FORMALIZED MATHEMATICS
Volume 8, Number 1, 1999
University of Białystok

Properties of Left and Right Components

Artur Korniłowicz[1]
University of Białystok

MML Identifier: GOBRD14.

The notation and terminology used here have been introduced in the following papers: [33], [42], [43], [6], [7], [41], [5], [16], [35], [1], [30], [38], [31], [17], [27], [8], [19], [39], [18], [20], [15], [4], [2], [3], [40], [32], [29], [44], [12], [28], [11], [13], [14], [21], [22], [25], [34], [10], [24], [23], [37], [36], [26], and [9].

1. Components

For simplicity, we adopt the following rules: r denotes a real number, i, j, n denote natural numbers, f denotes a non constant standard special circular sequence, g denotes a clockwise oriented non constant standard special circular sequence, p, q denote points of $\mathcal{E}^2_{\rm T}$, P, Q, R denote subsets of $\mathcal{E}^2_{\rm T}$, C denotes a compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$, and G denotes a Go-board.

Next we state several propositions:

(1) Let T be a topological space, A be a subset of the carrier of T, and B be a subset of T. If B is a component of A, then B is connected.

(2) Let A be a subset of the carrier of $\mathcal{E}^n_{\rm T}$ and B be a subset of $\mathcal{E}^n_{\rm T}$. If B is inside component of A, then B is connected.

(3) Let A be a subset of the carrier of $\mathcal{E}^n_{\rm T}$ and B be a subset of $\mathcal{E}^n_{\rm T}$. If B is outside component of A, then B is connected.

(4) For every subset A of the carrier of $\mathcal{E}^n_{\rm T}$ and for every subset B of $\mathcal{E}^n_{\rm T}$ such that B is a component of $A^{\rm c}$ holds $A \cap B = \emptyset$.

(5) If P is outside component of Q and R is inside component of Q, then $P \cap R = \emptyset$.

[1]This paper was written while the author visited Shinshu University, winter 1999.

ISSN 1426–2630

(6) Let A, B be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose A is outside component of $\widetilde{\mathcal{L}}(f)$ and B is outside component of $\widetilde{\mathcal{L}}(f)$. Then $A = B$.

(7) Let p be a point of $\mathcal{E}^2$. Suppose $p = 0_{\mathcal{E}^2_{\mathrm{T}}}$ and P is outside component of $\widetilde{\mathcal{L}}(f)$. Then there exists a real number r such that $r > 0$ and $\mathrm{Ball}(p, r)^{\mathrm{c}} \subseteq P$.

Let C be a closed subset of $\mathcal{E}^2_{\mathrm{T}}$. Observe that $\mathrm{BDD}\, C$ is open and $\mathrm{UBD}\, C$ is open.

Let C be a compact subset of $\mathcal{E}^2_{\mathrm{T}}$. Observe that $\mathrm{UBD}\, C$ is connected.

2. Go-Boards

One can prove the following proposition

(8) For every finite sequence f of elements of $\mathcal{E}^n_{\mathrm{T}}$ such that $\widetilde{\mathcal{L}}(f) \neq \emptyset$ holds $2 \leqslant \mathrm{len}\, f$.

Let n be a natural number and let a, b be points of $\mathcal{E}^n_{\mathrm{T}}$. The functor $\rho(a, b)$ yields a real number and is defined by:

(Def. 1) There exist points p, q of $\mathcal{E}^n$ such that $p = a$ and $q = b$ and $\rho(a, b) = \rho(p, q)$.

Let us notice that the functor $\rho(a, b)$ is commutative.

The following propositions are true:

(9) $\rho(p, q) = \sqrt{(p_{\mathbf{1}} - q_{\mathbf{1}})^{\mathbf{2}} + (p_{\mathbf{2}} - q_{\mathbf{2}})^{\mathbf{2}}}$.

(10) For every point p of $\mathcal{E}^n_{\mathrm{T}}$ holds $\rho(p, p) = 0$.

(11) For all points p, q, r of $\mathcal{E}^n_{\mathrm{T}}$ holds $\rho(p, r) \leqslant \rho(p, q) + \rho(q, r)$.

(12) Let x_1, x_2, y_1, y_2 be real numbers and a, b be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $x_1 \leqslant a_{\mathbf{1}}$ and $a_{\mathbf{1}} \leqslant x_2$ and $y_1 \leqslant a_{\mathbf{2}}$ and $a_{\mathbf{2}} \leqslant y_2$ and $x_1 \leqslant b_{\mathbf{1}}$ and $b_{\mathbf{1}} \leqslant x_2$ and $y_1 \leqslant b_{\mathbf{2}}$ and $b_{\mathbf{2}} \leqslant y_2$. Then $\rho(a, b) \leqslant |x_2 - x_1| + |y_2 - y_1|$.

(13) If $1 \leqslant i$ and $i < \mathrm{len}\, G$ and $1 \leqslant j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G, i, j) = \prod[1 \longmapsto [(G_{i,1})_{\mathbf{1}}, (G_{i+1,1})_{\mathbf{1}}], 2 \longmapsto [(G_{1,j})_{\mathbf{2}}, (G_{1,j+1})_{\mathbf{2}}]]$.

(14) If $1 \leqslant i$ and $i < \mathrm{len}\, G$ and $1 \leqslant j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G, i, j)$ is compact.

(15) If $\langle i,\, j\rangle \in$ the indices of G and $\langle i + n,\, j\rangle \in$ the indices of G, then $\rho(G_{i,j}, G_{i+n,j}) = (G_{i+n,j})_{\mathbf{1}} - (G_{i,j})_{\mathbf{1}}$.

(16) If $\langle i,\, j\rangle \in$ the indices of G and $\langle i,\, j + n\rangle \in$ the indices of G, then $\rho(G_{i,j}, G_{i,j+n}) = (G_{i,j+n})_{\mathbf{2}} - (G_{i,j})_{\mathbf{2}}$.

(17) $3 \leqslant \mathrm{len}\, \mathrm{Gauge}(C, n) -' 1$.

(18) Suppose $i \leqslant j$. Let a, b be natural numbers. Suppose $2 \leqslant a$ and $a \leqslant \mathrm{len}\, \mathrm{Gauge}(C, i) - 1$ and $2 \leqslant b$ and $b \leqslant \mathrm{len}\, \mathrm{Gauge}(C, i) - 1$. Then there exist natural numbers c, d such that

$2 \leqslant c$ and $c \leqslant \operatorname{len}\operatorname{Gauge}(C,j)-1$ and $2 \leqslant d$ and $d \leqslant \operatorname{len}\operatorname{Gauge}(C,j)-1$ and $\langle c, d\rangle \in$ the indices of $\operatorname{Gauge}(C,j)$ and $(\operatorname{Gauge}(C,i))_{a,b} = (\operatorname{Gauge}(C,j))_{c,d}$ and $c = 2+2^{j-'i}\cdot(a-'2)$ and $d = 2+2^{j-'i}\cdot(b-'2)$.

(19) If $\langle i, j\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$ and $\langle i, j+1\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$, then $\rho((\operatorname{Gauge}(C,n))_{i,j}, (\operatorname{Gauge}(C,n))_{i,j+1}) = \frac{\text{N-bound}\,C-\text{S-bound}\,C}{2^n}$.

(20) If $\langle i, j\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$ and $\langle i+1, j\rangle \in$ the indices of $\operatorname{Gauge}(C,n)$, then $\rho((\operatorname{Gauge}(C,n))_{i,j}, (\operatorname{Gauge}(C,n))_{i+1,j}) = \frac{\text{E-bound}\,C-\text{W-bound}\,C}{2^n}$.

(21) If $r > 0$, then there exists a natural number n such that $\rho((\operatorname{Gauge}(C,n))_{1,1}, (\operatorname{Gauge}(C,n))_{1,2}) < r$ and $\rho((\operatorname{Gauge}(C,n))_{1,1}, (\operatorname{Gauge}(C,n))_{2,1}) < r$.

3. LeftComp and RightComp

One can prove the following propositions:

(22) For every subset P of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ such that P is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ holds $P = \operatorname{RightComp}(f)$ or $P = \operatorname{LeftComp}(f)$.

(23) Let A_1, A_2 be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} = A_1 \cup A_2$,

(ii) $A_1 \cap A_2 = \emptyset$, and

(iii) for all subsets C_1, C_2 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ such that $C_1 = A_1$ and $C_2 = A_2$ holds C_1 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and C_2 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

Then $A_1 = \operatorname{RightComp}(f)$ and $A_2 = \operatorname{LeftComp}(f)$ or $A_1 = \operatorname{LeftComp}(f)$ and $A_2 = \operatorname{RightComp}(f)$.

(24) $\operatorname{LeftComp}(f) \cap \operatorname{RightComp}(f) = \emptyset$.

(25) $\widetilde{\mathcal{L}}(f) \cup \operatorname{RightComp}(f) \cup \operatorname{LeftComp}(f) =$ the carrier of $\mathcal{E}^2_{\mathrm{T}}$.

(26) $p \in \widetilde{\mathcal{L}}(f)$ iff $p \notin \operatorname{LeftComp}(f)$ and $p \notin \operatorname{RightComp}(f)$.

(27) $p \in \operatorname{LeftComp}(f)$ iff $p \notin \widetilde{\mathcal{L}}(f)$ and $p \notin \operatorname{RightComp}(f)$.

(28) $p \in \operatorname{RightComp}(f)$ iff $p \notin \widetilde{\mathcal{L}}(f)$ and $p \notin \operatorname{LeftComp}(f)$.

(29) $\widetilde{\mathcal{L}}(f) = \overline{\operatorname{RightComp}(f)} \setminus \operatorname{RightComp}(f)$.

(30) $\widetilde{\mathcal{L}}(f) = \overline{\operatorname{LeftComp}(f)} \setminus \operatorname{LeftComp}(f)$.

(31) $\overline{\operatorname{RightComp}(f)} = \operatorname{RightComp}(f) \cup \widetilde{\mathcal{L}}(f)$.

(32) $\overline{\operatorname{LeftComp}(f)} = \operatorname{LeftComp}(f) \cup \widetilde{\mathcal{L}}(f)$.

Let f be a non constant standard special circular sequence. One can verify that $\widetilde{\mathcal{L}}(f)$ is Jordan.

The following propositions are true:

(33) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{W-bound}\,\widetilde{\mathcal{L}}(g) < p_{\mathbf{1}}$.

(34) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{E-bound}\,\widetilde{\mathcal{L}}(g) > p_{\mathbf{1}}$.

(35) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{N-bound}\,\widetilde{\mathcal{L}}(g) > p_{\mathbf{2}}$.

(36) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{S-bound}\,\widetilde{\mathcal{L}}(g) < p_{\mathbf{2}}$.

(37) If $p \in \text{RightComp}(f)$ and $q \in \text{LeftComp}(f)$, then $\mathcal{L}(p, q) \cap \widetilde{\mathcal{L}}(f) \neq \emptyset$.

(38) $\overline{\text{RightComp}(\text{SpStSeq}\,C)} = \prod[1 \longmapsto [\text{W-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C),$ $\text{E-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C)], 2 \longmapsto [\text{S-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C),$ $\text{N-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C)]]$.

(39) $(\text{proj1})^{\circ}\widetilde{\mathcal{L}}(f) \subseteq (\text{proj1})^{\circ}\overline{\text{RightComp}(f)}$ and if $\pi_1 f = \text{N-min}\,\widetilde{\mathcal{L}}(f)$ and f is clockwise oriented, then $(\text{proj1})^{\circ}\overline{\text{RightComp}(f)} = (\text{proj1})^{\circ}\widetilde{\mathcal{L}}(f)$.

(40) $(\text{proj2})^{\circ}\widetilde{\mathcal{L}}(f) \subseteq (\text{proj2})^{\circ}\overline{\text{RightComp}(f)}$ and if $\pi_1 f = \text{N-min}\,\widetilde{\mathcal{L}}(f)$ and f is clockwise oriented, then $(\text{proj2})^{\circ}\overline{\text{RightComp}(f)} = (\text{proj2})^{\circ}\widetilde{\mathcal{L}}(f)$.

(41) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{RightComp}(g) \subseteq \text{RightComp}(\text{SpStSeq}\,\widetilde{\mathcal{L}}(g))$.

(42) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\overline{\text{RightComp}(g)}$ is compact.

(43) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{LeftComp}(g)$ is non Bounded.

(44) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{LeftComp}(g)$ is outside component of $\widetilde{\mathcal{L}}(g)$.

(45) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{RightComp}(g)$ is inside component of $\widetilde{\mathcal{L}}(g)$.

(46) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{UBD}\,\widetilde{\mathcal{L}}(g) = \text{LeftComp}(g)$.

(47) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{BDD}\,\widetilde{\mathcal{L}}(g) = \text{RightComp}(g)$.

(48) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and P is outside component of $\widetilde{\mathcal{L}}(g)$, then $P = \text{LeftComp}(g)$.

(49) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and P is inside component of $\widetilde{\mathcal{L}}(g)$, then $P \cap \text{RightComp}(g) \neq \emptyset$.

(50) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and P is inside component of $\widetilde{\mathcal{L}}(g)$, then $P = \text{BDD}\,\widetilde{\mathcal{L}}(g)$.

(51) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{W-bound}\,\widetilde{\mathcal{L}}(g) = \text{W-bound}\,\text{RightComp}(g)$.

(52) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{E-bound}\,\widetilde{\mathcal{L}}(g) = \text{E-bound}\,\text{RightComp}(g)$.

(53) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{N-bound}\,\widetilde{\mathcal{L}}(g) = \text{N-bound}\,\text{RightComp}(g)$.

(54) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{S-bound}\,\widetilde{\mathcal{L}}(g) = \text{S-bound}\,\text{RightComp}(g)$.

Acknowledgments

I would like to thank Professor Yatsuka Nakamura for his help in the preparation of the article.

References

[1] Grzegorz Bancerek. The fundamental properties of natural numbers. *Formalized Mathematics*, 1(**1**):41–46, 1990.

[2] Grzegorz Bancerek. König's theorem. *Formalized Mathematics*, 1(**3**):589–593, 1990.

[3] Grzegorz Bancerek. Countable sets and Hessenberg's theorem. *Formalized Mathematics*, 2(**1**):65–69, 1991.
[4] Grzegorz Bancerek and Krzysztof Hryniewiecki. Segments of natural numbers and finite sequences. *Formalized Mathematics*, 1(**1**):107–114, 1990.
[5] Czesław Byliński. Binary operations. *Formalized Mathematics*, 1(**1**):175–180, 1990.
[6] Czesław Byliński. Functions and their basic properties. *Formalized Mathematics*, 1(**1**):55–65, 1990.
[7] Czesław Byliński. Functions from a set to a set. *Formalized Mathematics*, 1(**1**):153–164, 1990.
[8] Czesław Byliński. Products and coproducts in categories. *Formalized Mathematics*, 2(**5**):701–709, 1991.
[9] Czesław Byliński. Gauges. *Formalized Mathematics*, 8(**1**):25–27, 1999.
[10] Czesław Byliński and Piotr Rudnicki. Bounding boxes for compact sets in $\mathcal{E}^2$. *Formalized Mathematics*, 6(**3**):427–440, 1997.
[11] Agata Darmochwał. Compact spaces. *Formalized Mathematics*, 1(**2**):383–386, 1990.
[12] Agata Darmochwał. Families of subsets, subspaces and mappings in topological spaces. *Formalized Mathematics*, 1(**2**):257–261, 1990.
[13] Agata Darmochwał. The Euclidean space. *Formalized Mathematics*, 2(**4**):599–603, 1991.
[14] Agata Darmochwał and Yatsuka Nakamura. The topological space $\mathcal{E}^2_{\mathrm{T}}$. Arcs, line segments and special polygonal arcs. *Formalized Mathematics*, 2(**5**):617–621, 1991.
[15] Alicia de la Cruz. Totally bounded metric spaces. *Formalized Mathematics*, 2(**4**):559–562, 1991.
[16] Krzysztof Hryniewiecki. Basic properties of real numbers. *Formalized Mathematics*, 1(**1**):35–40, 1990.
[17] Katarzyna Jankowska. Matrices. Abelian group of matrices. *Formalized Mathematics*, 2(**4**):475–480, 1991.
[18] Stanisława Kanas, Adam Lecko, and Mariusz Startek. Metric spaces. *Formalized Mathematics*, 1(**3**):607–610, 1990.
[19] Jarosław Kotowicz. Convergent real sequences. Upper and lower bound of sets of real numbers. *Formalized Mathematics*, 1(**3**):477–481, 1990.
[20] Jarosław Kotowicz. The limit of a real function at infinity. *Formalized Mathematics*, 2(**1**):17–28, 1991.
[21] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part I. *Formalized Mathematics*, 3(**1**):107–115, 1992.
[22] Jarosław Kotowicz and Yatsuka Nakamura. Introduction to Go-board - part II. *Formalized Mathematics*, 3(**1**):117–121, 1992.
[23] Yatsuka Nakamura. Graph theoretical properties of arcs in the plane and Fashoda Meet Theorem. *Formalized Mathematics*, 7(**2**):193–201, 1998.
[24] Yatsuka Nakamura and Czesław Byliński. Extremal properties of vertices on special polygons. Part I. *Formalized Mathematics*, 5(**1**):97–102, 1996.
[25] Yatsuka Nakamura and Andrzej Trybulec. Decomposing a Go-board into cells. *Formalized Mathematics*, 5(**3**):323–328, 1996.
[26] Yatsuka Nakamura, Andrzej Trybulec, and Czesław Byliński. Bounded domains and unbounded domains. *Formalized Mathematics*, 8(**1**):1–13, 1999.
[27] Takaya Nishiyama and Yasuho Mizuhara. Binary arithmetics. *Formalized Mathematics*, 4(**1**):83–86, 1993.
[28] Beata Padlewska. Connected spaces. *Formalized Mathematics*, 1(**1**):239–244, 1990.
[29] Beata Padlewska and Agata Darmochwał. Topological spaces and continuous functions. *Formalized Mathematics*, 1(**1**):223–230, 1990.
[30] Jan Popiołek. Some properties of functions modul and signum. *Formalized Mathematics*, 1(**2**):263–264, 1990.
[31] Konrad Raczkowski and Andrzej Nędzusiak. Real exponents and logarithms. *Formalized Mathematics*, 2(**2**):213–216, 1991.
[32] Konrad Raczkowski and Paweł Sadowski. Topological properties of subsets in real numbers. *Formalized Mathematics*, 1(**4**):777–780, 1990.
[33] Andrzej Trybulec. Tarski Grothendieck set theory. *Formalized Mathematics*, 1(**1**):9–11, 1990.
[34] Andrzej Trybulec. Left and right component of the complement of a special closed curve. *Formalized Mathematics*, 5(**4**):465–468, 1996.

[35] Andrzej Trybulec and Czesław Byliński. Some properties of real numbers. *Formalized Mathematics*, 1(**3**):445–449, 1990.
[36] Andrzej Trybulec and Yatsuka Nakamura. On the order on a special polygon. *Formalized Mathematics*, 6(**4**):541–548, 1997.
[37] Andrzej Trybulec and Yatsuka Nakamura. On the rectangular finite sequences of the points of the plane. *Formalized Mathematics*, 6(**4**):531–539, 1997.
[38] Michał J. Trybulec. Integers. *Formalized Mathematics*, 1(**3**):501–505, 1990.
[39] Wojciech A. Trybulec. Groups. *Formalized Mathematics*, 1(**5**):821–827, 1990.
[40] Wojciech A. Trybulec. Pigeon hole principle. *Formalized Mathematics*, 1(**3**):575–579, 1990.
[41] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.
[42] Zinaida Trybulec and Halina Święczkowska. Boolean properties of sets. *Formalized Mathematics*, 1(**1**):17–23, 1990.
[43] Edmund Woronowicz. Relations and their basic properties. *Formalized Mathematics*, 1(**1**):73–83, 1990.
[44] Mirosław Wysocki and Agata Darmochwał. Subsets of topological spaces. *Formalized Mathematics*, 1(**1**):231–237, 1990.

Received May 5, 1999