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# Algebra of Morphisms

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The papers [22], [27], [15], [2], [23], [21], [28], [11], [12], [14], [9], [10], [19], [3], [26], [1], [4], [24], [18], [25], [17], [20], [6], [16], [5], [7], [8], and [13] provide the notation and terminology for this paper.

## 1. Preliminaries

Let $I$ be a set and let $A$, $f$ be functions. The functor $f \upharpoonright_I A$ yielding a many sorted function indexed by $I$ is defined by:

(Def. 1) For every set $i$ such that $i \in I$ holds $(f \upharpoonright_I A)(i) = f \upharpoonright A(i)$.

One can prove the following propositions:

(1) For every set $I$ and for every many sorted set $A$ indexed by $I$ holds $\mathrm{id}_{\mathrm{Union}\,A} \upharpoonright_I A = \mathrm{id}_A$.

(2) Let $I$ be a set, $A$, $B$ be many sorted sets indexed by $I$, and $f$, $g$ be functions. If $\mathrm{rng}_\kappa (f \upharpoonright_I A)(\kappa) \subseteq B$, then $(g \cdot f) \upharpoonright_I A = (g \upharpoonright_I B) \circ (f \upharpoonright_I A)$.

(3) Let $f$ be a function, $I$ be a set, and $A$, $B$ be many sorted sets indexed by $I$. Suppose that for every set $i$ such that $i \in I$ holds $A(i) \subseteq \mathrm{dom}\, f$ and $f^\circ A(i) \subseteq B(i)$. Then $f \upharpoonright_I A$ is a many sorted function from $A$ into $B$.

(4) Let $A$ be a set, $i$ be a natural number, and $p$ be a finite sequence. Then $p \in A^i$ if and only if $\mathrm{len}\, p = i$ and $\mathrm{rng}\, p \subseteq A$.

(5) Let $A$ be a set, $i$ be a natural number, and $p$ be a finite sequence of elements of $A$. Then $p \in A^i$ if and only if $\mathrm{len}\, p = i$.

(6) For every set $A$ and for every natural number $i$ holds $A^i \subseteq A^*$.

(7) For every set $A$ and for every natural number $i$ holds $i \neq 0$ and $A = \emptyset$ iff $A^i = \emptyset$.




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(8) For all sets $A$, $x$ holds $x \in A^1$ iff there exists a set $a$ such that $a \in A$ and $x = \langle a \rangle$.

(9) For all sets $A$, $a$ such that $\langle a \rangle \in A^1$ holds $a \in A$.

(10) For all sets $A$, $x$ holds $x \in A^2$ iff there exist sets $a$, $b$ such that $a \in A$ and $b \in A$ and $x = \langle a, b \rangle$.

(11) For all sets $A$, $a$, $b$ such that $\langle a, b \rangle \in A^2$ holds $a \in A$ and $b \in A$.

(12) For all sets $A$, $x$ holds $x \in A^3$ iff there exist sets $a$, $b$, $c$ such that $a \in A$ and $b \in A$ and $c \in A$ and $x = \langle a, b, c \rangle$.

(13) For all sets $A$, $a$, $b$, $c$ such that $\langle a, b, c \rangle \in A^3$ holds $a \in A$ and $b \in A$ and $c \in A$.

Let $A$ be a function. We say that $A$ is mutually-disjoint if and only if:

(Def. 2) For all sets $x$, $y$ such that $x \neq y$ holds $A(x)$ misses $A(y)$.

Let $S$ be a non empty many sorted signature and let $A$ be an algebra over $S$. We say that $A$ is empty if and only if:

(Def. 3) The sorts of $A$ are empty yielding.

We say that $A$ is disjoint if and only if:

(Def. 4) The sorts of $A$ are mutually-disjoint.

Let $S$ be a non empty many sorted signature. Note that every algebra over $S$ which is non-empty is also non empty.

Let $S$ be a non empty non void many sorted signature and let $X$ be a non-empty many sorted set indexed by the carrier of $S$. One can check that $\mathrm{Free}(X)$ is disjoint.

Let $S$ be a non empty non void many sorted signature. Observe that there exists an algebra over $S$ which is strict, non-empty, and disjoint.

Let $S$ be a non empty non void many sorted signature and let $A$ be a non empty algebra over $S$. One can verify that the sorts of $A$ is non empty yielding.

One can verify that there exists a function which is non empty yielding.

## 2. Signature of a category

Let $A$ be a set. The functor $\mathrm{CatSign}(A)$ yielding a strict many sorted signature is defined by the conditions (Def. 5).

(Def. 5)(i) The carrier of $\mathrm{CatSign}(A) = [\!:\{0\}, A^2:\!]$,

(ii) the operation symbols of $\mathrm{CatSign}(A) = [\!:\{1\}, A^1:\!] \cup [\!:\{2\}, A^3:\!]$,

(iii) for every set $a$ such that $a \in A$ holds (the arity of $\mathrm{CatSign}(A)$)($\langle 1, \langle a \rangle \rangle) = \varepsilon$ and (the result sort of $\mathrm{CatSign}(A)$)($\langle 1, \langle a \rangle \rangle) = \langle 0, \langle a, a \rangle \rangle$, and

(iv) for all sets $a$, $b$, $c$ such that $a \in A$ and $b \in A$ and $c \in A$ holds (the arity of $\mathrm{CatSign}(A)$)($\langle 2, \langle a, b, c \rangle \rangle) = \langle \langle 0, \langle b, c \rangle \rangle, \langle 0, \langle a, b \rangle \rangle \rangle$ and (the result sort of $\mathrm{CatSign}(A)$)($\langle 2, \langle a, b, c \rangle \rangle) = \langle 0, \langle a, c \rangle \rangle$.

Let $A$ be a set. Observe that $\mathrm{CatSign}(A)$ is feasible.

Let $A$ be a non empty set. Observe that CatSign($A$) is non empty and non void.

Instead of a feasible many sorted signature we will use a signature.

Let $S$ be a signature. We say that $S$ is categorial if and only if:

(Def. 6) There exists a set $A$ such that CatSign($A$) is a subsignature of $S$ and the carrier of $S = [\!:\{0\}, A^2:\!]$.

Let us note that every non empty signature which is categorial is also non void.

One can check that there exists a signature which is categorial, non empty, and strict.

A cat-signature is a categorial signature.

Let $A$ be a set. A signature is said to be a cat-signature of $A$ if:

(Def. 7) CatSign($A$) is a subsignature of it and the carrier of it $= [\!:\{0\}, A^2:\!]$.

One can prove the following proposition

(14) For all sets $A_1$, $A_2$ and for every cat-signature $S$ of $A_1$ such that $S$ is a cat-signature of $A_2$ holds $A_1 = A_2$.

Let $A$ be a set. Note that every cat-signature of $A$ is categorial.

Let $A$ be a non empty set. Note that every cat-signature of $A$ is non empty.

Let $A$ be a set. Observe that there exists a cat-signature of $A$ which is strict.

Let $A$ be a set. Then CatSign($A$) is a strict cat-signature of $A$.

Let $S$ be a many sorted signature. The functor underlay $S$ is defined by the condition (Def. 8).

(Def. 8) Let $x$ be a set. Then $x \in$ underlay $S$ if and only if there exists a set $a$ and there exists a function $f$ such that $\langle a, f\rangle \in$ (the carrier of $S$) $\cup$ (the operation symbols of $S$) and $x \in \operatorname{rng} f$.

One can prove the following proposition

(15) For every set $A$ holds underlay CatSign($A$) $= A$.

Let $S$ be a many sorted signature. We say that $S$ is $\delta$-concrete if and only if the condition (Def. 9) is satisfied.

(Def. 9) There exists a function $f$ from $\mathbb{N}$ into $\mathbb{N}$ such that

(i) for every set $s$ such that $s \in$ the carrier of $S$ there exists a natural number $i$ and there exists a finite sequence $p$ such that $s = \langle i, p\rangle$ and $\operatorname{len} p = f(i)$ and $[\!:\{i\}, (\text{underlay } S)^{f(i)}:\!] \subseteq$ the carrier of $S$, and

(ii) for every set $o$ such that $o \in$ the operation symbols of $S$ there exists a natural number $i$ and there exists a finite sequence $p$ such that $o = \langle i, p\rangle$ and $\operatorname{len} p = f(i)$ and $[\!:\{i\}, (\text{underlay } S)^{f(i)}:\!] \subseteq$ the operation symbols of $S$.

Let $A$ be a set. One can check that CatSign($A$) is $\delta$-concrete.

Observe that there exists a cat-signature which is $\delta$-concrete, non empty, and strict. Let $A$ be a set. One can check that there exists a cat-signature of $A$ which is $\delta$-concrete and strict.

The following propositions are true:

(16) Let $S$ be a $\delta$-concrete many sorted signature and $x$ be a set. Suppose $x \in$ the carrier of $S$ or $x \in$ the operation symbols of $S$. Then there exists a natural number $i$ and there exists a finite sequence $p$ such that $x = \langle i, p \rangle$ and $\operatorname{rng} p \subseteq \operatorname{underlay} S$.

(17) Let $S$ be a $\delta$-concrete many sorted signature, $i$ be a set, and $p_1$, $p_2$ be finite sequences. Suppose that

(i) $\langle i, p_1 \rangle \in$ the carrier of $S$ and $\langle i, p_2 \rangle \in$ the carrier of $S$, or

(ii) $\langle i, p_1 \rangle \in$ the operation symbols of $S$ and $\langle i, p_2 \rangle \in$ the operation symbols of $S$.

Then $\operatorname{len} p_1 = \operatorname{len} p_2$.

(18) Let $S$ be a $\delta$-concrete many sorted signature, $i$ be a set, and $p_1$, $p_2$ be finite sequences such that $\operatorname{len} p_2 = \operatorname{len} p_1$ and $\operatorname{rng} p_2 \subseteq \operatorname{underlay} S$. Then

(i) if $\langle i, p_1 \rangle \in$ the carrier of $S$, then $\langle i, p_2 \rangle \in$ the carrier of $S$, and

(ii) if $\langle i, p_1 \rangle \in$ the operation symbols of $S$, then $\langle i, p_2 \rangle \in$ the operation symbols of $S$.

(19) Every $\delta$-concrete categorial non empty signature $S$ is a cat-signature of underlay $S$.

## 3. Symbols of categorial signatures

Let $S$ be a non empty cat-signature and let $s$ be a sort symbol of $S$. Note that $s_{\mathbf{2}}$ is relation-like and function-like.

Let $S$ be a non empty $\delta$-concrete many sorted signature and let $s$ be a sort symbol of $S$. Observe that $s_{\mathbf{2}}$ is relation-like and function-like.

Let $S$ be a non void $\delta$-concrete many sorted signature and let $o$ be an element of the operation symbols of $S$. One can verify that $o_{\mathbf{2}}$ is relation-like and function-like.

Let $S$ be a non empty cat-signature and let $s$ be a sort symbol of $S$. One can verify that $s_{\mathbf{2}}$ is finite sequence-like.

Let $S$ be a non empty $\delta$-concrete many sorted signature and let $s$ be a sort symbol of $S$. Observe that $s_{\mathbf{2}}$ is finite sequence-like.

Let $S$ be a non void $\delta$-concrete many sorted signature and let $o$ be an element of the operation symbols of $S$. Observe that $o_{\mathbf{2}}$ is finite sequence-like.

Let $a$ be a set. The functor $\operatorname{idsym} a$ is defined as follows:

(Def. 10) $\operatorname{idsym} a = \langle 1, \langle a \rangle \rangle$.

Let $b$ be a set. The functor $\operatorname{homsym}(a, b)$ is defined as follows:

(Def. 11) $\operatorname{homsym}(a, b) = \langle 0, \langle a, b \rangle \rangle$.

Let $c$ be a set. The functor $\operatorname{compsym}(a, b, c)$ is defined as follows:

(Def. 12) $\operatorname{compsym}(a, b, c) = \langle 2, \langle a, b, c \rangle \rangle$.

Next we state the proposition

(20) Let $A$ be a non empty set, $S$ be a cat-signature of $A$, and $a$ be an element of $A$. Then

(i) idsym $a \in$ the operation symbols of $S$, and

(ii) for every element $b$ of $A$ holds homsym$(a, b) \in$ the carrier of $S$ and for every element $c$ of $A$ holds compsym$(a, b, c) \in$ the operation symbols of $S$.

Let $A$ be a non empty set and let $a$ be an element of $A$. Then idsym $a$ is an operation symbol of CatSign$(A)$. Let $b$ be an element of $A$. Then homsym$(a, b)$ is a sort symbol of CatSign$(A)$. Let $c$ be an element of $A$. Then compsym$(a, b, c)$ is an operation symbol of CatSign$(A)$.

We now state several propositions:

(21) For all sets $a$, $b$ such that idsym $a =$ idsym $b$ holds $a = b$.

(22) For all sets $a_1$, $b_1$, $a_2$, $b_2$ such that homsym$(a_1, a_2) =$ homsym$(b_1, b_2)$ holds $a_1 = b_1$ and $a_2 = b_2$.

(23) For all sets $a_1$, $b_1$, $a_2$, $b_2$, $a_3$, $b_3$ such that compsym$(a_1, a_2, a_3) =$ compsym$(b_1, b_2, b_3)$ holds $a_1 = b_1$ and $a_2 = b_2$ and $a_3 = b_3$.

(24) Let $A$ be a non empty set, $S$ be a cat-signature of $A$ and $s$ be a sort symbol of $S$. Then there exist elements $a$, $b$ of $A$ such that $s =$ homsym$(a, b)$.

(25) For every non empty set $A$ and for every operation symbol $o$ of CatSign$(A)$ holds $o_{\mathbf{1}} = 1$ and len$(o_{\mathbf{2}}) = 1$ or $o_{\mathbf{1}} = 2$ and len$(o_{\mathbf{2}}) = 3$.

(26) Let $A$ be a non empty set and $o$ be an operation symbol of CatSign$(A)$. If $o_{\mathbf{1}} = 1$ or len$(o_{\mathbf{2}}) = 1$, then there exists an element $a$ of $A$ such that $o =$ idsym $a$.

(27) Let $A$ be a non empty set and $o$ be an operation symbol of CatSign$(A)$. If $o_{\mathbf{1}} = 2$ or len$(o_{\mathbf{2}}) = 3$, then there exist elements $a$, $b$, $c$ of $A$ such that $o =$ compsym$(a, b, c)$.

(28) For every non empty set $A$ and for every element $a$ of $A$ holds Arity(idsym $a$) $= \varepsilon$ and the result sort of idsym $a =$ homsym$(a, a)$.

(29) For every non empty set $A$ and for all elements $a$, $b$, $c$ of $A$ holds Arity(compsym$(a, b, c)$) $= \langle$homsym$(b, c)$, homsym$(a, b)\rangle$ and the result sort of compsym$(a, b, c) =$ homsym$(a, c)$.

## 4. Signature homomorphism generated by a functor

Let $C_1$, $C_2$ be categories and let $F$ be a functor from $C_1$ to $C_2$. The functor $\Upsilon_F$ yields a function from the carrier of CatSign(the objects of $C_1$) into the carrier of CatSign(the objects of $C_2$) and is defined as follows:

(Def. 13) For every sort symbol $s$ of CatSign(the objects of $C_1$) holds $\Upsilon_F(s) = \langle 0$, Obj $F \cdot s_{\mathbf{2}}\rangle$.

The functor $\Psi_F$ yields a function from the operation symbols of CatSign(the objects of $C_1$) into the operation symbols of CatSign(the objects of $C_2$) and is defined as follows:

(Def. 14) For every operation symbol $o$ of CatSign(the objects of $C_1$) holds $\Psi_F(o) = \langle o_{\mathbf{1}}, \mathrm{Obj}\, F \cdot o_{\mathbf{2}} \rangle$.

The following propositions are true:

(30) For all categories $C_1$, $C_2$ and for every functor $F$ from $C_1$ to $C_2$ and for all objects $a$, $b$ of $C_1$ holds $\Upsilon_F(\mathrm{homsym}(a, b)) = \mathrm{homsym}(F(a), F(b))$.

(31) For all categories $C_1$, $C_2$ and for every functor $F$ from $C_1$ to $C_2$ and for every object $a$ of $C_1$ holds $\Psi_F(\mathrm{idsym}\, a) = \mathrm{idsym}\, F(a)$.

(32) Let $C_1$, $C_2$ be categories, $F$ be a functor from $C_1$ to $C_2$, and $a$, $b$, $c$ be objects of $C_1$. Then $\Psi_F(\mathrm{compsym}(a, b, c)) = \mathrm{compsym}(F(a), F(b), F(c))$.

(33) Let $C_1, C_2$ be categories and $F$ be a functor from $C_1$ to $C_2$. Then $\Upsilon_F$ and $\Psi_F$ form morphism between CatSign(the objects of $C_1$) and CatSign(the objects of $C_2$).

## 5. Algebra of morphisms

Next we state the proposition

(34) For every non empty set $C$ and for every algebra $A$ over CatSign($C$) and for every element $a$ of $C$ holds $\mathrm{Args}(\mathrm{idsym}\, a, A) = \{\varepsilon\}$.

The scheme *CatAlgEx* deals with non empty sets $\mathcal{A}$, $\mathcal{B}$, a binary functor $\mathcal{F}$ yielding a set, a 5-ary functor $\mathcal{G}$ yielding a set, and a unary functor $\mathcal{H}$ yielding a set, and states that:

There exists a strict algebra $A$ over CatSign($\mathcal{A}$) such that

(i) for all elements $a$, $b$ of $\mathcal{A}$ holds (the sorts of $A$)$(\mathrm{homsym}(a, b)) = \mathcal{F}(a, b)$,

(ii) for every element $a$ of $\mathcal{A}$ holds $(\mathrm{Den}(\mathrm{idsym}\, a, A))(\varepsilon) = \mathcal{H}(a)$, and

(iii) for all elements $a$, $b$, $c$ of $\mathcal{A}$ and for all elements $f$, $g$ of $\mathcal{B}$ such that $f \in \mathcal{F}(a, b)$ and $g \in \mathcal{F}(b, c)$ holds $(\mathrm{Den}(\mathrm{compsym}(a, b, c), A))(\langle g, f \rangle) = \mathcal{G}(a, b, c, g, f)$

provided the parameters have the following properties:

- For all elements $a$, $b$ of $\mathcal{A}$ holds $\mathcal{F}(a, b) \subseteq \mathcal{B}$,
- For every element $a$ of $\mathcal{A}$ holds $\mathcal{H}(a) \in \mathcal{F}(a, a)$,
- For all elements $a$, $b$, $c$ of $\mathcal{A}$ and for all elements $f$, $g$ of $\mathcal{B}$ such that $f \in \mathcal{F}(a, b)$ and $g \in \mathcal{F}(b, c)$ holds $\mathcal{G}(a, b, c, g, f) \in \mathcal{F}(a, c)$.

Let $C$ be a category. The functor MSAlg($C$) yielding a strict algebra over CatSign(the objects of $C$) is defined by the conditions (Def. 15).

(Def. 15)(i) For all objects $a, b$ of $C$ holds (the sorts of MSAlg($C$))$(\mathrm{homsym}(a, b)) = \mathrm{hom}(a, b)$,

(ii) for every object $a$ of $C$ holds $(\mathrm{Den}(\mathrm{idsym}\, a, \mathrm{MSAlg}(C)))(\varepsilon) = \mathrm{id}_a$, and

(iii) for all objects $a$, $b$, $c$ of $C$ and for all morphisms $f$, $g$ of $C$ such that $\mathrm{dom}\, f = a$ and $\mathrm{cod}\, f = b$ and $\mathrm{dom}\, g = b$ and $\mathrm{cod}\, g = c$ holds $(\mathrm{Den}(\mathrm{compsym}(a, b, c), \mathrm{MSAlg}(C)))(\langle g, f \rangle) = g \cdot f$.

The following propositions are true:

(35) For every category $A$ and for all objects $a$, $b$ of $A$ holds (the sorts of $\mathrm{MSAlg}(A)$)($\mathrm{homsym}(a,b)$) $= \mathrm{hom}(a,b)$.

(36) For every category $A$ and for every object $a$ of $A$ holds $\mathrm{Result}(\mathrm{idsym}\,a, \mathrm{MSAlg}(A)) = \mathrm{hom}(a,a)$.

(37) For every category $A$ and for all objects $a$, $b$, $c$ of $A$ holds $\mathrm{Args}(\mathrm{compsym}(a,b,c), \mathrm{MSAlg}(A)) = \prod\langle \mathrm{hom}(b,c), \mathrm{hom}(a,b)\rangle$ and $\mathrm{Result}(\mathrm{compsym}(a,b,c), \mathrm{MSAlg}(A)) = \mathrm{hom}(a,c)$.

Let $C$ be a category. Note that $\mathrm{MSAlg}(C)$ is disjoint and feasible.

One can prove the following propositions:

(38) Let $C_1$, $C_2$ be categories and $F$ be a functor from $C_1$ to $C_2$. Then $F\restriction_{\text{the carrier of CatSign(the objects of } C_1)}$the sorts of $\mathrm{MSAlg}(C_1)$ is a many sorted function from $\mathrm{MSAlg}(C_1)$ into $\mathrm{MSAlg}(C_2)\restriction_{(\Upsilon_F,\Psi_F)}$ CatSign(the objects of $C_1$).

(39) Let $C$ be a category, $a$, $b$, $c$ be objects of $C$, and $x$ be a set. Then $x \in \mathrm{Args}(\mathrm{compsym}(a,b,c), \mathrm{MSAlg}(C))$ if and only if there exist morphisms $g$, $f$ of $C$ such that $x = \langle g, f\rangle$ and $\mathrm{dom}\, f = a$ and $\mathrm{cod}\, f = b$ and $\mathrm{dom}\, g = b$ and $\mathrm{cod}\, g = c$.

(40) Let $C_1$, $C_2$ be categories, $F$ be a functor from $C_1$ to $C_2$, $a$, $b$, $c$ be objects of $C_1$, and $f$, $g$ be morphisms of $C_1$. Suppose $f \in \mathrm{hom}(a,b)$ and $g \in \mathrm{hom}(b,c)$. Let $x$ be an element of $\mathrm{Args}(\mathrm{compsym}(a,b,c), \mathrm{MSAlg}(C_1))$. Suppose $x = \langle g, f\rangle$. Let $H$ be a many sorted function from $\mathrm{MSAlg}(C_1)$ into $\mathrm{MSAlg}(C_2)\restriction_{(\Upsilon_F,\Psi_F)}$ CatSign(the objects of $C_1$). Suppose $H = F\restriction_{\text{the carrier of CatSign(the objects of } C_1)}$the sorts of $\mathrm{MSAlg}(C_1)$. Then $H\#x = \langle F(g), F(f)\rangle$.

(41) For every category $C$ and for every object $a$ of $C$ holds $(\mathrm{Den}(\mathrm{idsym}\,a, \mathrm{MSAlg}(C)))(\emptyset) = \mathrm{id}_a$.

(42) Let $C$ be a category, $a$, $b$, $c$ be objects of $C$, and $f$, $g$ be morphisms of $C$. If $f \in \mathrm{hom}(a,b)$ and $g \in \mathrm{hom}(b,c)$, then $(\mathrm{Den}(\mathrm{compsym}(a,b,c), \mathrm{MSAlg}(C)))(\langle g, f\rangle) = g \cdot f$.

(43) Let $C$ be a category, $a$, $b$, $c$, $d$ be objects of $C$, and $f$, $g$, $h$ be morphisms of $C$. Suppose $f \in \mathrm{hom}(a,b)$ and $g \in \mathrm{hom}(b,c)$ and $h \in \mathrm{hom}(c,d)$. Then $(\mathrm{Den}(\mathrm{compsym}(a,c,d), \mathrm{MSAlg}(C)))(\langle h, (\mathrm{Den}(\mathrm{compsym}(a,b,c), \mathrm{MSAlg}(C)))(\langle g, f\rangle)\rangle) = (\mathrm{Den}(\mathrm{compsym}(a,b,d), \mathrm{MSAlg}(C)))(\langle (\mathrm{Den}(\mathrm{compsym}(b,c,d), \mathrm{MSAlg}(C)))(\langle h, g\rangle), f\rangle)$.

(44) Let $C$ be a category, $a$, $b$ be objects of $C$, and $f$ be a morphism of $C$. If $f \in \mathrm{hom}(a,b)$, then $(\mathrm{Den}(\mathrm{compsym}(a,b,b), \mathrm{MSAlg}(C)))(\langle \mathrm{id}_b, f\rangle) = f$ and $(\mathrm{Den}(\mathrm{compsym}(a,a,b), \mathrm{MSAlg}(C)))(\langle f, \mathrm{id}_a\rangle) = f$.

(45) Let $C_1$, $C_2$ be categories and $F$ be a functor from $C_1$ to $C_2$. Then there exists a many sorted function $H$ from $\mathrm{MSAlg}(C_1)$ into $\mathrm{MSAlg}(C_2)\restriction_{(\Upsilon_F,\Psi_F)}$ CatSign(the objects of $C_1$) such that

(i) $H = F\restriction_{\text{the carrier of CatSign(the objects of } C_1)}$the sorts of $\mathrm{MSAlg}(C_1)$, and

(ii) $H$ is a homomorphism of MSAlg($C_1$) into MSAlg($C_2$)$\restriction_{(\Upsilon_F, \Psi_F)}$ CatSign(the objects of $C_1$).

## References


[1] Grzegorz Bancerek. Curried and uncurried functions. *Formalized Mathematics*, 1(**3**):537–541, 1990.
[2] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.
[3] Grzegorz Bancerek. König's theorem. *Formalized Mathematics*, 1(**3**):589–593, 1990.
[4] Grzegorz Bancerek. Cartesian product of functions. *Formalized Mathematics*, 2(**4**):547–552, 1991.
[5] Grzegorz Bancerek. Minimal signature for partial algebra. *Formalized Mathematics*, 5(**3**):405–414, 1996.
[6] Grzegorz Bancerek. Terms over many sorted universal algebra. *Formalized Mathematics*, 5(**2**):191–198, 1996.
[7] Grzegorz Bancerek. Translations, endomorphisms, and stable equational theories. *Formalized Mathematics*, 5(**4**):553–564, 1996.
[8] Grzegorz Bancerek. Institution of many sorted algebras. Part I: Signature reduct of an algebra. *Formalized Mathematics*, 6(**2**):279–287, 1997.
[9] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[10] Czesław Byliński. Finite sequences and tuples of elements of a non-empty sets. *Formalized Mathematics*, 1(**3**):529–536, 1990.
[11] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[12] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[13] Czesław Byliński. Introduction to categories and functors. *Formalized Mathematics*, 1(**2**):409–420, 1990.
[14] Czesław Byliński. Partial functions. *Formalized Mathematics*, 1(**2**):357–367, 1990.
[15] Czesław Byliński. Some basic properties of sets. *Formalized Mathematics*, 1(**1**):47–53, 1990.
[16] Artur Korniłowicz. Extensions of mappings on generator set. *Formalized Mathematics*, 5(**2**):269–272, 1996.
[17] Małgorzata Korolkiewicz. Homomorphisms of many sorted algebras. *Formalized Mathematics*, 5(**1**):61–65, 1996.
[18] Beata Madras. Product of family of universal algebras. *Formalized Mathematics*, 4(**1**):103–108, 1993.
[19] Andrzej Nędzusiak. $\sigma$-fields and probability. *Formalized Mathematics*, 1(**2**):401–407, 1990.
[20] Beata Perkowska. Free many sorted universal algebra. *Formalized Mathematics*, 5(**1**):67–74, 1996.
[21] Andrzej Trybulec. Enumerated sets. *Formalized Mathematics*, 1(**1**):25–34, 1990.
[22] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[23] Andrzej Trybulec. Tuples, projections and Cartesian products. *Formalized Mathematics*, 1(**1**):97–105, 1990.
[24] Andrzej Trybulec. Many-sorted sets. *Formalized Mathematics*, 4(**1**):15–22, 1993.
[25] Andrzej Trybulec. Many sorted algebras. *Formalized Mathematics*, 5(**1**):37–42, 1996.
[26] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[27] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[28] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.


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