrm{FSA}}}$ holds $i \in \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

(17) For every integer i such that $\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}}(i) = \mathbb{Z}^*$ holds $i \in \mathrm{Data}^*\text{-}\mathrm{Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$.

An $\mathbf{SCM}_{\mathrm{FSA}}$-state is an element of $\prod(\mathrm{OK}_{\mathrm{SCM}_{\mathrm{FSA}}})$.

Next we state two propositions:

(18) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every element I of $\mathrm{Instr}_{\mathrm{SCM}}$ holds $s \restriction \mathbb{N} {+\cdot} (\mathrm{Instr\text{-}Loc}_{\mathrm{SCM}} \longmapsto I)$ is a state $_{\mathrm{SCM}}$.

(19) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every state $_{\mathrm{SCM}}$ s' holds $s {+\cdot} s' {+\cdot} s \restriction \mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$ is an $\mathbf{SCM}_{\mathrm{FSA}}$-state.

In the sequel s is an $\mathbf{SCM}_{\mathrm{FSA}}$-state.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state and let u be an element of $\mathrm{Instr\text{-}Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$. The functor $\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s, u)$ yields an $\mathbf{SCM}_{\mathrm{FSA}}$-state and is defined as follows:

(Def. 7) $\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s,u) = s{+\cdot}(0 \dot{\longmapsto} u)$.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, let t be an element of $\text{Data-Loc}_{\mathrm{SCM_{FSA}}}$, and let u be an integer. The functor $\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s,t,u)$ yielding an $\mathbf{SCM}_{\mathrm{FSA}}$-state is defined as follows:

(Def. 8) $\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s,t,u) = s{+\cdot}(t \dot{\longmapsto} u)$.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, let t be an element of $\text{Data}^{*}\text{-Loc}_{\mathrm{SCM_{FSA}}}$, and let u be a finite sequence of elements of $\mathbb{Z}$ The functor $\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s,t,u)$ yielding an $\mathbf{SCM}_{\mathrm{FSA}}$-state is defined as follows:

(Def. 9) $\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s,t,u) = s{+\cdot}(t \dot{\longmapsto} u)$.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state and let a be an element of $\text{Data-Loc}_{\mathrm{SCM_{FSA}}}$. Then $s(a)$ is an integer.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state and let a be an element of $\text{Data}^{*}\text{-Loc}_{\mathrm{SCM_{FSA}}}$. Then $s(a)$ is a finite sequence of elements of $\mathbb{Z}$.

Let x be an element of $\mathrm{Instr}_{\mathrm{SCM_{FSA}}}$. Let us assume that there exist c, f, b, J such that $x = \langle\!\langle J, \langle c, f, b\rangle\rangle\!\rangle$. The functor x int-addr$_1$ yielding an element of $\text{Data-Loc}_{\mathrm{SCM_{FSA}}}$ is defined by:

(Def. 10) There exist c, f, b such that $\langle c, f, b\rangle = x_{\mathbf{2}}$ and x int-addr$_1 = c$.

The functor x int-addr$_2$ yielding an element of $\text{Data-Loc}_{\mathrm{SCM_{FSA}}}$ is defined as follows:

(Def. 11) There exist c, f, b such that $\langle c, f, b\rangle = x_{\mathbf{2}}$ and x int-addr$_2 = b$.

The functor x coll-addr$_1$ yields an element of $\text{Data}^{*}\text{-Loc}_{\mathrm{SCM_{FSA}}}$ and is defined as follows:

(Def. 12) There exist c, f, b such that $\langle c, f, b\rangle = x_{\mathbf{2}}$ and x coll-addr$_1 = f$.

Let x be an element of $\mathrm{Instr}_{\mathrm{SCM_{FSA}}}$. Let us assume that there exist c, f, J such that $x = \langle\!\langle J, \langle c, f\rangle\rangle\!\rangle$. The functor x int-addr$_3$ yielding an element of $\text{Data-Loc}_{\mathrm{SCM_{FSA}}}$ is defined as follows:

(Def. 13) There exist c, f such that $\langle c, f\rangle = x_{\mathbf{2}}$ and x int-addr$_3 = c$.

The functor x coll-addr$_2$ yields an element of $\text{Data}^{*}\text{-Loc}_{\mathrm{SCM_{FSA}}}$ and is defined as follows:

(Def. 14) There exist c, f such that $\langle c, f\rangle = x_{\mathbf{2}}$ and x coll-addr$_2 = f$.

Let l be an element of $\text{Instr-Loc}_{\mathrm{SCM_{FSA}}}$. The functor $\mathrm{Next}(l)$ yielding an element of $\text{Instr-Loc}_{\mathrm{SCM_{FSA}}}$ is defined as follows:

(Def. 15) There exists an element L of $\text{Instr-Loc}_{\mathrm{SCM}}$ such that $L = l$ and $\mathrm{Next}(l) = \mathrm{Next}(L)$.

Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state. The functor $\mathbf{IC}_s$ yielding an element of $\text{Instr-Loc}_{\mathrm{SCM_{FSA}}}$ is defined by:

(Def. 16) $\mathbf{IC}_s = s(0)$.

Let x be an element of $\mathrm{Instr}_{\mathrm{SCM_{FSA}}}$ and let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state. The functor $\text{Exec-Res}_{\mathrm{SCM_{FSA}}}(x,s)$ yielding an $\mathbf{SCM}_{\mathrm{FSA}}$-state is defined by:

(Def. 17) (i) There exists an element x' of $\mathrm{Instr}_{\mathrm{SCM}}$ and there exists a state $_{\mathrm{SCM}}$ s' such that $x = x'$ and $s' = s \restriction \mathbb{N}{+\cdot}(\text{Instr-Loc}_{\mathrm{SCM}} \longmapsto x')$ and

Exec-Res$_{\mathrm{SCM_{FSA}}}(x, s) = s +\cdot$ Exec-Res$_{\mathrm{SCM}}(x', s') +\cdot s \upharpoonright$ Instr-Loc$_{\mathrm{SCM_{FSA}}}$ if InsCode$(x) \leq 8$,

(ii) there exists an integer i and there exists k such that $k = |s(x \text{ int-addr}_2)|$ and $i = \pi_k s(x \text{ coll-addr}_1)$ and Exec-Res$_{\mathrm{SCM_{FSA}}}(x, s) =$ Chg$_{\mathrm{SCM_{FSA}}}($Chg$_{\mathrm{SCM_{FSA}}}(s, x \text{ int-addr}_1, i), \mathrm{Next}(\mathbf{IC}_s))$ if InsCode$(x) = 9$,

(iii) there exists a finite sequence f of elements of $\mathbb{Z}$ and there exists k such that $k = |s(x \text{ int-addr}_2)|$ and $f = s(x \text{ coll-addr}_1) +\cdot (k, s(x \text{ int-addr}_1))$ and Exec-Res$_{\mathrm{SCM_{FSA}}}(x, s) =$ Chg$_{\mathrm{SCM_{FSA}}}($Chg$_{\mathrm{SCM_{FSA}}}(s, x \text{ coll-addr}_1, f), \mathrm{Next}(\mathbf{IC}_s))$ if InsCode$(x) = 10$,

(iv) Exec-Res$_{\mathrm{SCM_{FSA}}}(x, s) =$ Chg$_{\mathrm{SCM_{FSA}}}($Chg$_{\mathrm{SCM_{FSA}}}(s, x \text{ int-addr}_3, \mathrm{len}\, s(x \text{ coll-addr}_2)), \mathrm{Next}(\mathbf{IC}_s))$ if InsCode$(x) = 11$,

(v) there exists a finite sequence f of elements of $\mathbb{Z}$ and there exists k such that $k = |s(x \text{ int-addr}_3)|$ and $f = k \mapsto 0$ and Exec-Res$_{\mathrm{SCM_{FSA}}}(x, s) =$ Chg$_{\mathrm{SCM_{FSA}}}($Chg$_{\mathrm{SCM_{FSA}}}(s, x \text{ coll-addr}_2, f), \mathrm{Next}(\mathbf{IC}_s))$ if InsCode$(x) = 12$,

(vi) Exec-Res$_{\mathrm{SCM_{FSA}}}(x, s) = s$, otherwise.

The function Exec$_{\mathrm{SCM_{FSA}}}$ from Instr$_{\mathrm{SCM_{FSA}}}$ into $(\prod(\mathrm{OK_{SCM_{FSA}}}))^{\prod(\mathrm{OK_{SCM_{FSA}}})}$ is defined by:

(Def. 18) For every element x of Instr$_{\mathrm{SCM_{FSA}}}$ and for every $\mathbf{SCM}_{\mathrm{FSA}}$-state y holds (Exec$_{\mathrm{SCM_{FSA}}}(x)$ **qua** element of $(\prod(\mathrm{OK_{SCM_{FSA}}}))^{\prod(\mathrm{OK_{SCM_{FSA}}})})(y) =$ Exec-Res$_{\mathrm{SCM_{FSA}}}(x, y)$.

One can prove the following propositions:

(20) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every element u of Instr-Loc$_{\mathrm{SCM_{FSA}}}$ holds $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, u))(0) = u$.

(21) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every element u of Instr-Loc$_{\mathrm{SCM_{FSA}}}$ and for every element m_1 of Data-Loc$_{\mathrm{SCM_{FSA}}}$ holds $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, u))(m_1) = s(m_1)$.

(22) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every element u of Instr-Loc$_{\mathrm{SCM_{FSA}}}$ and for every element p of Data*-Loc$_{\mathrm{SCM_{FSA}}}$ holds $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, u))(p) = s(p)$.

(23) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for all elements u, v of Instr-Loc$_{\mathrm{SCM_{FSA}}}$ holds $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, u))(v) = s(v)$.

(24) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every element t of Data-Loc$_{\mathrm{SCM_{FSA}}}$ and for every integer u holds $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, t, u))(0) = s(0)$.

(25) For every $\mathbf{SCM}_{\mathrm{FSA}}$-state s and for every element t of Data-Loc$_{\mathrm{SCM_{FSA}}}$ and for every integer u holds $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, t, u))(t) = u$.

(26) Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, and let t be an element of Data-Loc$_{\mathrm{SCM_{FSA}}}$, and let u be an integer, and let m_1 be an element of Data-Loc$_{\mathrm{SCM_{FSA}}}$. If $m_1 \neq t$, then $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, t, u))(m_1) = s(m_1)$.

(27) Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, and let t be an element of Data-Loc$_{\mathrm{SCM_{FSA}}}$, and let u be an integer, and let f be an element of Data*-Loc$_{\mathrm{SCM_{FSA}}}$. Then $(\mathrm{Chg}_{\mathrm{SCM_{FSA}}}(s, t, u))(f) = s(f)$.

(28) Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, and let t be an element of $\text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and let u be an integer, and let v be an element of $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$. Then $(\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s,t,u))(v) = s(v)$.

(29) Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, and let t be an element of $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and let u be a finite sequence of elements of $\mathbb{Z}$. Then $(\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s,t,u))(t) = u$.

(30) Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, and let t be an element of $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and let u be a finite sequence of elements of $\mathbb{Z}$, and let m_1 be an element of $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$. If $m_1 \neq t$, then $(\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s,t,u))(m_1) = s(m_1)$.

(31) Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, and let t be an element of $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and let u be a finite sequence of elements of $\mathbb{Z}$, and let a be an element of $\text{Data-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$. Then $(\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s,t,u))(a) = s(a)$.

(32) Let s be an $\mathbf{SCM}_{\mathrm{FSA}}$-state, and let t be an element of $\text{Data}^*\text{-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$, and let u be a finite sequence of elements of $\mathbb{Z}$, and let v be an element of $\text{Instr-Loc}_{\mathrm{SCM}_{\mathrm{FSA}}}$. Then $(\mathrm{Chg}_{\mathrm{SCM}_{\mathrm{FSA}}}(s,t,u))(v) = s(v)$.

References

[1] Grzegorz Bancerek. König's theorem. *Formalized Mathematics*, 1(**3**):589–593, 1990.
[2] Grzegorz Bancerek. The reflection theorem. *Formalized Mathematics*, 1(**5**):973–977, 1990.
[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[4] Grzegorz Bancerek and Andrzej Trybulec. Miscellaneous facts about functions. *Formalized Mathematics*, 5(**4**):485–492, 1996.
[5] Czesław Byliński. A classical first order language. *Formalized Mathematics*, 1(**4**):669–676, 1990.
[6] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[7] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[8] Czesław Byliński. The modification of a function by a function and the iteration of the composition of a function. *Formalized Mathematics*, 1(**3**):521–527, 1990.
[9] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[10] Czesław Byliński. Subcategories and products of categories. *Formalized Mathematics*, 1(**4**):725–732, 1990.
[11] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[12] Yatsuka Nakamura and Andrzej Trybulec. A mathematical model of CPU. *Formalized Mathematics*, 3(**2**):151–160, 1992.
[13] Yatsuka Nakamura and Andrzej Trybulec. On a mathematical model of programs. *Formalized Mathematics*, 3(**2**):241–250, 1992.
[14] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.
[15] Dariusz Surowik. Cyclic groups and some of their properties - part I. *Formalized Mathematics*, 2(**5**):623–627, 1991.
[16] Andrzej Trybulec. Binary operations applied to functions. *Formalized Mathematics*, 1(**2**):329–334, 1990.
[17] Andrzej Trybulec. Function domains and Frænkel operator. *Formalized Mathematics*, 1(**3**):495–500, 1990.
[18] Andrzej Trybulec. Many-sorted sets. *Formalized Mathematics*, 4(**1**):15–22, 1993.

[19] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[20] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.
[21] Andrzej Trybulec and Yatsuka Nakamura. Some remarks on the simple concrete model of computer. *Formalized Mathematics*, 4(**1**):51–56, 1993.
[22] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[23] Wojciech A. Trybulec. Binary operations on finite sequences. *Formalized Mathematics*, 1(**5**):979–981, 1990.
[24] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[25] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[26] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

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