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Examples of Category Structures

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Summary. This article contains definitions of two category structures: the category of many sorted signatures and the category of many sorted algebras. Some facts about these structures are proved.

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The papers [22], [10], [23], [24], [7], [8], [17], [25], [9], [6], [2], [5], [18], [1], [21], [15], [20], [14], [12], [19], [16], [13], [3], [4], and [11] provide the terminology and notation for this paper.

1. Category of Many Sorted Signatures

In this paper A denotes a non empty set, S denotes a non void non empty many sorted signature, and x denotes a set.

Let us consider A. The functor MSSCat(A) yields a strict non empty category structure and is defined by the conditions (Def. 1).

(Def. 1) (i) The carrier of MSSCat(A) = MSS-set(A),

(ii) for all elements i, j of MSS-set(A) holds (the arrows of MSSCat(A))(i, j) = MSS-morph(i, j), and

(iii) for all objects i, j, k of MSSCat(A) such that $i \in$ MSS-set(A) and $j \in$ MSS-set(A) and $k \in$ MSS-set(A) and for all functions f_1, f_2, g_1, g_2 such that $\langle f_1, f_2 \rangle \in$ (the arrows of MSSCat(A))(i, j) and $\langle g_1, g_2 \rangle \in$ (the arrows of MSSCat(A))(j, k) holds (the composition of MSSCat(A))(i, j, k)($\langle g_1, g_2 \rangle$, $\langle f_1, f_2 \rangle$) = $\langle g_1 \cdot f_1, g_2 \cdot f_2 \rangle$.

Let us consider A. Note that MSSCat(A) is transitive and associative and has units.

The following proposition is true

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(1) For every category C such that $C = \mathrm{MSSCat}(A)$ holds every object of C is a non empty non void many sorted signature.

Let us consider S. Note that there exists an algebra over S which is strict and feasible.

Let us consider S, A. The functor $\mathrm{MSAlg\_set}(S, A)$ is defined by the condition (Def. 2).

(Def. 2) $x \in \mathrm{MSAlg\_set}(S, A)$ if and only if there exists a strict feasible algebra M over S such that $x = M$ and for every component C of the sorts of M holds $C \subseteq A$.

Let us consider S, A. Observe that $\mathrm{MSAlg\_set}(S, A)$ is non empty.

2. Category of Many Sorted Algebras

In the sequel o is an operation symbol of S.

One can prove the following four propositions:

(2) Let x be an algebra over S. Suppose $x \in \mathrm{MSAlg\_set}(S, A)$. Then the sorts of $x \in (2^A)^{\text{the carrier of } S}$ and the characteristics of $x \in ((\mathbb{N} \dot{\to} A) \dot{\to} A)^{\text{the operation symbols of } S}$.

(3) Let U_1, U_2 be algebras over S. Suppose the sorts of U_1 is transformable to the sorts of U_2 and $\mathrm{Args}(o, U_1) \neq \emptyset$. Then $\mathrm{Args}(o, U_2) \neq \emptyset$.

(4) Let U_1, U_2, U_3 be feasible algebras over S, and let F be a many sorted function from U_1 into U_2, and let G be a many sorted function from U_2 into U_3, and let x be an element of $\mathrm{Args}(o, U_1)$. Suppose that

(i) $\mathrm{Args}(o, U_1) \neq \emptyset$,

(ii) the sorts of U_1 is transformable to the sorts of U_2, and

(iii) the sorts of U_2 is transformable to the sorts of U_3.

Then there exists a many sorted function G_1 from U_1 into U_3 such that $G_1 = G \circ F$ and $G_1 \# x = G \# (F \# x)$.

(5) Let U_1, U_2, U_3 be feasible algebras over S, and let F be a many sorted function from U_1 into U_2, and let G be a many sorted function from U_2 into U_3. Suppose that

(i) the sorts of U_1 is transformable to the sorts of U_2,

(ii) the sorts of U_2 is transformable to the sorts of U_3,

(iii) F is a homomorphism of U_1 into U_2, and

(iv) G is a homomorphism of U_2 into U_3.

Then there exists a many sorted function G_1 from U_1 into U_3 such that $G_1 = G \circ F$ and G_1 is a homomorphism of U_1 into U_3.

Let us consider S, A and let i, j be sets. Let us assume that $i \in \mathrm{MSAlg\_set}(S, A)$ and $j \in \mathrm{MSAlg\_set}(S, A)$. The functor $\mathrm{MSAlg\_morph}(S, A, i, j)$ is defined by the condition (Def. 3).

(Def. 3) $x \in \mathrm{MSAlg\_morph}(S, A, i, j)$ if and only if there exist strict feasible algebras M, N over S and there exists a many sorted function f from M

into N such that $M = i$ and $N = j$ and $f = x$ and the sorts of M is transformable to the sorts of N and f is a homomorphism of M into N.

Let us consider S, A. The functor MSAlgCat(S, A) yields a strict non empty category structure and is defined by the conditions (Def. 4).

(Def. 4) (i) The carrier of MSAlgCat(S, A) = MSAlg_set(S, A),

(ii) for all elements i, j of MSAlg_set(S, A) holds (the arrows of MSAlgCat(S, A))(i, j) = MSAlg_morph(S, A, i, j), and

(iii) for all objects i, j, k of MSAlgCat(S, A) and for all function yielding functions f, g such that $f \in$ (the arrows of MSAlgCat(S, A))(i, j) and $g \in$ (the arrows of MSAlgCat(S, A))(j, k) holds (the composition of MSAlgCat(S, A))$(i, j, k)(g, f) = g \circ f$.

Let us consider S, A. One can verify that MSAlgCat(S, A) is transitive and associative and has units.

One can prove the following proposition

(6) For every category C such that C = MSAlgCat(S, A) holds every object of C is a strict feasible algebra over S.

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