

# Bubble Sort on $\mathbf{SCM}_{\text{FSA}}$

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**Summary.** We present the bubble sorting algorithm using macro instructions such as the if-Macro (conditional branch macro instructions) and the Times-Macro (for-loop macro instructions) etc. The correctness proof of the program should include the proof of autonomic, halting and the correctness of the program result. In the three terms, we justify rigorously the correctness of the bubble sorting algorithm. In order to prove it is autonomic, we use the following theorem: if all variables used by the program are initialized, it is autonomic. This justification method probably reveals that autonomic concept is not important.

MML Identifier: SCMBSORT.

The articles [18], [24], [21], [19], [31], [7], [9], [12], [22], [10], [13], [29], [14], [15], [11], [28], [8], [32], [17], [26], [5], [6], [3], [1], [2], [4], [27], [25], [16], [20], [30], and [23] provide the terminology and notation for this paper.

## 1. PRELIMINARIES

For simplicity, we adopt the following rules:  $p$  is a programmed finite partial state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $i_1$  is an instruction of  $\mathbf{SCM}_{\text{FSA}}$ ,  $i, j, k$  are natural numbers,  $f_1, f$  are finite sequence locations,  $a, b, d_1, d_2$  are integer locations,  $l, l_1$  are instructions-locations of  $\mathbf{SCM}_{\text{FSA}}$ , and  $s_1$  is a state of  $\mathbf{SCM}_{\text{FSA}}$ .

We now state a number of propositions:

- (1) Let  $I, J$  be macro instructions and  $a, b$  be integer locations. Suppose  $I$  does not destroy  $b$  and  $J$  does not destroy  $b$ . Then **if**  $a > 0$  **then**  $I$  **else**  $J$  does not destroy  $b$ .

- (2) Let  $I, J$  be macro instructions and  $a, b$  be integer locations. Suppose  $I$  does not destroy  $b$  and  $J$  does not destroy  $b$ . Then **if**  $a = 0$  **then**  $I$  **else**  $J$  does not destroy  $b$ .
- (3) Let  $I$  be a macro instruction and  $a, b$  be integer locations. If  $I$  does not destroy  $b$  and  $a \neq b$ , then  $\text{Times}(a, I)$  does not destroy  $b$ .
- (4) For every function  $f$  and for all sets  $n, m$  holds  
 $(f + \cdot (n \dot{\rightarrow} m) + \cdot (m \dot{\rightarrow} n))(m) = n$ .
- (5) For every function  $f$  and for all sets  $n, m$  holds  
 $(f + \cdot (n \dot{\rightarrow} m) + \cdot (m \dot{\rightarrow} n))(n) = m$ .
- (6) For every function  $f$  and for all sets  $n, m, x$  such that  $x \in \text{dom } f$  and  $x \neq m$  and  $x \neq n$  holds  $(f + \cdot (n \dot{\rightarrow} m) + \cdot (m \dot{\rightarrow} n))(x) = f(x)$ .
- (7) Let  $f, g$  be functions and  $m, n$  be sets. Suppose that
- (i)  $f(m) = g(n)$ ,
  - (ii)  $f(n) = g(m)$ ,
  - (iii)  $m \in \text{dom } f$ ,
  - (iv)  $n \in \text{dom } f$ ,
  - (v)  $\text{dom } f = \text{dom } g$ , and
  - (vi) for every set  $k$  such that  $k \neq m$  and  $k \neq n$  and  $k \in \text{dom } f$  holds  $f(k) = g(k)$ .
- Then  $f$  and  $g$  are fiberwise equipotent.
- (8) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $f$  be a finite sequence location, and  $a, b$  be integer locations. Then  $(\text{Exec}(b := f_a, s))(b) = \pi_{|s(a)|} s(f)$ .
- (9) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $f$  be a finite sequence location, and  $a, b$  be integer locations. Then  $(\text{Exec}(f_a := b, s))(f) = s(f) + \cdot (|s(a)|, s(b))$ .
- (10) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $f$  be a finite sequence location,  $m, n$  be natural numbers, and  $a$  be an integer location. If  $m \neq n + 1$ , then  $(\text{Exec}(\text{intloc}(m) := f_a, \text{Initialize}(s)))(\text{intloc}(n + 1)) = s(\text{intloc}(n + 1))$ .
- (11) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $m, n$  be natural numbers, and  $a$  be an integer location. If  $m \neq n + 1$ , then  $(\text{Exec}(\text{intloc}(m) := a, \text{Initialize}(s)))(\text{intloc}(n + 1)) = s(\text{intloc}(n + 1))$ .
- (12) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $f$  be a finite sequence location, and  $a$  be a read-write integer location. Then  $(\text{IExec}(\text{Stop}_{\text{SCM}_{\text{FSA}}}, s))(a) = s(a)$  and  $(\text{IExec}(\text{Stop}_{\text{SCM}_{\text{FSA}}}, s))(f) = s(f)$ .

In the sequel  $n$  denotes a natural number.

One can prove the following propositions:

- (13) If  $n \leq 10$ , then  $n = 0$  or  $n = 1$  or  $n = 2$  or  $n = 3$  or  $n = 4$  or  $n = 5$  or  $n = 6$  or  $n = 7$  or  $n = 8$  or  $n = 9$  or  $n = 10$ .
- (14) Suppose  $n \leq 12$ . Then  $n = 0$  or  $n = 1$  or  $n = 2$  or  $n = 3$  or  $n = 4$  or  $n = 5$  or  $n = 6$  or  $n = 7$  or  $n = 8$  or  $n = 9$  or  $n = 10$  or  $n = 11$  or  $n = 12$ .

- (15) Let  $f, g$  be functions and  $X$  be a set. If  $\text{dom } f = \text{dom } g$  and for every set  $x$  such that  $x \in X$  holds  $f(x) = g(x)$ , then  $f \upharpoonright X = g \upharpoonright X$ .
- (16) If  $i_1 \in \text{rng } p$  and if  $i_1 = a := b$  or  $i_1 = \text{AddTo}(a, b)$  or  $i_1 = \text{SubFrom}(a, b)$  or  $i_1 = \text{MultBy}(a, b)$  or  $i_1 = \text{Divide}(a, b)$ , then  $a \in \text{UsedIntLoc}(p)$  and  $b \in \text{UsedIntLoc}(p)$ .
- (17) If  $i_1 \in \text{rng } p$  and if  $i_1 = \mathbf{if } a = 0 \mathbf{ goto } l_1$  or  $i_1 = \mathbf{if } a > 0 \mathbf{ goto } l_1$ , then  $a \in \text{UsedIntLoc}(p)$ .
- (18) If  $i_1 \in \text{rng } p$  and if  $i_1 = b := f_{1a}$  or  $i_1 = f_{1a} := b$ , then  $a \in \text{UsedIntLoc}(p)$  and  $b \in \text{UsedIntLoc}(p)$ .
- (19) If  $i_1 \in \text{rng } p$  and if  $i_1 = b := f_{1a}$  or  $i_1 = f_{1a} := b$ , then  $f_1 \in \text{UsedInt}^* \text{Loc}(p)$ .
- (20) If  $i_1 \in \text{rng } p$  and if  $i_1 = a := \text{len } f_1$  or  $i_1 = f_1 := \underbrace{\langle 0, \dots, 0 \rangle}_a$ , then  $a \in \text{UsedIntLoc}(p)$ .
- (21) If  $i_1 \in \text{rng } p$  and if  $i_1 = a := \text{len } f_1$  or  $i_1 = f_1 := \underbrace{\langle 0, \dots, 0 \rangle}_a$ , then  $f_1 \in \text{UsedInt}^* \text{Loc}(p)$ .
- (22) Let  $p$  be a macro instruction,  $s_2, s_3$  be states of  $\mathbf{SCM}_{\text{FSA}}$ , and given  $i$ . If  $p \subseteq s_2$  and  $p \subseteq s_3$ , then  $(\text{Computation}(s_2))(i) \upharpoonright \text{dom } p = (\text{Computation}(s_3))(i) \upharpoonright \text{dom } p$ .
- (23) Let  $t$  be a finite partial state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $p$  be a macro instruction, and  $x$  be a set. Suppose  $\text{dom } t \subseteq \text{Int-Locations} \cup \text{FinSeq-Locations}$  and  $x \in \text{dom } t \cup \text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p)$ . Then  $x$  is an integer location or a finite sequence location.
- (24) For every  $f_1$  holds  $(\text{Exec}(\text{Divide}(d_1, d_2), s_1))(f_1) = s_1(f_1)$  and  $(\text{Exec}(\text{Divide}(d_1, d_2), s_1))(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}) = \text{Next}(\mathbf{IC}_{(s_1)})$ .
- (25) Let  $i, k$  be natural numbers,  $t$  be a finite partial state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $p$  be a macro instruction, and  $s_2, s_3$  be states of  $\mathbf{SCM}_{\text{FSA}}$ . Suppose that
- (i)  $k \leq i$ ,
  - (ii)  $p \subseteq s_2$ ,
  - (iii)  $p \subseteq s_3$ ,
  - (iv)  $\text{dom } t \subseteq \text{Int-Locations} \cup \text{FinSeq-Locations}$ ,
  - (v) for every  $j$  holds  $\mathbf{IC}_{(\text{Computation}(s_2))(j)} \in \text{dom } p$  and  $\mathbf{IC}_{(\text{Computation}(s_3))(j)} \in \text{dom } p$ ,
  - (vi)  $(\text{Computation}(s_2))(k)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}) = (\text{Computation}(s_3))(k)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}})$ ,  
and
  - (vii)  $(\text{Computation}(s_2))(k) \upharpoonright (\text{dom } t \cup \text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p)) = (\text{Computation}(s_3))(k) \upharpoonright (\text{dom } t \cup \text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p))$ .
- Then  $(\text{Computation}(s_2))(i)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}) = (\text{Computation}(s_3))(i)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}})$   
and  $(\text{Computation}(s_2))(i) \upharpoonright (\text{dom } t \cup \text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p)) = (\text{Computation}(s_3))(i) \upharpoonright (\text{dom } t \cup \text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p))$ .

- (26) Let  $i, k$  be natural numbers,  $p$  be a macro instruction, and  $s_2, s_3$  be states of  $\mathbf{SCM}_{\text{FSA}}$ . Suppose  $k \leq i$  and  $p \subseteq s_2$  and  $p \subseteq s_3$  and for every  $j$  holds  $\mathbf{IC}_{(\text{Computation}(s_2))(j)} \in \text{dom } p$  and  $\mathbf{IC}_{(\text{Computation}(s_3))(j)} \in \text{dom } p$  and  $(\text{Computation}(s_2))(k)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}) = (\text{Computation}(s_3))(k)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}})$  and  $(\text{Computation}(s_2))(k) \upharpoonright (\text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p)) = (\text{Computation}(s_3))(k) \upharpoonright (\text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p))$ .  
Then  $(\text{Computation}(s_2))(i)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}) = (\text{Computation}(s_3))(i)(\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}})$  and  $(\text{Computation}(s_2))(i) \upharpoonright (\text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p)) = (\text{Computation}(s_3))(i) \upharpoonright (\text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p))$ .
- (27)  $\text{UsedIntLoc}(\text{Stop}_{\mathbf{SCM}_{\text{FSA}}}) = \emptyset$ .
- (28)  $\text{UsedIntLoc}(\text{Goto}(l)) = \emptyset$ .
- (29) For all macro instructions  $I, J$  and for every integer location  $a$  holds  $\text{UsedIntLoc}(\mathbf{if } a = 0 \mathbf{ then } I \mathbf{ else } J) = \{a\} \cup \text{UsedIntLoc}(I) \cup \text{UsedIntLoc}(J)$  and  $\text{UsedIntLoc}(\mathbf{if } a > 0 \mathbf{ then } I \mathbf{ else } J) = \{a\} \cup \text{UsedIntLoc}(I) \cup \text{UsedIntLoc}(J)$ .
- (30) For every macro instruction  $I$  and for every instruction-location  $l$  of  $\mathbf{SCM}_{\text{FSA}}$  holds  $\text{UsedIntLoc}(\text{Directed}(I, l)) = \text{UsedIntLoc}(I)$ .
- (31) For every integer location  $a$  and for every macro instruction  $I$  holds  $\text{UsedIntLoc}(\text{Times}(a, I)) = \text{UsedIntLoc}(I) \cup \{a, \text{intloc}(0)\}$ .
- (32) For all sets  $x_1, x_2, x_3$  holds  $\{x_2, x_1\} \cup \{x_3, x_1\} = \{x_1, x_2, x_3\}$ .
- (33)  $\text{UsedInt}^* \text{Loc}(\text{Stop}_{\mathbf{SCM}_{\text{FSA}}}) = \emptyset$ .
- (34)  $\text{UsedInt}^* \text{Loc}(\text{Goto}(l)) = \emptyset$ .
- (35) For all macro instructions  $I, J$  and for every integer location  $a$  holds  $\text{UsedInt}^* \text{Loc}(\mathbf{if } a = 0 \mathbf{ then } I \mathbf{ else } J) = \text{UsedInt}^* \text{Loc}(I) \cup \text{UsedInt}^* \text{Loc}(J)$  and  $\text{UsedInt}^* \text{Loc}(\mathbf{if } a > 0 \mathbf{ then } I \mathbf{ else } J) = \text{UsedInt}^* \text{Loc}(I) \cup \text{UsedInt}^* \text{Loc}(J)$ .
- (36) For every macro instruction  $I$  and for every instruction-location  $l$  of  $\mathbf{SCM}_{\text{FSA}}$  holds  $\text{UsedInt}^* \text{Loc}(\text{Directed}(I, l)) = \text{UsedInt}^* \text{Loc}(I)$ .
- (37) For every integer location  $a$  and for every macro instruction  $I$  holds  $\text{UsedInt}^* \text{Loc}(\text{Times}(a, I)) = \text{UsedInt}^* \text{Loc}(I)$ .

Let  $f$  be a finite sequence location and let  $t$  be a finite sequence of elements of  $\mathbb{Z}$ . Then  $f \mapsto t$  is a finite partial state of  $\mathbf{SCM}_{\text{FSA}}$ .

One can prove the following propositions:

- (38) Every finite sequence of elements of  $\mathbb{Z}$  is a finite sequence of elements of  $\mathbb{R}$ .
- (39) Let  $t$  be a finite sequence of elements of  $\mathbb{Z}$ . Then there exists a finite sequence  $u$  of elements of  $\mathbb{R}$  such that  $t$  and  $u$  are fiberwise equipotent and  $u$  is a finite sequence of elements of  $\mathbb{Z}$  and non-increasing.
- (40)  $\text{dom}((\text{intloc}(0) \mapsto 1) + \cdot \text{Start-At}(\text{insloc}(0))) = \{\text{intloc}(0), \mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}\}$ .

- (41) For every macro instruction  $I$  holds  $\text{dom Initialized}(I) = \text{dom } I \cup \{\text{intloc}(0), \mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}\}$ .
- (42) Let  $w$  be a finite sequence of elements of  $\mathbb{Z}$ ,  $f$  be a finite sequence location, and  $I$  be a macro instruction. Then  $\text{dom}(\text{Initialized}(I) + \cdot (f \mapsto w)) = \text{dom } I \cup \{\text{intloc}(0), \mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}, f\}$ .
- (43) For every instruction-location  $l$  of  $\mathbf{SCM}_{\text{FSA}}$  holds  $\mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}} \neq l$ .
- (44) For every integer location  $a$  and for every macro instruction  $I$  holds  $\text{card Times}(a, I) = \text{card } I + 12$ .
- (45) For all instructions  $i_2, i_3, i_4$  of  $\mathbf{SCM}_{\text{FSA}}$  holds  $\text{card}(i_2; i_3; i_4) = 6$ , where  $i_2 = b_4 := b_3$ ,  $b_4 = \text{intloc}(3 + 1)$ ,  $b_3 = \text{intloc}(2 + 1)$ ,  $i_3 = \text{SubFrom}(b_3, a_0)$ ,  $a_0 = \text{intloc}(0)$ ,  $i_4 = b_5 := f_{0b_3}$ ,  $b_5 = \text{intloc}(4 + 1)$ , and  $f_0 = \text{fsloc}(0)$ .
- (46) Let  $t$  be a finite sequence of elements of  $\mathbb{Z}$ ,  $f$  be a finite sequence location, and  $I$  be a macro instruction. Then  $\text{dom Initialized}(I) \cap \text{dom}(f \mapsto t) = \emptyset$ .
- (47) Let  $w$  be a finite sequence of elements of  $\mathbb{Z}$ ,  $f$  be a finite sequence location, and  $I$  be a macro instruction. Then  $\text{Initialized}(I) + \cdot (f \mapsto w)$  starts at  $\text{insloc}(0)$ .
- (48) Let  $I, J$  be macro instructions,  $k$  be a natural number, and  $i$  be an instruction of  $\mathbf{SCM}_{\text{FSA}}$ . If  $k < \text{card } J$  and  $i = J(\text{insloc}(k))$ , then  $(I; J)(\text{insloc}(\text{card } I + k)) = \text{IncAddr}(i, \text{card } I)$ .
- (49) Suppose that
- (i)  $i_1 = a := b$ , or
  - (ii)  $i_1 = \text{AddTo}(a, b)$ , or
  - (iii)  $i_1 = \text{SubFrom}(a, b)$ , or
  - (iv)  $i_1 = \text{MultBy}(a, b)$ , or
  - (v)  $i_1 = \text{Divide}(a, b)$ , or
  - (vi)  $i_1 = \text{goto } l_1$ , or
  - (vii)  $i_1 = \mathbf{if } a = 0 \mathbf{ goto } l_1$ , or
  - (viii)  $i_1 = \mathbf{if } a > 0 \mathbf{ goto } l_1$ , or
  - (ix)  $i_1 = b := f_a$ , or
  - (x)  $i_1 = f_a := b$ , or
  - (xi)  $i_1 = a := \text{len } f$ , or
  - (xii)  $i_1 = f := \underbrace{\langle 0, \dots, 0 \rangle}_a$ .

Then  $i_1 \neq \mathbf{halts}_{\mathbf{SCM}_{\text{FSA}}}$ .

- (50) Let  $I, J$  be macro instructions,  $k$  be a natural number, and  $i$  be an instruction of  $\mathbf{SCM}_{\text{FSA}}$ . Suppose for every natural number  $n$  holds  $\text{IncAddr}(i, n) = i$  and  $i \neq \mathbf{halts}_{\mathbf{SCM}_{\text{FSA}}}$  and  $k = \text{card } I$ . Then  $(I; i; J)(\text{insloc}(k)) = i$  and  $(I; i; J)(\text{insloc}(k + 1)) = \text{goto insloc}(\text{card } I + 2)$ .
- (51) Let  $I, J$  be macro instructions and  $k$  be a natural number. If  $k = \text{card } I$ , then  $(I; (a := b); J)(\text{insloc}(k)) = a := b$  and  $(I; (a := b); J)(\text{insloc}(k + 1)) =$

goto insloc(card  $I + 2$ ).

- (52) Let  $I, J$  be macro instructions and  $k$  be a natural number. If  $k = \text{card } I$ , then  $(I; (a := \text{len } f); J)(\text{insloc}(k)) = a := \text{len } f$  and  $(I; (a := \text{len } f); J)(\text{insloc}(k + 1)) = \text{goto insloc}(\text{card } I + 2)$ .
- (53) Let  $w$  be a finite sequence of elements of  $\mathbb{Z}$ ,  $f$  be a finite sequence location,  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ , and  $I$  be a macro instruction. If  $\text{Initialized}(I) + \cdot (f \mapsto w) \subseteq s$ , then  $I \subseteq s$ .
- (54) Let  $w$  be a finite sequence of elements of  $\mathbb{Z}$ ,  $f$  be a finite sequence location,  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ , and  $I$  be a macro instruction. If  $\text{Initialized}(I) + \cdot (f \mapsto w) \subseteq s$ , then  $s(f) = w$  and  $s(\text{intloc}(0)) = 1$ .
- (55) For every finite sequence location  $f$  and for every integer location  $a$  and for every state  $s$  of  $\mathbf{SCM}_{\text{FSA}}$  holds  $\{a, \mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}, f\} \subseteq \text{dom } s$ .
- (56) For every macro instruction  $p$  and for every state  $s$  of  $\mathbf{SCM}_{\text{FSA}}$  holds  $\text{UsedInt}^* \text{Loc}(p) \cup \text{UsedIntLoc}(p) \subseteq \text{dom } s$ .
- (57) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ ,  $I$  be a macro instruction, and  $f$  be a finite sequence location. Then  $(\text{Result}(s + \cdot \text{Initialized}(I)))(f) = (\text{IExec}(I, s))(f)$ .

## 2. THE PROGRAM CODE FOR BUBLE SORT

Let  $f$  be a finite sequence location. The functor  $\text{bubble-sort}(f)$  yields a macro instruction and is defined as follows:

- (Def. 1)  $\text{bubble-sort}(f) = i_5$ ;  
 $(a_1 := \text{len } f)$ ;  
 $\text{Times}(a_1,$   
 $(a_2 := a_1)$ ;  
 $\text{SubFrom}(a_2, a_0)$ ;  
 $(a_3 := \text{len } f)$ ;  
 $\text{Times}(a_2,$   
 $(a_4 := a_3)$ ;  
 $\text{SubFrom}(a_3, a_0)$ ;  
 $(a_5 := f_{a_3})$ ;  
 $(a_6 := f_{a_4})$ ;  
 $\text{SubFrom}(a_6, a_5)$ ;  
 $(\text{if } a_6 > 0 \text{ then } (a_6 := f_{a_4}); (f_{a_3} := a_6); (f_{a_4} := a_5) \text{ else } (\text{Stop}_{\mathbf{SCM}_{\text{FSA}}}))$ ),  
 where  $i_5 = (a_2 := a_0); (a_3 := a_0); (a_4 := a_0); (a_5 := a_0); (a_6 := a_0)$ ,  
 $a_2 = \text{intloc}(2)$ ,  $a_0 = \text{intloc}(0)$ ,  $a_3 = \text{intloc}(3)$ ,  $a_4 = \text{intloc}(4)$ ,  $a_5 = \text{intloc}(5)$ ,  $a_6 = \text{intloc}(6)$ , and  $a_1 = \text{intloc}(1)$ .

The macro instruction the bubble sort algorithm is defined by:

- (Def. 2) The bubble sort algorithm =  $\text{bubble-sort}(\text{fsloc}(0))$ .

The following propositions are true:

- (58) For every finite sequence location  $f$  holds  $\text{UsedIntLoc}(\text{bubble-sort}(f)) = \{a_0, a_1, a_2, a_3, a_4, a_5, a_6\}$ , where  $a_0 = \text{intloc}(0)$ ,  $a_1 = \text{intloc}(1)$ ,  $a_2 = \text{intloc}(2)$ ,  $a_3 = \text{intloc}(3)$ ,  $a_4 = \text{intloc}(4)$ ,  $a_5 = \text{intloc}(5)$ , and  $a_6 = \text{intloc}(6)$ .
- (59) For every finite sequence location  $f$  holds  $\text{UsedInt}^* \text{Loc}(\text{bubble-sort}(f)) = \{f\}$ .

### 3. DEFINING RELATIONSHIP BETWEEN THE INPUT AND OUTPUT OF SORTING ALGORITHMS

The partial function *Sorting-Function* from  $\text{FinPartSt}(\mathbf{SCM}_{\text{FSA}})$  to  $\text{FinPartSt}(\mathbf{SCM}_{\text{FSA}})$  is defined by the condition (Def. 3).

- (Def. 3) Let  $p, q$  be finite partial states of  $\mathbf{SCM}_{\text{FSA}}$ . Then  $\langle p, q \rangle \in \text{Sorting-Function}$  if and only if there exists a finite sequence  $t$  of elements of  $\mathbb{Z}$  and there exists a finite sequence  $u$  of elements of  $\mathbb{R}$  such that  $t$  and  $u$  are fiberwise equipotent and  $u$  is a finite sequence of elements of  $\mathbb{Z}$  and non-increasing and  $p = \text{fsloc}(0) \mapsto t$  and  $q = \text{fsloc}(0) \mapsto u$ .

We now state two propositions:

- (60) For every set  $p$  holds  $p \in \text{dom } \text{Sorting-Function}$  iff there exists a finite sequence  $t$  of elements of  $\mathbb{Z}$  such that  $p = \text{fsloc}(0) \mapsto t$ .
- (61) Let  $t$  be a finite sequence of elements of  $\mathbb{Z}$ . Then there exists a finite sequence  $u$  of elements of  $\mathbb{R}$  such that
- (i)  $t$  and  $u$  are fiberwise equipotent,
  - (ii)  $u$  is non-increasing and a finite sequence of elements of  $\mathbb{Z}$ , and
  - (iii)  $(\text{Sorting-Function})(\text{fsloc}(0) \mapsto t) = \text{fsloc}(0) \mapsto u$ .

### 4. THE BASIC PROPERTY OF BUBLE SORT

Next we state several propositions:

- (62) For every finite sequence location  $f$  holds  $\text{card } \text{bubble-sort}(f) = 63$ .
- (63) For every finite sequence location  $f$  and for every natural number  $k$  such that  $k < 63$  holds  $\text{insloc}(k) \in \text{dom } \text{bubble-sort}(f)$ .
- (64)  $\text{bubble-sort}(\text{fsloc}(0))$  is  $\text{keepInt0 } 1$  and  $\text{InitHalting}$ .
- (65) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ . Then
- (i)  $s(f_0)$  and  $(\text{IExec}(\text{bubble-sort}(f_0), s))(f_0)$  are fiberwise equipotent, and
  - (ii) for all natural numbers  $i, j$  such that  $i \geq 1$  and  $j \leq \text{len } s(f_0)$  and  $i < j$  and for all integers  $x_1, x_2$  such that  $x_1 = (\text{IExec}(\text{bubble-sort}(f_0), s))(f_0)(i)$  and  $x_2 = (\text{IExec}(\text{bubble-sort}(f_0), s))(f_0)(j)$  holds  $x_1 \geq x_2$ ,

where  $f_0 = \text{fsloc}(0)$ .

- (66) Let  $i$  be a natural number,  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$ , and  $w$  be a finite sequence of elements of  $\mathbb{Z}$ . Suppose  $\text{Initialized}(\text{the bubble sort algorithm}) + \cdot(\text{fsloc}(0) \vdash \rightarrow w) \subseteq s$ . Then  $\mathbf{IC}_{(\text{Computation}(s))(i)} \in \text{dom}(\text{the bubble sort algorithm})$ .
- (67) Let  $s$  be a state of  $\mathbf{SCM}_{\text{FSA}}$  and  $t$  be a finite sequence of elements of  $\mathbb{Z}$ . Suppose  $\text{Initialized}(\text{the bubble sort algorithm}) + \cdot(\text{fsloc}(0) \vdash \rightarrow t) \subseteq s$ . Then there exists a finite sequence  $u$  of elements of  $\mathbb{R}$  such that
- (i)  $t$  and  $u$  are fiberwise equipotent,
  - (ii)  $u$  is non-increasing and a finite sequence of elements of  $\mathbb{Z}$ , and
  - (iii)  $(\text{Result}(s))(\text{fsloc}(0)) = u$ .

## 5. THE CORRECTNESS AND AUTONOMOUSNESS OF BUBLE SORT ALGORITHM

We now state two propositions:

- (68) For every finite sequence  $w$  of elements of  $\mathbb{Z}$  holds  $\text{Initialized}(\text{the bubble sort algorithm}) + \cdot(\text{fsloc}(0) \vdash \rightarrow w)$  is autonomic.
- (69)  $\text{Initialized}(\text{the bubble sort algorithm})$  computes Sorting-Function.

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*Received June 17, 1998*

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