FORMALIZED MATHEMATICS
Volume 7, Number 2, 1998
University of Białystok

Introduction to Meet-Continuous Topological Lattices[1]

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MML Identifier: YELLOW13.

The papers [20], [14], [6], [7], [4], [17], [1], [18], [8], [13], [19], [15], [11], [10], [21], [3], [2], [5], [12], [9], [22], and [16] provide the notation and terminology for this paper.

1. Preliminaries

Let S be a finite 1-sorted structure. One can verify that the carrier of S is finite.

Let S be a trivial 1-sorted structure. One can check that the carrier of S is trivial.

One can check that every set which is trivial is also finite.

One can verify that every 1-sorted structure which is trivial is also finite.

Let us mention that every 1-sorted structure which is non trivial is also non empty.

One can check the following observations:

* there exists a 1-sorted structure which is strict, non empty, and trivial,
* there exists a relational structure which is strict, non empty, and trivial, and
* there exists a FR-structure which is strict, non empty, and trivial.

We now state the proposition

(1) For every T_1 non empty topological space T holds every finite subset of T is closed.

[1]This work has been supported by KBN Grant 8 T11C 018 12.

ISSN 1426–2630

Let T be a compact topological structure. Observe that Ω_T is compact.

Let us observe that there exists a topological space which is strict, non empty, and trivial.

Let us mention that every non empty topological space which is finite and T_1 is also discrete.

Let us observe that every topological space which is finite is also compact.

One can prove the following propositions:

(2) Every discrete non empty topological space is a T_4 space.

(3) Every discrete non empty topological space is a T_3 space.

(4) Every discrete non empty topological space is a T_2 space.

(5) Every discrete non empty topological space is a T_1 space.

One can check that every non empty topological space which is T_4 and T_1 is also T_3.

Let us observe that every non empty topological space which is T_3 and T_1 is also T_2.

Let us note that every topological space which is T_2 is also T_1.

One can check that every topological space which is T_1 is also T_0.

Next we state three propositions:

(6) Let S be a reflexive relational structure, T be a reflexive transitive relational structure, f be a map from S into T, and X be a subset of S. Then $\downarrow(f^\circ X) \subseteq \downarrow(f^\circ \downarrow X)$.

(7) Let S be a reflexive relational structure, T be a reflexive transitive relational structure, f be a map from S into T, and X be a subset of S. If f is monotone, then $\downarrow(f^\circ X) = \downarrow(f^\circ \downarrow X)$.

(8) For every non empty poset N holds IdsMap(N) is one-to-one.

One can prove the following proposition

(9) For every finite lattice N holds SupMap(N) is one-to-one.

We now state three propositions:

(10) For every finite lattice N holds N and $\langle \mathrm{Ids}(N), \subseteq \rangle$ are isomorphic.

(11) Let N be a complete non empty poset, x be an element of N, and X be a non empty subset of N. Then $x \sqcap \Box$ preserves inf of X.

(12) For every complete non empty poset N and for every element x of N holds $x \sqcap \Box$ is meet-preserving.

2. On the Basis of Topological Spaces

Next we state several propositions:

(13) Let T be an anti-discrete non empty topological structure and p be a point of T. Then {the carrier of T} is a basis of p.

(14) Let T be an anti-discrete non empty topological structure, p be a point of T, and D be a basis of p. Then $D =$ {the carrier of T}.

(15) Let T be a non empty topological space, P be a basis of T, and p be a point of T. Then $\{A; A$ ranges over subsets of T: $A \in P \ \wedge \ p \in A\}$ is a basis of p.

(16) Let T be a non empty topological structure, A be a subset of T, and p be a point of T. Then $p \in \overline{A}$ if and only if for every basis K of p and for every subset Q of T such that $Q \in K$ holds $A \cap Q \neq \emptyset$.

(17) Let T be a non empty topological structure, A be a subset of T, and p be a point of T. Then $p \in \overline{A}$ if and only if there exists a basis K of p such that for every subset Q of T such that $Q \in K$ holds $A \cap Q \neq \emptyset$.

Let T be a topological structure and let p be a point of T. A family of subsets of T is said to be a generalized basis of p if:

(Def. 1) For every subset A of T such that $p \in \operatorname{Int} A$ there exists a subset P of T such that $P \in$ it and $p \in \operatorname{Int} P$ and $P \subseteq A$.

Let T be a non empty topological space and let p be a point of T. Let us note that the generalized basis of p can be characterized by the following (equivalent) condition:

(Def. 2) For every neighbourhood A of p there exists a neighbourhood P of p such that $P \in$ it and $P \subseteq A$.

The following propositions are true:

(18) Let T be a topological structure and p be a point of T. Then $2^{\text{the carrier of } T}$ is a generalized basis of p.

(19) For every non empty topological space T and for every point p of T holds every generalized basis of p is non empty.

Let T be a topological structure and let p be a point of T. Observe that there exists a generalized basis of p which is non empty.

Let T be a topological structure, let p be a point of T, and let P be a generalized basis of p. We say that P is correct if and only if:

(Def. 3) For every subset A of T holds $A \in P$ iff $p \in \operatorname{Int} A$.

Let T be a topological structure and let p be a point of T. Note that there exists a generalized basis of p which is correct.

One can prove the following proposition

(20) Let T be a topological structure and p be a point of T. Then $\{A; A$ ranges over subsets of T: $p \in \operatorname{Int} A\}$ is a correct generalized basis of p.

Let T be a non empty topological space and let p be a point of T. Observe that there exists a generalized basis of p which is non empty and correct.

One can prove the following three propositions:

(21) Let T be an anti-discrete non empty topological structure and p be a point of T. Then {the carrier of T} is a correct generalized basis of p.

(22) Let T be an anti-discrete non empty topological structure, p be a point of T, and D be a correct generalized basis of p. Then $D =$ {the carrier of T}.

(23) For every non empty topological space T and for every point p of T holds every basis of p is a generalized basis of p.

Let T be a topological structure. A family of subsets of T is said to be a generalized basis of T if:

(Def. 4) For every point p of T holds it is a generalized basis of p.

Next we state two propositions:

(24) For every topological structure T holds $2^{\text{the carrier of } T}$ is a generalized basis of T.

(25) For every non empty topological space T holds every generalized basis of T is non empty.

Let T be a topological structure. Note that there exists a generalized basis of T which is non empty.

Next we state two propositions:

(26) For every non empty topological space T and for every generalized basis P of T holds the topology of $T \subseteq \operatorname{UniCl}(\operatorname{Int} P)$.

(27) For every topological space T holds every basis of T is a generalized basis of T.

Let T be a non empty topological space-like FR-structure. We say that T is topological semilattice if and only if:

(Def. 5) For every map f from $[\!: T, (T$ **qua** topological space$)\, :\!]$ into T such that $f = \sqcap_T$ holds f is continuous.

Let us note that every non empty topological space-like FR-structure which is reflexive and trivial is also topological semilattice.

Let us mention that there exists a FR-structure which is reflexive, trivial, non empty, and topological space-like.

We now state the proposition

(28) Let T be a topological semilattice non empty topological space-like FR-structure and x be an element of T. Then $x \sqcap \square$ is continuous.

References

[1] Grzegorz Bancerek. Complete lattices. *Formalized Mathematics*, 2(**5**):719–725, 1991.
[2] Grzegorz Bancerek. Directed sets, nets, ideals, filters, and maps. *Formalized Mathematics*, 6(**1**):93–107, 1997.
[3] Józef Białas and Yatsuka Nakamura. Dyadic numbers and T_4 topological spaces. *Formalized Mathematics*, 5(**3**):361–366, 1996.
[4] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[5] Czesław Byliński. Galois connections. *Formalized Mathematics*, 6(**1**):131–143, 1997.
[6] Agata Darmochwał. Compact spaces. *Formalized Mathematics*, 1(**2**):383–386, 1990.
[7] Agata Darmochwał. Finite sets. *Formalized Mathematics*, 1(**1**):165–167, 1990.
[8] Adam Grabowski. On the category of posets. *Formalized Mathematics*, 5(**4**):501–505, 1996.
[9] Adam Grabowski and Robert Milewski. Boolean posets, posets under inclusion and products of relational structures. *Formalized Mathematics*, 6(**1**):117–121, 1997.
[10] Zbigniew Karno. The lattice of domains of an extremally disconnected space. *Formalized Mathematics*, 3(**2**):143–149, 1992.
[11] Zbigniew Karno and Toshihiko Watanabe. Completeness of the lattices of domains of a topological space. *Formalized Mathematics*, 3(**1**):71–79, 1992.
[12] Artur Korniłowicz. On the topological properties of meet-continuous lattices. *Formalized Mathematics*, 6(**2**):269–277, 1997.
[13] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[14] Alexander Yu. Shibakov and Andrzej Trybulec. The Cantor set. *Formalized Mathematics*, 5(**2**):233–236, 1996.
[15] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[16] Andrzej Trybulec. Baire spaces, Sober spaces. *Formalized Mathematics*, 6(**2**):289–294, 1997.
[17] Wojciech A. Trybulec. Groups. *Formalized Mathematics*, 1(**5**):821–827, 1990.
[18] Wojciech A. Trybulec. Partially ordered sets. *Formalized Mathematics*, 1(**2**):313–319, 1990.
[19] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[20] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[21] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.
[22] Mariusz Żynel and Czesław Byliński. Properties of relational structures, posets, lattices and maps. *Formalized Mathematics*, 6(**1**):123–130, 1997.

Received November 3, 1998