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Projections in n-Dimensional Euclidean Space to Each Coordinates

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Summary. In the n-dimensional Euclidean space $\mathcal{E}^n_{\rm T}$, a projection operator to each coordinate is defined. It is proven that such an operator is linear. Moreover, it is continuous as a mapping from $\mathcal{E}^n_{\rm T}$ to R^1, the carrier of which is a set of all reals. If n is 1, the projection becomes a homeomorphism, which means that $\mathcal{E}^1_{\rm T}$ is homeomorphic to R^1.

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The notation and terminology used in this paper are introduced in the following articles: [30], [35], [34], [20], [1], [37], [33], [27], [12], [29], [11], [26], [23], [36], [2], [8], [9], [5], [32], [3], [18], [17], [25], [15], [10], [14], [31], [16], [19], [22], [7], [24], [13], [21], [4], [6], and [28].

1. Projections

For simplicity, we use the following convention: a, b, s, s_1, r, r_1, r_2 denote real numbers, n, i denote natural numbers, X denotes a non empty topological space, p, p_1, p_2, q denote points of $\mathcal{E}^n_{\rm T}$, P denotes a subset of the carrier of $\mathcal{E}^n_{\rm T}$, and f denotes a map from $\mathcal{E}^n_{\rm T}$ into $\mathbb{R}^{\mathbf{1}}$.

Let n, i be natural numbers and let p be an element of the carrier of $\mathcal{E}^n_{\rm T}$. The functor $\mathrm{Proj}(p,i)$ yielding a real number is defined as follows:

(Def. 1) For every finite sequence g of elements of $\mathbb{R}$ such that $g = p$ holds $\mathrm{Proj}(p,i) = \pi_i g$.

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The following propositions are true:

(1) For every i there exists a map f from $\mathcal{E}^n_{\rm T}$ into $\mathbb{R}^{\bf 1}$ such that for every element p of the carrier of $\mathcal{E}^n_{\rm T}$ holds $f(p) = \mathrm{Proj}(p, i)$.

(2) For every i such that $i \in \mathrm{Seg}\, n$ holds $\langle \underbrace{0, \ldots, 0}_{n} \rangle(i) = 0$.

(3) For every i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(0_{\mathcal{E}^n_{\rm T}}, i) = 0$.

(4) For all r, p, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(r \cdot p, i) = r \cdot \mathrm{Proj}(p, i)$.

(5) For all p, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(-p, i) = -\mathrm{Proj}(p, i)$.

(6) For all p_1, p_2, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(p_1 + p_2, i) = \mathrm{Proj}(p_1, i) + \mathrm{Proj}(p_2, i)$.

(7) For all p_1, p_2, i such that $i \in \mathrm{Seg}\, n$ holds $\mathrm{Proj}(p_1 - p_2, i) = \mathrm{Proj}(p_1, i) - \mathrm{Proj}(p_2, i)$.

(8) $\mathrm{len} \langle \underbrace{0, \ldots, 0}_{n} \rangle = n$.

(9) For every i such that $i \leqslant n$ holds $\langle \underbrace{0, \ldots, 0}_{n} \rangle \restriction i = \langle \underbrace{0, \ldots, 0}_{i} \rangle$.

(10) For every i holds $\langle \underbrace{0, \ldots, 0}_{n} \rangle_{\lfloor i} = \langle \underbrace{0, \ldots, 0}_{n -' i} \rangle$.

(11) For every i holds $\sum \langle \underbrace{0, \ldots, 0}_{i} \rangle = 0$.

(12) For every finite sequence w and for all r, i holds $\mathrm{len}(w \mathbin{+\cdot} (i, r)) = \mathrm{len}\, w$.

(13) For every finite sequence w of elements of $\mathbb{R}$ and for all r, i such that $i \in \mathrm{Seg}\, \mathrm{len}\, w$ holds $w \mathbin{+\cdot} (i, r) = (w \restriction i -' 1) \mathbin{^\frown} \langle r \rangle \mathbin{^\frown} (w_{\lfloor i})$.

(14) For all i, r such that $i \in \mathrm{Seg}\, n$ holds $\sum(\langle \underbrace{0, \ldots, 0}_{n} \rangle \mathbin{+\cdot} (i, r)) = r$.

(15) For every element q of $\mathcal{R}^n$ and for all p, i such that $i \in \mathrm{Seg}\, n$ and $q = p$ holds $\mathrm{Proj}(p, i) \leqslant |q|$ and $(\mathrm{Proj}(p, i))^{\bf 2} \leqslant |q|^{\bf 2}$.

2. Continuity of Projections

Next we state several propositions:

(16) For all s_1, P, i such that $P = \{p : s_1 > \mathrm{Proj}(p, i)\}$ and $i \in \mathrm{Seg}\, n$ holds P is open.

(17) For all s_1, P, i such that $P = \{p : s_1 < \mathrm{Proj}(p, i)\}$ and $i \in \mathrm{Seg}\, n$ holds P is open.

(18) Let P be a subset of the carrier of $\mathcal{E}^n_{\rm T}$, a, b be real numbers, and given i. Suppose $P = \{p; p$ ranges over elements of the carrier of $\mathcal{E}^n_{\rm T}$: $a < \mathrm{Proj}(p, i) \ \wedge\ \mathrm{Proj}(p, i) < b\}$ and $i \in \mathrm{Seg}\, n$. Then P is open.

(19) Let a, b be real numbers, f be a map from $\mathcal{E}^n_{\rm T}$ into $\mathbb{R}^{\mathbf{1}}$, and given i. Suppose that for every element p of the carrier of $\mathcal{E}^n_{\rm T}$ holds $f(p) = {\rm Proj}(p, i)$. Then $f^{-1}(\{s : a < s \ \wedge\ s < b\}) = \{p; p$ ranges over elements of the carrier of $\mathcal{E}^n_{\rm T}$: $a < {\rm Proj}(p, i) \ \wedge\ {\rm Proj}(p, i) < b\}$.

(20) Let M be a metric space and f be a map from X into $M_{\rm top}$. Suppose that for every real number r and for every element u of the carrier of M and for every subset P of the carrier of $M_{\rm top}$ such that $r > 0$ and $P = {\rm Ball}(u, r)$ holds $f^{-1}(P)$ is open. Then f is continuous.

(21) Let u be a point of the metric space of real numbers and r, u_1 be real numbers. If $u_1 = u$ and $r > 0$, then ${\rm Ball}(u, r) = \{s : u_1 - r < s \ \wedge\ s < u_1 + r\}$.

(22) Let f be a map from $\mathcal{E}^n_{\rm T}$ into $\mathbb{R}^{\mathbf{1}}$ and given i. Suppose $i \in {\rm Seg}\, n$ and for every element p of the carrier of $\mathcal{E}^n_{\rm T}$ holds $f(p) = {\rm Proj}(p, i)$. Then f is continuous.

3. 1-dimensional and 2-dimensional Cases

The following three propositions are true:

(23) For every s holds $|\langle s\rangle| = \langle |s|\rangle$.

(24) For every element p of the carrier of $\mathcal{E}^1_{\rm T}$ there exists r such that $p = \langle r\rangle$.

(25) For every element w of the carrier of $\mathcal{E}^1$ there exists r such that $w = \langle r\rangle$.

Let us consider r. The functor $|[r]|$ yields a point of $\mathcal{E}^1_{\rm T}$ and is defined by:

(Def. 2) $|[r]| = \langle r\rangle$.

The following propositions are true:

(26) For all r, s holds $s \cdot |[r]| = |[s \cdot r]|$.

(27) For all r_1, r_2 holds $|[r_1 + r_2]| = |[r_1]| + |[r_2]|$.

(28) $|[0]| = 0_{\mathcal{E}^1_{\rm T}}$.

(29) For all r_1, r_2 such that $|[r_1]| = |[r_2]|$ holds $r_1 = r_2$.

(30) For every subset P of the carrier of $\mathbb{R}^{\mathbf{1}}$ and for every real number b such that $P = \{s : s < b\}$ holds P is open.

(31) For every subset P of the carrier of $\mathbb{R}^{\mathbf{1}}$ and for every real number a such that $P = \{s : a < s\}$ holds P is open.

(32) For every subset P of the carrier of $\mathbb{R}^{\mathbf{1}}$ and for all real numbers a, b such that $P = \{s : a < s \ \wedge\ s < b\}$ holds P is open.

(33) For every point u of $\mathcal{E}^1$ and for all real numbers r, u_1 such that $\langle u_1\rangle = u$ and $r > 0$ holds ${\rm Ball}(u, r) = \{\langle s\rangle : u_1 - r < s \ \wedge\ s < u_1 + r\}$.

(34) Let f be a map from $\mathcal{E}^1_{\rm T}$ into $\mathbb{R}^{\mathbf{1}}$. Suppose that for every element p of the carrier of $\mathcal{E}^1_{\rm T}$ holds $f(p) = {\rm Proj}(p, 1)$. Then f is a homeomorphism.

(35) For every element p of the carrier of $\mathcal{E}^2_{\rm T}$ holds $\mathrm{Proj}(p, 1) = p_{\mathbf{1}}$ and $\mathrm{Proj}(p, 2) = p_{\mathbf{2}}$.

(36) For every element p of the carrier of $\mathcal{E}^2_{\rm T}$ holds $\mathrm{Proj}(p, 1) = (\mathrm{proj1})(p)$ and $\mathrm{Proj}(p, 2) = (\mathrm{proj2})(p)$.

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