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Propositional Calculus for Boolean Valued Functions. Part IV

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Summary. In this paper, we have proved some elementary propositional calculus formulae for Boolean valued functions.

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The notation and terminology used here are introduced in the following articles: [6], [7], [8], [2], [3], [5], [1], and [4].

In this paper Y denotes a non empty set.

One can prove the following propositions:

(1) For all elements a, b, c, d of BVF(Y) holds $a \Rightarrow b \wedge c \wedge d = (a \Rightarrow b) \wedge (a \Rightarrow c) \wedge (a \Rightarrow d)$.

(2) For all elements a, b, c, d of BVF(Y) holds $a \Rightarrow b \vee c \vee d = (a \Rightarrow b) \vee (a \Rightarrow c) \vee (a \Rightarrow d)$.

(3) For all elements a, b, c, d of BVF(Y) holds $a \wedge b \wedge c \Rightarrow d = (a \Rightarrow d) \vee (b \Rightarrow d) \vee (c \Rightarrow d)$.

(4) For all elements a, b, c, d of BVF(Y) holds $a \vee b \vee c \Rightarrow d = (a \Rightarrow d) \wedge (b \Rightarrow d) \wedge (c \Rightarrow d)$.

(5) For all elements a, b, c of BVF(Y) holds $(a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) = (a \Rightarrow b) \wedge (b \Rightarrow c) \wedge (c \Rightarrow a) \wedge (b \Rightarrow a) \wedge (a \Rightarrow c)$.

(6) For all elements a, b of BVF(Y) holds $a = a \wedge b \vee a \wedge \neg b$.

(7) For all elements a, b of BVF(Y) holds $a = (a \vee b) \wedge (a \vee \neg b)$.

(8) For all elements a, b, c of BVF(Y) holds $a = a \wedge b \wedge c \vee a \wedge b \wedge \neg c \vee a \wedge \neg b \wedge c \vee a \wedge \neg b \wedge \neg c$.

(9) For all elements a, b, c of BVF(Y) holds $a = (a \vee b \vee c) \wedge (a \vee b \vee \neg c) \wedge (a \vee \neg b \vee c) \wedge (a \vee \neg b \vee \neg c)$.

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(10) For all elements a, b of BVF(Y) holds $a \wedge b = a \wedge (\neg a \vee b)$.

(11) For all elements a, b of BVF(Y) holds $a \vee b = a \vee \neg a \wedge b$.

(12) For all elements a, b of BVF(Y) holds $a \oplus b = \neg(a \Leftrightarrow b)$.

(13) For all elements a, b of BVF(Y) holds $a \oplus b = (a \vee b) \wedge (\neg a \vee \neg b)$.

(14) For every element a of BVF(Y) holds $a \oplus \mathit{true}(Y) = \neg a$.

(15) For every element a of BVF(Y) holds $a \oplus \mathit{false}(Y) = a$.

(16) For all elements a, b of BVF(Y) holds $a \oplus b = \neg a \oplus \neg b$.

(17) For all elements a, b of BVF(Y) holds $\neg(a \oplus b) = a \oplus \neg b$.

(18) For all elements a, b of BVF(Y) holds $a \Leftrightarrow b = (a \vee \neg b) \wedge (\neg a \vee b)$.

(19) For all elements a, b of BVF(Y) holds $a \Leftrightarrow b = a \wedge b \vee \neg a \wedge \neg b$.

(20) For every element a of BVF(Y) holds $a \Leftrightarrow \mathit{true}(Y) = a$.

(21) For every element a of BVF(Y) holds $a \Leftrightarrow \mathit{false}(Y) = \neg a$.

(22) For all elements a, b of BVF(Y) holds $\neg(a \Leftrightarrow b) = a \Leftrightarrow \neg b$.

(23) For all elements a, b of BVF(Y) holds $\neg a \Subset a \Rightarrow b \Leftrightarrow \neg a$.

(24) For all elements a, b of BVF(Y) holds $\neg a \Subset b \Rightarrow a \Leftrightarrow \neg b$.

(25) For all elements a, b of BVF(Y) holds $a \Subset a \vee b \Leftrightarrow b \vee a \Leftrightarrow a$.

(26) For every element a of BVF(Y) holds $a \Rightarrow \neg a \Leftrightarrow \neg a = \mathit{true}(Y)$.

(27) For all elements a, b of BVF(Y) holds $a \Rightarrow b \Rightarrow a \Rightarrow a = \mathit{true}(Y)$.

(28) For all elements a, b, c, d of BVF(Y) holds $(a \Rightarrow c) \wedge (b \Rightarrow d) \wedge (\neg c \vee \neg d) \Rightarrow \neg a \vee \neg b = \mathit{true}(Y)$.

(29) For all elements a, b, c of BVF(Y) holds $a \Rightarrow b \Rightarrow a \Rightarrow b \Rightarrow c \Rightarrow a \Rightarrow c = \mathit{true}(Y)$.

References

[1] Grzegorz Bancerek and Andrzej Trybulec. Miscellaneous facts about functions. *Formalized Mathematics*, 5(**4**):485–492, 1996.

[2] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.

[3] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.

[4] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Formalized Mathematics*, 7(**2**):249–254, 1998.

[5] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.

[6] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

[7] Edmund Woronowicz. Many–argument relations. *Formalized Mathematics*, 1(**4**):733–737, 1990.

[8] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.

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