FORMALIZED MATHEMATICS
Volume 5, Number 4, 1996
Warsaw University - Białystok

Some Multi Instructions Defined by Sequence of Instructions of $\mathbf{SCM}_{\rm FSA}$

Noriko Asamoto
Ochanomizu University
Tokyo

MML Identifier: SCMFSA_7.

The terminology and notation used in this paper are introduced in the following papers: [10], [2], [14], [13], [18], [22], [6], [16], [21], [1], [15], [3], [9], [7], [20], [4], [19], [8], [5], [11], [12], and [17].

In this paper m will be a natural number.

Let us note that every finite partial state of $\mathbf{SCM}_{\rm FSA}$ is finite.

Let p be a finite sequence and let x, y be arbitrary. Note that $p +\cdot (x, y)$ is finite sequence-like.

Let i be an integer. Then $|i|$ is a natural number.

Let D be a set. Note that D^* is non empty.

The following four propositions are true:

(1) For every natural number k holds $|k| = k$.

(2) For all natural numbers a, b, c such that $a \geq c$ and $b \geq c$ and $a -' c = b -' c$ holds $a = b$.

(3) For all natural numbers a, b such that $a \geq b$ holds $a -' b = a - b$.

(4) For all integers a, b such that $a < b$ holds $a \leq b - 1$.

The scheme *CardMono”* concerns a set $\mathcal{A}$, a non empty set $\mathcal{B}$, and a unary functor $\mathcal{F}$ yielding arbitrary, and states that:

$\mathcal{A} \approx \{\mathcal{F}(d) : d$ ranges over elements of $\mathcal{B}$, $d \in \mathcal{A}\}$

provided the parameters satisfy the following conditions:

- $\mathcal{A} \subseteq \mathcal{B}$,
- For all elements d_1, d_2 of $\mathcal{B}$ such that $d_1 \in \mathcal{A}$ and $d_2 \in \mathcal{A}$ and $\mathcal{F}(d_1) = \mathcal{F}(d_2)$ holds $d_1 = d_2$.

One can prove the following propositions:

(5) For all finite sequences p_1, p_2, q such that $p_1 \subseteq q$ and $p_2 \subseteq q$ and $\operatorname{len} p_1 = \operatorname{len} p_2$ holds $p_1 = p_2$.

ISSN 1426–2630

(6) For all finite sequences p, q such that $p \frown q = p$ holds $q = \varepsilon$.

(7) For every finite sequence p and for arbitrary x holds $\operatorname{len}(p \frown \langle x \rangle) = \operatorname{len} p + 1$.

(8) For all finite sequences p, q such that $p \subseteq q$ holds $\operatorname{len} p \leq \operatorname{len} q$.

(9) For all finite sequences p, q and for every natural number i such that $1 \leq i$ and $i \leq \operatorname{len} p$ holds $(p \frown q)(i) = p(i)$.

(10) For all finite sequences p, q and for every natural number i such that $1 \leq i$ and $i \leq \operatorname{len} q$ holds $(p \frown q)(\operatorname{len} p + i) = q(i)$.

(11) For every finite sequence p and for every natural number i holds $i \in \operatorname{dom} p$ iff $1 \leq i$ and $i \leq \operatorname{len} p$.

(12) For every finite sequence p such that $p \neq \varepsilon$ holds $\operatorname{len} p \in \operatorname{dom} p$.

(13) For every set D holds $\operatorname{Flat}(\varepsilon_{D^*}) = \varepsilon_D$.

(14) For every set D and for all finite sequences F, G of elements of D^* holds $\operatorname{Flat}(F \frown G) = \operatorname{Flat}(F) \frown \operatorname{Flat}(G)$.

(15) For every set D and for all elements p, q of D^* holds $\operatorname{Flat}(\langle p, q \rangle) = p \frown q$.

(16) For every set D and for all elements p, q, r of D^* holds $\operatorname{Flat}(\langle p, q, r \rangle) = p \frown q \frown r$.

(17) Let D be a non empty set and let p, q be finite sequences of elements of D. If $p \subseteq q$, then there exists a finite sequence p' of elements of D such that $p \frown p' = q$.

(18) Let D be a non empty set, and let p, q be finite sequences of elements of D, and let i be a natural number. If $p \subseteq q$ and $1 \leq i$ and $i \leq \operatorname{len} p$, then $q(i) = p(i)$.

(19) For every set D and for all finite sequences F, G of elements of D^* such that $F \subseteq G$ holds $\operatorname{Flat}(F) \subseteq \operatorname{Flat}(G)$.

(20) For every finite sequence p holds $p \restriction \operatorname{Seg} 0 = \varepsilon$.

(21) For all finite sequences f, g holds $f \restriction \operatorname{Seg} 0 = g \restriction \operatorname{Seg} 0$.

(22) For every non empty set D and for every element x of D holds $\langle x \rangle$ is a finite sequence of elements of D.

(23) Let D be a set and let p, q be finite sequences of elements of D. Then $p \frown q$ is a finite sequence of elements of D.

Let f be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$. The functor $\operatorname{Load}(f)$ yielding a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ is defined by:

(Def. 1) $\operatorname{dom} \operatorname{Load}(f) = \{\operatorname{insloc}(m -' 1) : m \in \operatorname{dom} f\}$ and for every natural number k such that $\operatorname{insloc}(k) \in \operatorname{dom} \operatorname{Load}(f)$ holds $(\operatorname{Load}(f))(\operatorname{insloc}(k)) = \pi_{k+1} f$.

The following propositions are true:

(24) Let f be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\mathrm{FSA}}$ and let k be a natural number. Then $\operatorname{dom} \operatorname{Load}(f) = \{\operatorname{insloc}(m -' 1) : m \in \operatorname{dom} f\}$.

(25) For every finite sequence f of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ holds $\mathrm{card\,Load}(f) = \mathrm{len}\, f$.

(26) Let p be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ and let k be a natural number. Then $\mathrm{insloc}(k) \in \mathrm{dom\,Load}(p)$ if and only if $k + 1 \in \mathrm{dom}\, p$.

(27) For all natural numbers k, n holds $k < n$ iff $0 < k + 1$ and $k + 1 \leq n$.

(28) For all natural numbers k, n holds $k < n$ iff $1 \leq k + 1$ and $k + 1 \leq n$.

(29) Let p be a finite sequence of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ and let k be a natural number. Then $\mathrm{insloc}(k) \in \mathrm{dom\,Load}(p)$ if and only if $k < \mathrm{len}\, p$.

(30) For every non empty finite sequence f of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ holds $1 \in \mathrm{dom}\, f$ and $\mathrm{insloc}(0) \in \mathrm{dom\,Load}(f)$.

(31) For all finite sequences p, q of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ holds $\mathrm{Load}(p) \subseteq \mathrm{Load}(p \frown q)$.

(32) For all finite sequences p, q of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ such that $p \subseteq q$ holds $\mathrm{Load}(p) \subseteq \mathrm{Load}(q)$.

Let a be an integer location and let k be an integer. The functor $a{:=}k$ yields a finite partial state of $\mathbf{SCM}_{\rm FSA}$ and is defined as follows:

(Def. 2) (i) There exists a natural number k_1 such that $k_1 + 1 = k$ and $a{:=}k = \mathrm{Load}(\langle a{:=}\,\mathrm{intloc}(0)\rangle \frown (k_1 \mapsto \mathrm{AddTo}(a, \mathrm{intloc}(0))) \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\rm FSA}}\rangle)$ if $k > 0$,

(ii) there exists a natural number k_1 such that $k_1 + k = 1$ and $a{:=}k = \mathrm{Load}(\langle a{:=}\,\mathrm{intloc}(0)\rangle \frown (k_1 \mapsto \mathrm{SubFrom}(a, \mathrm{intloc}(0))) \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\rm FSA}}\rangle)$, otherwise.

Let a be an integer location and let k be an integer. The functor $\mathrm{aSeq}(a, k)$ yielding a finite sequence of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ is defined by:

(Def. 3) (i) There exists a natural number k_1 such that $k_1 + 1 = k$ and $\mathrm{aSeq}(a, k) = \langle a{:=}\,\mathrm{intloc}(0)\rangle \frown (k_1 \mapsto \mathrm{AddTo}(a, \mathrm{intloc}(0)))$ if $k > 0$,

(ii) there exists a natural number k_1 such that $k_1 + k = 1$ and $\mathrm{aSeq}(a, k) = \langle a{:=}\,\mathrm{intloc}(0)\rangle \frown (k_1 \mapsto \mathrm{SubFrom}(a, \mathrm{intloc}(0)))$, otherwise.

One can prove the following proposition

(33) For every integer location a and for every integer k holds $a{:=}k = \mathrm{Load}((\mathrm{aSeq}(a, k)) \frown \langle \mathbf{halt}_{\mathbf{SCM}_{\rm FSA}}\rangle)$.

Let f be a finite sequence location and let p be a finite sequence of elements of $\mathbb{Z}$. The functor $\mathrm{aSeq}(f, p)$ yields a finite sequence of elements of the instructions of $\mathbf{SCM}_{\rm FSA}$ and is defined by the condition (Def. 4).

(Def. 4) There exists a finite sequence p_3 of elements of (the instructions of $\mathbf{SCM}_{\rm FSA})^*$ such that

(i) $\mathrm{len}\, p_3 = \mathrm{len}\, p$,

(ii) for every natural number k such that $1 \leq k$ and $k \leq \mathrm{len}\, p$ there exists an integer i such that $i = p(k)$ and $p_3(k) = (\mathrm{aSeq}(\mathrm{intloc}(1), k)) \frown$

aSeq(intloc(2), i) ⌢ ⟨$f_{\mathrm{intloc}(1)}$:= intloc(2)⟩, and

(iii) aSeq(f, p) = Flat(p_3).

Let f be a finite sequence location and let p be a finite sequence of elements of $\mathbb{Z}$ The functor f:=p yielding a finite partial state of $\mathbf{SCM}_{\mathrm{FSA}}$ is defined by:

(Def. 5) f:=p = Load((aSeq(intloc(1), len p)) ⌢ ⟨f:=⟨$\underbrace{0, \ldots, 0}_{\mathrm{intloc}(1)}$⟩⟩ ⌢ aSeq($f, p$) ⌢ ⟨$\mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$⟩).

Next we state several propositions:

(34) For every integer location a holds a:=1 = Load(⟨a:= intloc(0)⟩ ⌢ ⟨$\mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$⟩).

(35) For every integer location a holds a:=0 = Load(⟨a:= intloc(0)⟩ ⌢ ⟨SubFrom(a, intloc(0))⟩ ⌢ ⟨$\mathbf{halt}_{\mathbf{SCM}_{\mathrm{FSA}}}$⟩).

(36) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose s(intloc(0)) = 1. Let c_0 be a natural number. Suppose $\mathbf{IC}_s$ = insloc(c_0). Let a be an integer location and let k be an integer. Suppose $a \neq$ intloc(0) and for every natural number c such that $c \in$ dom aSeq(a, k) holds (aSeq(a, k))(c) = s(insloc((c_0+c)−′1)). Then

(i) for every natural number i such that $i \leq$ len aSeq(a, k) holds $\mathbf{IC}_{(\mathrm{Computation}(s))(i)}$ = insloc($c_0 + i$) and for every integer location b such that $b \neq a$ holds (Computation(s))(i)(b) = s(b) and for every finite sequence location f holds (Computation(s))(i)(f) = s(f), and

(ii) (Computation(s))(len aSeq(a, k))(a) = k.

(37) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s$ = insloc(0) and s(intloc(0)) = 1. Let a be an integer location and let k be an integer. Suppose Load(aSeq(a, k)) ⊆ s and $a \neq$ intloc(0). Then

(i) for every natural number i such that $i \leq$ len aSeq(a, k) holds $\mathbf{IC}_{(\mathrm{Computation}(s))(i)}$ = insloc(i) and for every integer location b such that $b \neq a$ holds (Computation(s))(i)(b) = s(b) and for every finite sequence location f holds (Computation(s))(i)(f) = s(f), and

(ii) (Computation(s))(len aSeq(a, k))(a) = k.

(38) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s$ = insloc(0) and s(intloc(0)) = 1. Let a be an integer location and let k be an integer. Suppose a:=k ⊆ s and $a \neq$ intloc(0). Then

(i) s is halting,

(ii) (Result(s))(a) = k,

(iii) for every integer location b such that $b \neq a$ holds (Result(s))(b) = s(b), and

(iv) for every finite sequence location f holds (Result(s))(f) = s(f).

(39) Let s be a state of $\mathbf{SCM}_{\mathrm{FSA}}$. Suppose $\mathbf{IC}_s$ = insloc(0) and s(intloc(0)) = 1. Let f be a finite sequence location and let p be a finite sequence of elements of $\mathbb{Z}$. Suppose f:=p ⊆ s. Then

(i) s is halting,

(ii) (Result(s))(f) = p,

(iii) for every integer location b such that $b \neq \mathrm{intloc}(1)$ and $b \neq \mathrm{intloc}(2)$ holds $(\mathrm{Result}(s))(b) = s(b)$, and

(iv) for every finite sequence location g such that $g \neq f$ holds $(\mathrm{Result}(s))(g) = s(g)$.

References

[1] Grzegorz Bancerek. Cardinal numbers. *Formalized Mathematics*, 1(**2**):377–382, 1990.
[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.
[3] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[4] Grzegorz Bancerek and Piotr Rudnicki. On defining functions on trees. *Formalized Mathematics*, 4(**1**):91–101, 1993.
[5] Grzegorz Bancerek and Andrzej Trybulec. Miscellaneous facts about functions. *Formalized Mathematics*, 5(**4**):485–492, 1996.
[6] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.
[7] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Formalized Mathematics*, 1(**3**):529–536, 1990.
[8] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[9] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[10] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[11] Yatsuka Nakamura and Andrzej Trybulec. A mathematical model of CPU. *Formalized Mathematics*, 3(**2**):151–160, 1992.
[12] Yatsuka Nakamura and Andrzej Trybulec. On a mathematical model of programs. *Formalized Mathematics*, 3(**2**):241–250, 1992.
[13] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.
[14] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.
[15] Andrzej Trybulec. Semilattice operations on finite subsets. *Formalized Mathematics*, 1(**2**):369–376, 1990.
[16] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[17] Andrzej Trybulec, Yatsuka Nakamura, and Piotr Rudnicki. The $\mathbf{SCM}_{\mathrm{FSA}}$ computer. *Formalized Mathematics*, 5(**4**):519–528, 1996.
[18] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[19] Wojciech A. Trybulec. Binary operations on finite sequences. *Formalized Mathematics*, 1(**5**):979–981, 1990.
[20] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[21] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[22] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

Received April 24, 1996