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On the Dividing Function of the Simple Closed Curve into Segments

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Summary. At the beginning, the concept of the segment of the simple closed curve in 2-dimensional Euclidean space is defined. Some properties of segments are shown in the succeeding theorems. At the end, the existence of the function which can divide the simple closed curve into segments is shown. We can make the diameter of segments as small as we want.

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The terminology and notation used in this paper are introduced in the following papers: [17], [5], [7], [2], [15], [3], [11], [12], [13], [1], [14], [4], [18], [16], [10], [8], [9], and [6].

1. Definition of the Segment and Its Property

In this paper p, p_1, q are points of $\mathcal{E}^2_{\rm T}$.

The following three propositions are true:

(1) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$. Suppose P is a simple closed curve. Then $\text{W-min}\,P \in \text{LowerArc}\,P$ and $\text{E-max}\,P \in \text{LowerArc}\,P$ and $\text{W-min}\,P \in \text{UpperArc}\,P$ and $\text{E-max}\,P \in \text{UpperArc}\,P$.

(2) For every compact non empty subset P of $\mathcal{E}^2_{\rm T}$ and for every q such that P is a simple closed curve and $\text{LE}(q, \text{W-min}\,P, P)$ holds $q = \text{W-min}\,P$.

(3) For every compact non empty subset P of $\mathcal{E}^2_{\rm T}$ and for every q such that P is a simple closed curve and $q \in P$ holds $\text{LE}(\text{W-min}\,P, q, P)$.

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Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and let q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$. The functor $\mathrm{Segment}(q_1, q_2, P)$ yields a subset of $\mathcal{E}^2_{\rm T}$ and is defined by:

$$\text{(Def. 1)}\quad \mathrm{Segment}(q_1, q_2, P) = \begin{cases} \{p : \mathrm{LE}(q_1, p, P) \ \land \ \mathrm{LE}(p, q_2, P)\}, \\ \text{ if } q_2 \neq \text{W-min}\, P, \\ \{p_1 : \mathrm{LE}(q_1, p_1, P) \ \lor \ q_1 \in P \ \land \ p_1 = \text{W-min}\, P\}, \\ \text{ otherwise.} \end{cases}$$

One can prove the following propositions:

(4) For every compact non empty subset P of $\mathcal{E}^2_{\rm T}$ such that P is a simple closed curve holds $\mathrm{Segment}(\text{W-min}\, P, \text{E-max}\, P, P) = \mathrm{UpperArc}\, P$ and $\mathrm{Segment}(\text{E-max}\, P, \text{W-min}\, P, P) = \mathrm{LowerArc}\, P$.

(5) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$. If P is a simple closed curve and $\mathrm{LE}(q_1, q_2, P)$, then $q_1 \in P$ and $q_2 \in P$.

(6) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$. If P is a simple closed curve and $\mathrm{LE}(q_1, q_2, P)$, then $q_1 \in \mathrm{Segment}(q_1, q_2, P)$ and $q_2 \in \mathrm{Segment}(q_1, q_2, P)$.

(7) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and q be a point of $\mathcal{E}^2_{\rm T}$. If P is a simple closed curve and $q \in P$ and $q \neq \text{W-min}\, P$, then $\mathrm{Segment}(q, q, P) = \{q\}$.

(8) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$. If P is a simple closed curve and $q_1 \neq \text{W-min}\, P$ and $q_2 \neq \text{W-min}\, P$, then $\text{W-min}\, P \notin \mathrm{Segment}(q_1, q_2, P)$.

(9) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and q_1, q_2, q_3 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is a simple closed curve and $\mathrm{LE}(q_1, q_2, P)$ and $\mathrm{LE}(q_2, q_3, P)$ and $q_1 = q_2$ and $q_1 = \text{W-min}\, P$ and $q_1 \neq q_3$ and $q_2 = q_3$ and $q_2 = \text{W-min}\, P$. Then $\mathrm{Segment}(q_1, q_2, P) \cap \mathrm{Segment}(q_2, q_3, P) = \{q_2\}$.

(10) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and q_1, q_2 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is a simple closed curve and $\mathrm{LE}(q_1, q_2, P)$ and $q_1 \neq q_2$ and $q_1 \neq \text{W-min}\, P$. Then $\mathrm{Segment}(q_2, \text{W-min}\, P, P) \cap \mathrm{Segment}(\text{W-min}\, P, q_1, P) = \{\text{W-min}\, P\}$.

(11) Let P be a compact non empty subset of $\mathcal{E}^2_{\rm T}$ and q_1, q_2, q_3, q_4 be points of $\mathcal{E}^2_{\rm T}$. Suppose P is a simple closed curve and $\mathrm{LE}(q_1, q_2, P)$ and $\mathrm{LE}(q_2, q_3, P)$ and $\mathrm{LE}(q_3, q_4, P)$ and $q_1 \neq q_2$ and $q_2 \neq q_3$. Then $\mathrm{Segment}(q_1, q_2, P) \cap \mathrm{Segment}(q_3, q_4, P) = \emptyset$.

2. A Function to Divide the Simple Closed Curve

In the sequel n is a natural number.

We now state three propositions:

(12) Let P be a non empty subset of the carrier of $\mathcal{E}^n_{\mathrm{T}}$ and f be a map from $\mathbb{I}$ into $(\mathcal{E}^n_{\mathrm{T}})\restriction P$. Suppose $P \neq \emptyset$ and f is a homeomorphism. Then there exists a map g from $\mathbb{I}$ into $\mathcal{E}^n_{\mathrm{T}}$ such that $f = g$ and g is continuous and one-to-one.

(13) For every finite sequence f of elements of $\mathbb{R}$ such that f is increasing holds f is one-to-one.

(14) Let P be a compact non empty subset of $\mathcal{E}^2_{\mathrm{T}}$ and e be a real number. Suppose P is a simple closed curve and $e > 0$. Then there exists a finite sequence h of elements of the carrier of $\mathcal{E}^2_{\mathrm{T}}$ such that

(i) $h(1) = \text{W-min}\, P$,

(ii) h is one-to-one,

(iii) $8 \leqslant \operatorname{len} h$,

(iv) $\operatorname{rng} h \subseteq P$,

(v) for every natural number i such that $1 \leqslant i$ and $i < \operatorname{len} h$ holds $\mathrm{LE}(\pi_i h, \pi_{i+1} h, P)$,

(vi) for every natural number i and for every subset W of the carrier of $\mathcal{E}^2$ such that $1 \leqslant i$ and $i < \operatorname{len} h$ and $W = \mathrm{Segment}(\pi_i h, \pi_{i+1} h, P)$ holds $\varnothing W < e$,

(vii) for every subset W of the carrier of $\mathcal{E}^2$ such that $W = \mathrm{Segment}(\pi_{\operatorname{len} h} h, \pi_1 h, P)$ holds $\varnothing W < e$,

(viii) for every natural number i such that $1 \leqslant i$ and $i + 1 < \operatorname{len} h$ holds $\mathrm{Segment}(\pi_i h, \pi_{i+1} h, P) \cap \mathrm{Segment}(\pi_{i+1} h, \pi_{i+2} h, P) = \{\pi_{i+1} h\}$,

(ix) $\mathrm{Segment}(\pi_{\operatorname{len} h} h, \pi_1 h, P) \cap \mathrm{Segment}(\pi_1 h, \pi_2 h, P) = \{\pi_1 h\}$, and

(x) for all natural numbers i, j such that $1 \leqslant i$ and $i < \operatorname{len} h$ and $1 \leqslant j$ and $j < \operatorname{len} h$ and $i \neq j$ and i and j are not adjacent holds $\mathrm{Segment}(\pi_i h, \pi_{i+1} h, P) \cap \mathrm{Segment}(\pi_j h, \pi_{j+1} h, P) = \emptyset$.

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