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Predicate Calculus for Boolean Valued Functions. Part V

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The papers [1], [2], [3], [5], and [4] provide the terminology and notation for this paper.

In this paper Y denotes a non empty set.

One can prove the following propositions:

(1) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(2) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(3) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(4) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(5) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

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(6) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(7) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(8) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(9) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(10) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(11) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B of Y holds $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(12) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(13) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(14) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(15) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(16) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(17) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(18) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(19) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and

A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(20) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(21) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(22) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg\exists_{a,B}G,A}G$.

(23) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\neg\exists_{a,B}G,A}G$.

(24) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(25) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(26) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(27) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(28) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(29) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\forall_{\neg a,B}G,A}G$.

(30) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(31) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(32) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$,

then $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(33) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(34) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(35) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(36) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(37) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(38) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(39) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(40) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\neg\exists_{a,B}G,A}G$.

(41) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\neg\exists_{a,B}G,A}G$.

(42) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(43) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(44) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(45) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(46) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(47) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\neg\exists_{a,A}G,B}G \Subset \forall_{\forall_{\neg a,B}G,A}G$.

(48) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(49) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(50) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(51) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(52) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(53) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(54) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(55) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(56) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(57) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(58) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\exists_{a,B}G,A}G$.

(59) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and

A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg \exists_{a,B}G,A}G$.

(61)[1] Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(62) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B be partitions of Y. If G is a coordinate and $G = \{A, B\}$ and $A \neq B$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

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[1] The proposition (60) has been removed.