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A Small Computer Model with Push-Down Stack[1]

Jing-Chao Chen
Shanghai Jiaotong University

Summary. The SCMFSA computer can prove the correctness of many algorithms. Unfortunately, it cannot prove the correctness of recursive algorithms. For this reason, this article improves the SCMFSA computer and presents a Small Computer Model with Push-Down Stack (called SCMPDS for short). In addition to conventional arithmetic and "goto" instructions, we increase two new instructions such as "return" and "save instruction-counter" in order to be able to design recursive programs.

MML Identifier: `SCMPDS_1`.

The articles [15], [21], [8], [13], [22], [5], [6], [20], [12], [16], [2], [17], [1], [3], [14], [19], [4], [7], [9], [11], [10], and [18] provide the terminology and notation for this paper.

1. Preliminaries

For simplicity, we follow the rules: x_1, x_2, x_3, x_4, x_5 are sets, i, j, k are natural numbers, I, I_2, I_3, I_4 are elements of $\mathbb{Z}_{14}$, i_1 is an element of Instr-Loc$_{\rm SCM}$, d_1, d_2, d_3, d_4, d_5 are elements of Data-Loc$_{\rm SCM}$, and k_1, k_2, k_3, k_4, k_5, k_6 are integers.

Let x_1, x_2, x_3, x_4 be sets. The functor $< *x_1, x_2, x_3, x_4* >$ yields a set and is defined as follows:

(Def. 1) $< *x_1, x_2, x_3, x_4* > = \langle x_1, x_2, x_3 \rangle \frown \langle x_4 \rangle$.

Let x_5 be a set. The functor $< *x_1, x_2, x_3, x_4, x_5* >$ yielding a set is defined by:

(Def. 2) $< *x_1, x_2, x_3, x_4, x_5* > = \langle x_1, x_2, x_3 \rangle \frown \langle x_4, x_5 \rangle$.

[1]This work was done while the author visited Shinshu University March–April 1999.

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Let x_1, x_2, x_3, x_4 be sets. One can verify that $< *x_1, x_2, x_3, x_4* >$ is function-like and relation-like. Let x_5 be a set. One can verify that $< *x_1, x_2, x_3, x_4, x_5* >$ is function-like and relation-like.

Let x_1, x_2, x_3, x_4 be sets. One can verify that $< *x_1, x_2, x_3, x_4* >$ is finite sequence-like. Let x_5 be a set. One can check that $< *x_1, x_2, x_3, x_4, x_5* >$ is finite sequence-like.

Let D be a non empty set and let x_1, x_2, x_3, x_4 be elements of D. Then $< *x_1, x_2, x_3, x_4* >$ is a finite sequence of elements of D.

Let D be a non empty set and let x_1, x_2, x_3, x_4, x_5 be elements of D. Then $< *x_1, x_2, x_3, x_4, x_5* >$ is a finite sequence of elements of D.

One can prove the following propositions:

(1) $< *x_1, x_2, x_3, x_4* >= \langle x_1, x_2, x_3 \rangle \frown \langle x_4 \rangle$ and $< *x_1, x_2, x_3, x_4* >= \langle x_1, x_2 \rangle \frown \langle x_3, x_4 \rangle$ and $< *x_1, x_2, x_3, x_4* >= \langle x_1 \rangle \frown \langle x_2, x_3, x_4 \rangle$ and $< *x_1, x_2, x_3, x_4* >= \langle x_1 \rangle \frown \langle x_2 \rangle \frown \langle x_3 \rangle \frown \langle x_4 \rangle$.

(2) $< *x_1, x_2, x_3, x_4, x_5* >= \langle x_1, x_2, x_3 \rangle \frown \langle x_4, x_5 \rangle$ and $< *x_1, x_2, x_3, x_4, x_5* > =< *x_1, x_2, x_3, x_4* > \frown \langle x_5 \rangle$ and $< *x_1, x_2, x_3, x_4, x_5* >= \langle x_1 \rangle \frown \langle x_2 \rangle \frown \langle x_3 \rangle \frown \langle x_4 \rangle \frown \langle x_5 \rangle$ and $< *x_1, x_2, x_3, x_4, x_5* >= \langle x_1, x_2 \rangle \frown \langle x_3, x_4, x_5 \rangle$ and $< *x_1, x_2, x_3, x_4, x_5* >= \langle x_1 \rangle \frown < *x_2, x_3, x_4, x_5* >$.

We adopt the following rules: N_1 is a non empty set, y_1, y_2, y_3, y_4, y_5 are elements of N_1, and p is a finite sequence.

We now state several propositions:

(3) $p =< *x_1, x_2, x_3, x_4* >$ iff $\operatorname{len} p = 4$ and $p(1) = x_1$ and $p(2) = x_2$ and $p(3) = x_3$ and $p(4) = x_4$.

(4) $\operatorname{dom} < *x_1, x_2, x_3, x_4* >= \operatorname{Seg} 4$.

(5) $p =< *x_1, x_2, x_3, x_4, x_5* >$ iff $\operatorname{len} p = 5$ and $p(1) = x_1$ and $p(2) = x_2$ and $p(3) = x_3$ and $p(4) = x_4$ and $p(5) = x_5$.

(6) $\operatorname{dom} < *x_1, x_2, x_3, x_4, x_5* >= \operatorname{Seg} 5$.

(7) $\pi_1 < *y_1, y_2, y_3, y_4* >= y_1$ and $\pi_2 < *y_1, y_2, y_3, y_4* >= y_2$ and $\pi_3 < *y_1, y_2, y_3, y_4* >= y_3$ and $\pi_4 < *y_1, y_2, y_3, y_4* >= y_4$.

(8) $\pi_1 < *y_1, y_2, y_3, y_4, y_5* >= y_1$ and $\pi_2 < *y_1, y_2, y_3, y_4, y_5* >= y_2$ and $\pi_3 < *y_1, y_2, y_3, y_4, y_5* >= y_3$ and $\pi_4 < *y_1, y_2, y_3, y_4, y_5* >= y_4$ and $\pi_5 < *y_1, y_2, y_3, y_4, y_5* >= y_5$.

(9) For every integer k holds $k \in \bigcup\{\mathbb{Z}\} \cup \mathbb{N}$.

(10) For every integer k holds $k \in \text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$.

(11) For every element d of $\text{Data-Loc}_{\text{SCM}}$ holds $d \in \text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$.

2. The Construction of SCM with Push-Down Stack

The subset SCMPDS − Instr of $[: \mathbb{Z}_{14}, (\bigcup\{\mathbb{Z}\} \cup \mathbb{N})^* :]$ is defined by the condition (Def. 3).

(Def. 3) $\mathrm{SCMPDS}-\mathrm{Instr} = \{\langle 0, \langle l\rangle\rangle : l$ ranges over integers$\} \cup \{\langle 1, \langle s_1\rangle\rangle : s_1$ ranges over elements of $\text{Data-Loc}_{\mathrm{SCM}}\} \cup \{\langle I, \langle v, c\rangle\rangle; I$ ranges over elements of $\mathbb{Z}_{14}$, v ranges over elements of $\text{Data-Loc}_{\mathrm{SCM}}$, c ranges over integers: $I \in \{2,3\}\} \cup \{\langle I, \langle v, c_1, c_2\rangle\rangle; I$ ranges over elements of $\mathbb{Z}_{14}$, v ranges over elements of $\text{Data-Loc}_{\mathrm{SCM}}$, c_1 ranges over integers, c_2 ranges over integers: $I \in \{4,5,6,7,8\}\} \cup \{\langle I, < *v_1, v_2, c_1, c_2* > \rangle; I$ ranges over elements of $\mathbb{Z}_{14}$, v_1 ranges over elements of $\text{Data-Loc}_{\mathrm{SCM}}$, v_2 ranges over elements of $\text{Data-Loc}_{\mathrm{SCM}}$, c_1 ranges over integers, c_2 ranges over integers: $I \in \{9,10,11,12,13\}\}$.

We now state two propositions:

(12) $\mathrm{SCMPDS}-\mathrm{Instr} = \{\langle 0, \langle k_1\rangle\rangle\} \cup \{\langle 1, \langle d_1\rangle\rangle\} \cup \{\langle I_2, \langle d_2, k_2\rangle\rangle : I_2 \in \{2,3\}\} \cup \{\langle I_3, \langle d_3, k_3, k_4\rangle\rangle : I_3 \in \{4,5,6,7,8\}\} \cup \{\langle I_4, < *d_4, d_5, k_5, k_6* > \rangle : I_4 \in \{9,10,11,12,13\}\}$.

(13) $\langle 0, \langle 0\rangle\rangle \in \mathrm{SCMPDS}-\mathrm{Instr}$.

One can verify that $\mathrm{SCMPDS}-\mathrm{Instr}$ is non empty.

We now state three propositions:

(14) $k = 0$ or there exists j such that $k = 2\cdot j + 1$ or there exists j such that $k = 2\cdot j + 2$.

(15) If $k = 0$, then it is not true that there exists j such that $k = 2\cdot j + 1$ and it is not true that there exists j such that $k = 2\cdot j + 2$.

(16)(i) If there exists j such that $k = 2\cdot j + 1$, then $k \neq 0$ and it is not true that there exists j such that $k = 2\cdot j + 2$, and

(ii) if there exists j such that $k = 2\cdot j + 2$, then $k \neq 0$ and it is not true that there exists j such that $k = 2\cdot j + 1$.

The function $\mathrm{SCMPDS}-\mathrm{OK}$ from $\mathbb{N}$ into $\{\mathbb{Z}\}\cup\{\mathrm{SCMPDS}-\mathrm{Instr}, \text{Instr-Loc}_{\mathrm{SCM}}\}$ is defined as follows:

(Def. 4) $(\mathrm{SCMPDS}-\mathrm{OK})(0) = \text{Instr-Loc}_{\mathrm{SCM}}$ and for every natural number k holds $(\mathrm{SCMPDS}-\mathrm{OK})(2\cdot k+1) = \mathbb{Z}$ and $(\mathrm{SCMPDS}-\mathrm{OK})(2\cdot k+2) = \mathrm{SCMPDS}-\mathrm{Instr}$.

A SCMPDS-State is an element of $\prod \mathrm{SCMPDS}-\mathrm{OK}$.

Next we state several propositions:

(17) $\text{Instr-Loc}_{\mathrm{SCM}} \neq \mathrm{SCMPDS}-\mathrm{Instr}$ and $\mathrm{SCMPDS}-\mathrm{Instr} \neq \mathbb{Z}$.

(18) $(\mathrm{SCMPDS}-\mathrm{OK})(i) = \text{Instr-Loc}_{\mathrm{SCM}}$ iff $i = 0$.

(19) $(\mathrm{SCMPDS}-\mathrm{OK})(i) = \mathbb{Z}$ iff there exists k such that $i = 2\cdot k + 1$.

(20) $(\mathrm{SCMPDS}-\mathrm{OK})(i) = \mathrm{SCMPDS}-\mathrm{Instr}$ iff there exists k such that $i = 2\cdot k + 2$.

(21) $(\mathrm{SCMPDS}-\mathrm{OK})(d_1) = \mathbb{Z}$.

(22) $(\mathrm{SCMPDS}-\mathrm{OK})(i_1) = \mathrm{SCMPDS}-\mathrm{Instr}$.

(23) $\pi_0 \prod \mathrm{SCMPDS}-\mathrm{OK} = \text{Instr-Loc}_{\mathrm{SCM}}$.

(24) $\pi_{d_1} \prod \text{SCMPDS} - \text{OK} = \mathbb{Z}$.

(25) $\pi_{i_1} \prod \text{SCMPDS} - \text{OK} = \text{SCMPDS} - \text{Instr}$.

Let s be a SCMPDS-State. The functor $\mathbf{IC}_s$ yielding an element of Instr-Loc$_{\text{SCM}}$ is defined as follows:

(Def. 5) $\mathbf{IC}_s = s(0)$.

Let s be a SCMPDS-State and let u be an element of Instr-Loc$_{\text{SCM}}$. The functor $\text{Chg}_{\text{SCM}}(s, u)$ yielding a SCMPDS-State is defined as follows:

(Def. 6) $\text{Chg}_{\text{SCM}}(s, u) = s +\cdot (0 \mapsto u)$.

We now state three propositions:

(26) For every SCMPDS-State s and for every element u of Instr-Loc$_{\text{SCM}}$ holds $(\text{Chg}_{\text{SCM}}(s, u))(0) = u$.

(27) For every SCMPDS-State s and for every element u of Instr-Loc$_{\text{SCM}}$ and for every element m_1 of Data-Loc$_{\text{SCM}}$ holds $(\text{Chg}_{\text{SCM}}(s, u))(m_1) = s(m_1)$.

(28) For every SCMPDS-State s and for all elements u, v of Instr-Loc$_{\text{SCM}}$ holds $(\text{Chg}_{\text{SCM}}(s, u))(v) = s(v)$.

Let s be a SCMPDS-State, let t be an element of Data-Loc$_{\text{SCM}}$, and let u be an integer. The functor $\text{Chg}_{\text{SCM}}(s, t, u)$ yields a SCMPDS-State and is defined as follows:

(Def. 7) $\text{Chg}_{\text{SCM}}(s, t, u) = s +\cdot (t \mapsto u)$.

The following propositions are true:

(29) For every SCMPDS-State s and for every element t of Data-Loc$_{\text{SCM}}$ and for every integer u holds $(\text{Chg}_{\text{SCM}}(s, t, u))(0) = s(0)$.

(30) For every SCMPDS-State s and for every element t of Data-Loc$_{\text{SCM}}$ and for every integer u holds $(\text{Chg}_{\text{SCM}}(s, t, u))(t) = u$.

(31) Let s be a SCMPDS-State, t be an element of Data-Loc$_{\text{SCM}}$, u be an integer, and m_1 be an element of Data-Loc$_{\text{SCM}}$. If $m_1 \neq t$, then $(\text{Chg}_{\text{SCM}}(s, t, u))(m_1) = s(m_1)$.

(32) Let s be a SCMPDS-State, t be an element of Data-Loc$_{\text{SCM}}$, u be an integer, and v be an element of Instr-Loc$_{\text{SCM}}$. Then $(\text{Chg}_{\text{SCM}}(s, t, u))(v) = s(v)$.

Let s be a SCMPDS-State and let a be an element of Data-Loc$_{\text{SCM}}$. Then $s(a)$ is an integer.

Let s be a SCMPDS-State, let a be an element of Data-Loc$_{\text{SCM}}$, and let n be an integer. The functor Address_Add(s, a, n) yields an element of Data-Loc$_{\text{SCM}}$ and is defined by:

(Def. 8) $\text{Address\_Add}(s, a, n) = 2 \cdot |s(a) + n| + 1$.

Let s be a SCMPDS-State and let n be an integer. The functor jump_address(s, n) yielding an element of Instr-Loc$_{\text{SCM}}$ is defined as follows:

(Def. 9) $\text{jump\_address}(s, n) = |((\mathbf{IC}_s\ \mathbf{qua}\ \text{natural number}) - 2) + 2 \cdot n| + 2$.

Let d be an element of $\text{Data-Loc}_{\text{SCM}}$ and let s be an integer. Then $\langle d, s\rangle$ is a finite sequence of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$.

Let x be an element of SCMPDS − Instr. Let us assume that there exist an element m_1 of $\text{Data-Loc}_{\text{SCM}}$ and I such that $x = \langle\!\langle I, \langle m_1\rangle\rangle\!\rangle$. The functor $x\,\text{address}_1$ yielding an element of $\text{Data-Loc}_{\text{SCM}}$ is defined as follows:

(Def. 10) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{address}_1 = \pi_1 f$.

The following proposition is true

(33) For every element x of SCMPDS − Instr and for every element m_1 of $\text{Data-Loc}_{\text{SCM}}$ such that $x = \langle\!\langle I, \langle m_1\rangle\rangle\!\rangle$ holds $x\,\text{address}_1 = m_1$.

Let x be an element of SCMPDS − Instr. Let us assume that there exist an integer r and I such that $x = \langle\!\langle I, \langle r\rangle\rangle\!\rangle$. The functor $x\,\text{const\_INT}$ yielding an integer is defined by:

(Def. 11) There exists a finite sequence f of elements of $\mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{const\_INT} = \pi_1 f$.

The following proposition is true

(34) For every element x of SCMPDS − Instr and for every integer k such that $x = \langle\!\langle I, \langle k\rangle\rangle\!\rangle$ holds $x\,\text{const\_INT} = k$.

Let x be an element of SCMPDS − Instr. Let us assume that there exist an element m_1 of $\text{Data-Loc}_{\text{SCM}}$, an integer r, and I such that $x = \langle\!\langle I, \langle m_1, r\rangle\rangle\!\rangle$. The functor $x\,\text{P21address}$ yielding an element of $\text{Data-Loc}_{\text{SCM}}$ is defined as follows:

(Def. 12) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P21address} = \pi_1 f$.

The functor $x\,\text{P22const}$ yielding an integer is defined as follows:

(Def. 13) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P22const} = \pi_2 f$.

The following proposition is true

(35) Let x be an element of SCMPDS − Instr, m_1 be an element of $\text{Data-Loc}_{\text{SCM}}$, and r be an integer. If $x = \langle\!\langle I, \langle m_1, r\rangle\rangle\!\rangle$, then $x\,\text{P21address} = m_1$ and $x\,\text{P22const} = r$.

Let x be an element of SCMPDS − Instr. Let us assume that there exist an element m_2 of $\text{Data-Loc}_{\text{SCM}}$, integers k_1, k_2, and I such that $x = \langle\!\langle I, \langle m_2, k_1, k_2\rangle\rangle\!\rangle$. The functor $x\,\text{P31address}$ yielding an element of $\text{Data-Loc}_{\text{SCM}}$ is defined as follows:

(Def. 14) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P31address} = \pi_1 f$.

The functor $x\,\text{P32const}$ yielding an integer is defined as follows:

(Def. 15) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P32const} = \pi_2 f$.

The functor $x\,$P33const yields an integer and is defined by:

(Def. 16) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P33const} = \pi_3 f$.

We now state the proposition

(36) Let x be an element of SCMPDS − Instr, d_1 be an element of $\text{Data-Loc}_{\text{SCM}}$, and k_1, k_2 be integers. If $x = \langle I, \langle d_1, k_1, k_2 \rangle \rangle$, then $x\,\text{P31address} = d_1$ and $x\,\text{P32const} = k_1$ and $x\,\text{P33const} = k_2$.

Let x be an element of SCMPDS − Instr. Let us assume that there exist elements m_2, m_3 of $\text{Data-Loc}_{\text{SCM}}$, integers k_1, k_2, and I such that $x = \langle I, < *m_2, m_3, k_1, k_2* > \rangle$. The functor $x\,$P41address yields an element of $\text{Data-Loc}_{\text{SCM}}$ and is defined by:

(Def. 17) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P41address} = \pi_1 f$.

The functor $x\,$P42address yields an element of $\text{Data-Loc}_{\text{SCM}}$ and is defined as follows:

(Def. 18) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P42address} = \pi_2 f$.

The functor $x\,$P43const yielding an integer is defined as follows:

(Def. 19) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P43const} = \pi_3 f$.

The functor $x\,$P44const yielding an integer is defined as follows:

(Def. 20) There exists a finite sequence f of elements of $\text{Data-Loc}_{\text{SCM}} \cup \mathbb{Z}$ such that $f = x_{\mathbf{2}}$ and $x\,\text{P44const} = \pi_4 f$.

We now state the proposition

(37) Let x be an element of SCMPDS − Instr, d_1, d_2 be elements of $\text{Data-Loc}_{\text{SCM}}$, and k_1, k_2 be integers. If $x = \langle I, < *d_1, d_2, k_1, k_2* > \rangle$, then $x\,\text{P41address} = d_1$ and $x\,\text{P42address} = d_2$ and $x\,\text{P43const} = k_1$ and $x\,\text{P44const} = k_2$.

Let s be a SCMPDS-State and let a be an element of $\text{Data-Loc}_{\text{SCM}}$. The functor PopInstrLoc(s, a) yielding an element of $\text{Instr-Loc}_{\text{SCM}}$ is defined as follows:

(Def. 21) $\text{PopInstrLoc}(s, a) = 2 \cdot (|s(a)| \div 2) + 4$.

The natural number RetSP is defined as follows:

(Def. 22) $\text{RetSP} = 0$.

The natural number RetIC is defined as follows:

(Def. 23) $\text{RetIC} = 1$.

Let x be an element of SCMPDS − Instr and let s be a SCMPDS-State. The functor $\text{Exec-Res}_{\text{SCM}}(x, s)$ yielding a SCMPDS-State is defined as follows:

(Def. 24) $\text{Exec-Res}_{\rm SCM}(x, s) =$

$$\begin{cases}
\mathrm{Chg}_{\rm SCM}(s, \text{jump_address}(s, x\,\text{const_INT})), & \text{if there exists } k_1 \text{ such that } x = \langle 0, \langle k_1 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, x\,\mathrm{P21address}, x\,\mathrm{P22const}), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, k_1 \text{ such that } x = \langle 2, \langle d_1, k_1 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P21address}, x\,\mathrm{P22const}), (\mathbf{IC}_s \textbf{ qua } \text{natural number})), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, k_1 \text{ such that } x = \langle 3, \langle d_1, k_1 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, x\,\mathrm{address}_1, s(\text{Address_Add}(s, x\,\mathrm{address}_1, \mathrm{RetSP}))), \mathrm{PopInstrLoc}(s, \text{Address_Add}(s, x\,\mathrm{address}_1, \mathrm{RetIC}))), & \text{if there exists } d_1 \text{ such that } x = \langle 1, \langle d_1 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(s, (s(\text{Address_Add}(s, x\,\mathrm{P31address}, x\,\mathrm{P32const})) = 0 \to \mathrm{Next}(\mathbf{IC}_s), \text{jump_address}(s, x\,\mathrm{P33const}))), & \text{if there exist } d_1, k_1, k_2 \text{ such that } x = \langle 4, \langle d_1, k_1, k_2 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(s, (s(\text{Address_Add}(s, x\,\mathrm{P31address}, x\,\mathrm{P32const})) > 0 \to \mathrm{Next}(\mathbf{IC}_s), \text{jump_address}(s, x\,\mathrm{P33const}))), & \text{if there exist } d_1, k_1, k_2 \text{ such that } x = \langle 5, \langle d_1, k_1, k_2 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(s, (0 > s(\text{Address_Add}(s, x\,\mathrm{P31address}, x\,\mathrm{P32const})) \to \mathrm{Next}(\mathbf{IC}_s), \text{jump_address}(s, x\,\mathrm{P33const}))), & \text{if there exist } d_1, k_1, k_2 \text{ such that } x = \langle 6, \langle d_1, k_1, k_2 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P31address}, x\,\mathrm{P32const}), x\,\mathrm{P33const}), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, k_1, k_2 \text{ such that } x = \langle 7, \langle d_1, k_1, k_2 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P31address}, x\,\mathrm{P32const}), s(\text{Address_Add}(s, x\,\mathrm{P31address}, x\,\mathrm{P32const})) + x\,\mathrm{P33const}), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, k_1, k_2 \text{ such that } x = \langle 8, \langle d_1, k_1, k_2 \rangle \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const}), s(\text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const})) + s(\text{Address_Add}(s, x\,\mathrm{P42address}, x\,\mathrm{P44const}))), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, d_2, k_1, k_2 \text{ such that } x = \langle 9, <*d_1, d_2, k_1, k_2*> \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const}), s(\text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const})) - s(\text{Address_Add}(s, x\,\mathrm{P42address}, x\,\mathrm{P44const}))), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, d_2, k_1, k_2 \text{ such that } x = \langle 10, <*d_1, d_2, k_1, k_2*> \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const}), s(\text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const})) \cdot s(\text{Address_Add}(s, x\,\mathrm{P42address}, x\,\mathrm{P44const}))), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, d_2, k_1, k_2 \text{ such that } x = \langle 11, <*d_1, d_2, k_1, k_2*> \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const}), s(\text{Address_Add}(s, x\,\mathrm{P42address}, x\,\mathrm{P44const}))), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, d_2, k_1, k_2 \text{ such that } x = \langle 13, <*d_1, d_2, k_1, k_2*> \rangle, \\
\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(\mathrm{Chg}_{\rm SCM}(s, \text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const}), s(\text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const})) \div s(\text{Address_Add}(s, x\,\mathrm{P42address}, x\,\mathrm{P44const}))), \text{Address_Add}(s, x\,\mathrm{P42address}, x\,\mathrm{P44const}), s(\text{Address_Add}(s, x\,\mathrm{P41address}, x\,\mathrm{P43const})) \bmod s(\text{Address_Add}(s, x\,\mathrm{P42address}, x\,\mathrm{P44const}))), \mathrm{Next}(\mathbf{IC}_s)), & \text{if there exist } d_1, d_2, k_1, k_2 \text{ such that } x = \langle 12, <*d_1, d_2, k_1, k_2*> \rangle, \\
s, & \text{otherwise.}
\end{cases}$$

Let f be a function from SCMPDS − Instr into $(\prod \text{SCMPDS} - \text{OK})^{\prod \text{SCMPDS}-\text{OK}}$ and let x be an element of SCMPDS − Instr. Note that $f(x)$ is function-like and relation-like.

The function SCMPDS − Exec from SCMPDS − Instr into

$(\prod \mathrm{SCMPDS} - \mathrm{OK})^{\Pi\,\mathrm{SCMPDS-OK}}$ is defined by:

(Def. 25) For every element x of SCMPDS − Instr and for every SCMPDS-State y holds $(\mathrm{SCMPDS} - \mathrm{Exec})(x)(y) = \text{Exec-Res}_{\mathrm{SCM}}(x, y)$.

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