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Adjacency Concept for Pairs of Natural Numbers

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Summary. First, we introduce the concept of adjacency for a pair of natural numbers. Second, we extend the concept for two pairs of natural numbers. The pairs represent points of a lattice in a plane. We show that if some property is infectious among adjacent points, and some points have the property, then all points have the property.

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The articles [8], [11], [10], [5], [1], [7], [12], [4], [3], [2], [9], and [6] provide the notation and terminology for this paper.

In this paper i, j, k, k_1, k_2, n, m, i_1, i_2, j_1, j_2 are natural numbers.

Let us consider i_1, i_2. We say that i_1 and i_2 are adjacent if and only if:

(Def. 1) $i_2 = i_1 + 1$ or $i_1 = i_2 + 1$.

One can prove the following propositions:

(1) For all i_1, i_2 such that i_1 and i_2 are adjacent holds $i_1 + 1$ and $i_2 + 1$ are adjacent.

(2) For all i_1, i_2 such that i_1 and i_2 are adjacent and $1 \leq i_1$ and $1 \leq i_2$ holds $i_1 -' 1$ and $i_2 -' 1$ are adjacent.

Let us consider i_1, j_1, i_2, j_2. We say that i_1, j_1, i_2, and j_2 are adjacent if and only if:

(Def. 2) i_1 and i_2 are adjacent and $j_1 = j_2$ or $i_1 = i_2$ and j_1 and j_2 are adjacent.

The following propositions are true:

(3) For all i_1, i_2, j_1, j_2 such that i_1, j_1, i_2, and j_2 are adjacent holds $i_1 + 1$, $j_1 + 1$, $i_2 + 1$, and $j_2 + 1$ are adjacent.

(4) Given i_1, i_2, j_1, j_2. Suppose i_1, j_1, i_2, and j_2 are adjacent and $1 \leq i_1$ and $1 \leq i_2$ and $1 \leq j_1$ and $1 \leq j_2$. Then $i_1 -' 1$, $j_1 -' 1$, $i_2 -' 1$, and $j_2 -' 1$ are adjacent.

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Let us consider i, n. The functor $\mathrm{Repeat}(i,n)$ yields a finite sequence of elements of $\mathbb{N}$ and is defined as follows:

(Def. 3) $\mathrm{len}\,\mathrm{Repeat}(i,n) = n$ and for every j such that $1 \leq j$ and $j \leq n$ holds $(\mathrm{Repeat}(i,n))(j) = i$.

Next we state four propositions:

(5) For every i holds $\mathrm{Repeat}(i,0) = \varepsilon$.

(6) Given n, i, j. Suppose $i \leq n$ and $j \leq n$. Then there exists a finite sequence f_1 of elements of $\mathbb{N}$ such that
- (i) $f_1(1) = i$,
- (ii) $f_1(\mathrm{len}\,f_1) = j$,
- (iii) $\mathrm{len}\,f_1 = i -' j + j -' i + 1$,
- (iv) for all k, k_1 such that $1 \leq k$ and $k \leq \mathrm{len}\,f_1$ and $k_1 = f_1(k)$ holds $k_1 \leq n$, and
- (v) for every i_1 such that $1 \leq i_1$ and $i_1 < \mathrm{len}\,f_1$ holds $f_1(i_1+1) = \pi_{i_1} f_1 + 1$ or $f_1(i_1) = \pi_{i_1+1} f_1 + 1$.

(7) Given n, i, j. Suppose $i \leq n$ and $j \leq n$. Then there exists a finite sequence f_1 of elements of $\mathbb{N}$ such that
- (i) $f_1(1) = i$,
- (ii) $f_1(\mathrm{len}\,f_1) = j$,
- (iii) $\mathrm{len}\,f_1 = i -' j + j -' i + 1$,
- (iv) for all k, k_1 such that $1 \leq k$ and $k \leq \mathrm{len}\,f_1$ and $k_1 = f_1(k)$ holds $k_1 \leq n$, and
- (v) for every i_1 such that $1 \leq i_1$ and $i_1 < \mathrm{len}\,f_1$ holds $\pi_{i_1} f_1$ and $\pi_{i_1+1} f_1$ are adjacent.

(8) Given n, m, i_1, j_1, i_2, j_2. Suppose $i_1 \leq n$ and $j_1 \leq m$ and $i_2 \leq n$ and $j_2 \leq m$. Then there exist finite sequences f_1, f_2 of elements of $\mathbb{N}$ such that
- (i) for all i, k_1, k_2 such that $i \in \mathrm{dom}\,f_1$ and $k_1 = f_1(i)$ and $k_2 = f_2(i)$ holds $k_1 \leq n$ and $k_2 \leq m$,
- (ii) $f_1(1) = i_1$,
- (iii) $f_1(\mathrm{len}\,f_1) = i_2$,
- (iv) $f_2(1) = j_1$,
- (v) $f_2(\mathrm{len}\,f_2) = j_2$,
- (vi) $\mathrm{len}\,f_1 = \mathrm{len}\,f_2$,
- (vii) $\mathrm{len}\,f_1 = i_1 -' i_2 + i_2 -' i_1 + j_1 -' j_2 + j_2 -' j_1 + 1$, and
- (viii) for every i such that $1 \leq i$ and $i < \mathrm{len}\,f_1$ holds $\pi_i f_1$, $\pi_i f_2$, $\pi_{i+1} f_1$, and $\pi_{i+1} f_2$ are adjacent.

In the sequel S is a set.

Next we state the proposition

(9) Let Y be a subset of S and let F be a matrix over 2^S of dimension $n \times m$. Suppose that
- (i) there exist i, j such that $i \in \mathrm{Seg}\,n$ and $j \in \mathrm{Seg}\,m$ and $F_{i,j} \subseteq Y$, and
- (ii) for all i_1, j_1, i_2, j_2 such that $i_1 \in \mathrm{Seg}\,n$ and $i_2 \in \mathrm{Seg}\,n$ and $j_1 \in \mathrm{Seg}\,m$ and $j_2 \in \mathrm{Seg}\,m$ and i_1, j_1, i_2, and j_2 are adjacent holds $F_{i_1,j_1} \subseteq Y$ iff

$F_{i_2,j_2} \subseteq Y$.

Given i, j. If $i \in \mathrm{Seg}\, n$ and $j \in \mathrm{Seg}\, m$, then $F_{i,j} \subseteq Y$.

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