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Predicate Calculus for Boolean Valued Functions. Part IV

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The terminology and notation used in this paper are introduced in the following papers: [1], [2], [3], [5], and [4].

In this paper Y is a non empty set.

The following propositions are true:

(1) For every element a of BVF(Y) and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\neg\forall_{\forall_{a,A}G,B}G = \exists_{\neg\forall_{a,A}G,B}G$.

(2) For every element a of BVF(Y) and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\neg\exists_{\forall_{a,A}G,B}G = \forall_{\neg\forall_{a,A}G,B}G$.

(3) For every element a of BVF(Y) and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\forall_{\neg\forall_{a,A}G,B}G = \forall_{\exists_{\neg a,A}G,B}G$.

(4) For every element a of BVF(Y) and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\forall_{\neg\exists_{a,A}G,B}G = \forall_{\forall_{\neg a,A}G,B}G$.

(5) For every element a of BVF(Y) and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\neg\forall_{\exists_{a,A}G,B}G = \exists_{\forall_{\neg a,A}G,B}G$.

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(6) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\neg\exists_{\forall_{a,A}G,B}G = \forall_{\exists_{\neg a,A}G,B}G$.
(7) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\neg\forall_{\forall_{a,A}G,B}G = \exists_{\exists_{\neg a,A}G,B}G$.
(8) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\exists_{\neg\forall_{a,A}G,B}G = \exists_{\exists_{\neg a,A}G,B}G$.
(9) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\exists_{\neg\exists_{a,A}G,B}G = \exists_{\forall_{\neg a,A}G,B}G$.
(10) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\neg\exists_{\exists_{a,A}G,B}G = \forall_{\neg\exists_{a,A}G,B}G$.
(11) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.
(12) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,A}G,B}G$.
(13) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,A}G,B}G$.
(14) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,A}G,B}G$.
(15) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,A}G,B}G$.
(16) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,A}G,B}G$.

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