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Properties of Left and Right Components

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The notation and terminology used here have been introduced in the following papers: [33], [42], [43], [6], [7], [41], [5], [16], [35], [1], [30], [38], [31], [17], [27], [8], [19], [39], [18], [20], [15], [4], [2], [3], [40], [32], [29], [44], [12], [28], [11], [13], [14], [21], [22], [25], [34], [10], [24], [23], [37], [36], [26], and [9].

1. Components

For simplicity, we adopt the following rules: r denotes a real number, i, j, n denote natural numbers, f denotes a non constant standard special circular sequence, g denotes a clockwise oriented non constant standard special circular sequence, p, q denote points of $\mathcal{E}^2_{\rm T}$, P, Q, R denote subsets of $\mathcal{E}^2_{\rm T}$, C denotes a compact non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$, and G denotes a Go-board.

Next we state several propositions:

(1) Let T be a topological space, A be a subset of the carrier of T, and B be a subset of T. If B is a component of A, then B is connected.

(2) Let A be a subset of the carrier of $\mathcal{E}^n_{\rm T}$ and B be a subset of $\mathcal{E}^n_{\rm T}$. If B is inside component of A, then B is connected.

(3) Let A be a subset of the carrier of $\mathcal{E}^n_{\rm T}$ and B be a subset of $\mathcal{E}^n_{\rm T}$. If B is outside component of A, then B is connected.

(4) For every subset A of the carrier of $\mathcal{E}^n_{\rm T}$ and for every subset B of $\mathcal{E}^n_{\rm T}$ such that B is a component of $A^{\rm c}$ holds $A \cap B = \emptyset$.

(5) If P is outside component of Q and R is inside component of Q, then $P \cap R = \emptyset$.

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(6) Let A, B be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose A is outside component of $\widetilde{\mathcal{L}}(f)$ and B is outside component of $\widetilde{\mathcal{L}}(f)$. Then $A = B$.

(7) Let p be a point of $\mathcal{E}^2$. Suppose $p = 0_{\mathcal{E}^2_{\mathrm{T}}}$ and P is outside component of $\widetilde{\mathcal{L}}(f)$. Then there exists a real number r such that $r > 0$ and $\mathrm{Ball}(p, r)^{\mathrm{c}} \subseteq P$.

Let C be a closed subset of $\mathcal{E}^2_{\mathrm{T}}$. Observe that $\mathrm{BDD}\,C$ is open and $\mathrm{UBD}\,C$ is open.

Let C be a compact subset of $\mathcal{E}^2_{\mathrm{T}}$. Observe that $\mathrm{UBD}\,C$ is connected.

2. Go-Boards

One can prove the following proposition

(8) For every finite sequence f of elements of $\mathcal{E}^n_{\mathrm{T}}$ such that $\widetilde{\mathcal{L}}(f) \neq \emptyset$ holds $2 \leqslant \mathrm{len}\, f$.

Let n be a natural number and let a, b be points of $\mathcal{E}^n_{\mathrm{T}}$. The functor $\rho(a, b)$ yields a real number and is defined by:

(Def. 1) There exist points p, q of $\mathcal{E}^n$ such that $p = a$ and $q = b$ and $\rho(a, b) = \rho(p, q)$.

Let us notice that the functor $\rho(a, b)$ is commutative.

The following propositions are true:

(9) $\rho(p, q) = \sqrt{(p_{\mathbf{1}} - q_{\mathbf{1}})^{\mathbf{2}} + (p_{\mathbf{2}} - q_{\mathbf{2}})^{\mathbf{2}}}$.

(10) For every point p of $\mathcal{E}^n_{\mathrm{T}}$ holds $\rho(p, p) = 0$.

(11) For all points p, q, r of $\mathcal{E}^n_{\mathrm{T}}$ holds $\rho(p, r) \leqslant \rho(p, q) + \rho(q, r)$.

(12) Let x_1, x_2, y_1, y_2 be real numbers and a, b be points of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $x_1 \leqslant a_{\mathbf{1}}$ and $a_{\mathbf{1}} \leqslant x_2$ and $y_1 \leqslant a_{\mathbf{2}}$ and $a_{\mathbf{2}} \leqslant y_2$ and $x_1 \leqslant b_{\mathbf{1}}$ and $b_{\mathbf{1}} \leqslant x_2$ and $y_1 \leqslant b_{\mathbf{2}}$ and $b_{\mathbf{2}} \leqslant y_2$. Then $\rho(a, b) \leqslant |x_2 - x_1| + |y_2 - y_1|$.

(13) If $1 \leqslant i$ and $i < \mathrm{len}\, G$ and $1 \leqslant j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G, i, j) = \prod[1 \longmapsto [(G_{i,1})_{\mathbf{1}}, (G_{i+1,1})_{\mathbf{1}}], 2 \longmapsto [(G_{1,j})_{\mathbf{2}}, (G_{1,j+1})_{\mathbf{2}}]]$.

(14) If $1 \leqslant i$ and $i < \mathrm{len}\, G$ and $1 \leqslant j$ and $j < \mathrm{width}\, G$, then $\mathrm{cell}(G, i, j)$ is compact.

(15) If $\langle i,\, j\rangle \in$ the indices of G and $\langle i + n,\, j\rangle \in$ the indices of G, then $\rho(G_{i,j}, G_{i+n,j}) = (G_{i+n,j})_{\mathbf{1}} - (G_{i,j})_{\mathbf{1}}$.

(16) If $\langle i,\, j\rangle \in$ the indices of G and $\langle i,\, j + n\rangle \in$ the indices of G, then $\rho(G_{i,j}, G_{i,j+n}) = (G_{i,j+n})_{\mathbf{2}} - (G_{i,j})_{\mathbf{2}}$.

(17) $3 \leqslant \mathrm{len}\, \mathrm{Gauge}(C, n) -' 1$.

(18) Suppose $i \leqslant j$. Let a, b be natural numbers. Suppose $2 \leqslant a$ and $a \leqslant \mathrm{len}\, \mathrm{Gauge}(C, i) - 1$ and $2 \leqslant b$ and $b \leqslant \mathrm{len}\, \mathrm{Gauge}(C, i) - 1$. Then there exist natural numbers c, d such that

$2 \leqslant c$ and $c \leqslant \operatorname{len} \mathrm{Gauge}(C, j) - 1$ and $2 \leqslant d$ and $d \leqslant \operatorname{len} \mathrm{Gauge}(C, j) - 1$ and $\langle c, d \rangle \in$ the indices of $\mathrm{Gauge}(C, j)$ and $(\mathrm{Gauge}(C, i))_{a,b} = (\mathrm{Gauge}(C, j))_{c,d}$ and $c = 2 + 2^{j -' i} \cdot (a -' 2)$ and $d = 2 + 2^{j -' i} \cdot (b -' 2)$.

(19) If $\langle i, j \rangle \in$ the indices of $\mathrm{Gauge}(C, n)$ and $\langle i, j + 1 \rangle \in$ the indices of $\mathrm{Gauge}(C, n)$, then $\rho((\mathrm{Gauge}(C, n))_{i,j}, (\mathrm{Gauge}(C, n))_{i,j+1}) = \frac{\text{N-bound}\, C - \text{S-bound}\, C}{2^n}$.

(20) If $\langle i, j \rangle \in$ the indices of $\mathrm{Gauge}(C, n)$ and $\langle i + 1, j \rangle \in$ the indices of $\mathrm{Gauge}(C, n)$, then $\rho((\mathrm{Gauge}(C, n))_{i,j}, (\mathrm{Gauge}(C, n))_{i+1,j}) = \frac{\text{E-bound}\, C - \text{W-bound}\, C}{2^n}$.

(21) If $r > 0$, then there exists a natural number n such that $\rho((\mathrm{Gauge}(C, n))_{1,1}, (\mathrm{Gauge}(C, n))_{1,2}) < r$ and $\rho((\mathrm{Gauge}(C, n))_{1,1}, (\mathrm{Gauge}(C, n))_{2,1}) < r$.

3. LeftComp and RightComp

One can prove the following propositions:

(22) For every subset P of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ such that P is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ holds $P = \mathrm{RightComp}(f)$ or $P = \mathrm{LeftComp}(f)$.

(23) Let A_1, A_2 be subsets of $\mathcal{E}^2_{\mathrm{T}}$. Suppose that

(i) $(\widetilde{\mathcal{L}}(f))^{\mathrm{c}} = A_1 \cup A_2$,

(ii) $A_1 \cap A_2 = \emptyset$, and

(iii) for all subsets C_1, C_2 of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ such that $C_1 = A_1$ and $C_2 = A_2$ holds C_1 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$ and C_2 is a component of $(\mathcal{E}^2_{\mathrm{T}}) \restriction (\widetilde{\mathcal{L}}(f))^{\mathrm{c}}$.

Then $A_1 = \mathrm{RightComp}(f)$ and $A_2 = \mathrm{LeftComp}(f)$ or $A_1 = \mathrm{LeftComp}(f)$ and $A_2 = \mathrm{RightComp}(f)$.

(24) $\mathrm{LeftComp}(f) \cap \mathrm{RightComp}(f) = \emptyset$.

(25) $\widetilde{\mathcal{L}}(f) \cup \mathrm{RightComp}(f) \cup \mathrm{LeftComp}(f) =$ the carrier of $\mathcal{E}^2_{\mathrm{T}}$.

(26) $p \in \widetilde{\mathcal{L}}(f)$ iff $p \notin \mathrm{LeftComp}(f)$ and $p \notin \mathrm{RightComp}(f)$.

(27) $p \in \mathrm{LeftComp}(f)$ iff $p \notin \widetilde{\mathcal{L}}(f)$ and $p \notin \mathrm{RightComp}(f)$.

(28) $p \in \mathrm{RightComp}(f)$ iff $p \notin \widetilde{\mathcal{L}}(f)$ and $p \notin \mathrm{LeftComp}(f)$.

(29) $\widetilde{\mathcal{L}}(f) = \overline{\mathrm{RightComp}(f)} \setminus \mathrm{RightComp}(f)$.

(30) $\widetilde{\mathcal{L}}(f) = \overline{\mathrm{LeftComp}(f)} \setminus \mathrm{LeftComp}(f)$.

(31) $\overline{\mathrm{RightComp}(f)} = \mathrm{RightComp}(f) \cup \widetilde{\mathcal{L}}(f)$.

(32) $\overline{\mathrm{LeftComp}(f)} = \mathrm{LeftComp}(f) \cup \widetilde{\mathcal{L}}(f)$.

Let f be a non constant standard special circular sequence. One can verify that $\widetilde{\mathcal{L}}(f)$ is Jordan.

The following propositions are true:

(33) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{W-bound}\,\widetilde{\mathcal{L}}(g) < p_{\mathbf{1}}$.

(34) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{E-bound}\,\widetilde{\mathcal{L}}(g) > p_{\mathbf{1}}$.

(35) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{N-bound}\,\widetilde{\mathcal{L}}(g) > p_{\mathbf{2}}$.

(36) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and $p \in \text{RightComp}(g)$, then $\text{S-bound}\,\widetilde{\mathcal{L}}(g) < p_{\mathbf{2}}$.

(37) If $p \in \text{RightComp}(f)$ and $q \in \text{LeftComp}(f)$, then $\mathcal{L}(p, q) \cap \widetilde{\mathcal{L}}(f) \neq \emptyset$.

(38) $\overline{\text{RightComp}(\text{SpStSeq}\,C)} = \prod[1 \longmapsto [\text{W-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C),$ $\text{E-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C)], 2 \longmapsto [\text{S-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C),$ $\text{N-bound}\,\widetilde{\mathcal{L}}(\text{SpStSeq}\,C)]]$.

(39) $(\text{proj1})^\circ\widetilde{\mathcal{L}}(f) \subseteq (\text{proj1})^\circ\overline{\text{RightComp}(f)}$ and if $\pi_1 f = \text{N-min}\,\widetilde{\mathcal{L}}(f)$ and f is clockwise oriented, then $(\text{proj1})^\circ\overline{\text{RightComp}(f)} = (\text{proj1})^\circ\widetilde{\mathcal{L}}(f)$.

(40) $(\text{proj2})^\circ\widetilde{\mathcal{L}}(f) \subseteq (\text{proj2})^\circ\overline{\text{RightComp}(f)}$ and if $\pi_1 f = \text{N-min}\,\widetilde{\mathcal{L}}(f)$ and f is clockwise oriented, then $(\text{proj2})^\circ\overline{\text{RightComp}(f)} = (\text{proj2})^\circ\widetilde{\mathcal{L}}(f)$.

(41) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{RightComp}(g) \subseteq \text{RightComp}(\text{SpStSeq}\,\widetilde{\mathcal{L}}(g))$.

(42) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\overline{\text{RightComp}(g)}$ is compact.

(43) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{LeftComp}(g)$ is non Bounded.

(44) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{LeftComp}(g)$ is outside component of $\widetilde{\mathcal{L}}(g)$.

(45) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{RightComp}(g)$ is inside component of $\widetilde{\mathcal{L}}(g)$.

(46) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{UBD}\,\widetilde{\mathcal{L}}(g) = \text{LeftComp}(g)$.

(47) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{BDD}\,\widetilde{\mathcal{L}}(g) = \text{RightComp}(g)$.

(48) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and P is outside component of $\widetilde{\mathcal{L}}(g)$, then $P = \text{LeftComp}(g)$.

(49) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and P is inside component of $\widetilde{\mathcal{L}}(g)$, then $P \cap \text{RightComp}(g) \neq \emptyset$.

(50) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$ and P is inside component of $\widetilde{\mathcal{L}}(g)$, then $P = \text{BDD}\,\widetilde{\mathcal{L}}(g)$.

(51) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{W-bound}\,\widetilde{\mathcal{L}}(g) = \text{W-bound}\,\text{RightComp}(g)$.

(52) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{E-bound}\,\widetilde{\mathcal{L}}(g) = \text{E-bound}\,\text{RightComp}(g)$.

(53) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{N-bound}\,\widetilde{\mathcal{L}}(g) = \text{N-bound}\,\text{RightComp}(g)$.

(54) If $\pi_1 g = \text{N-min}\,\widetilde{\mathcal{L}}(g)$, then $\text{S-bound}\,\widetilde{\mathcal{L}}(g) = \text{S-bound}\,\text{RightComp}(g)$.

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