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Predicate Calculus for Boolean Valued Functions. Part VII

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The articles [6], [1], [2], [4], [3], and [5] provide the terminology and notation for this paper.

In this paper Y is a non empty set.

Next we state a number of propositions:

(1) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, A, B, C be partitions of Y, and z, u be elements of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$ and $\mathrm{EqClass}(z, C) = \mathrm{EqClass}(u, C)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G)) \cap \mathrm{EqClass}(z, \mathrm{CompF}(B, G)) \neq \emptyset$.

(2) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(3) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\exists_{a,A}G,B}G = \exists_{\exists_{a,B}G,A}G$.

(4) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

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(5) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(6) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(7) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(8) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(9) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\forall_{\forall_{a,C}G,A}G,B}G \Subset \forall_{\exists_{\forall_{a,C}G,B}G,A}G$.

(10) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\forall_{\exists_{a,C}G,A}G,B}G \Subset \forall_{\exists_{\exists_{a,C}G,B}G,A}G$.

(11) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\exists_{\forall_{a,C}G,A}G,B}G = \exists_{\exists_{\forall_{a,C}G,B}G,A}G$.

(12) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\exists_{\exists_{a,C}G,A}G,B}G = \exists_{\exists_{\exists_{a,C}G,B}G,A}G$.

(13) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{\forall_{a,C}G,A}G,B}G \Subset \exists_{\forall_{\forall_{a,C}G,B}G,A}G$.

(14) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{\exists_{a,C}G,A}G,B}G \Subset \exists_{\forall_{\exists_{a,C}G,B}G,A}G$.

(15) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{\forall_{a,C}G,A}G,B}G \Subset \exists_{\exists_{\forall_{a,C}G,B}G,A}G$.

(16) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{\exists_{a,C}G,A}G,B}G \Subset \exists_{\exists_{\exists_{a,C}G,B}G,A}G$.

(17) Let a be an element of BVF(Y), G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{\forall_{a,C}G,A}G,B}G \Subset \forall_{\exists_{\forall_{a,C}G,B}G,A}G$.

(18) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\forall_{\exists_{a,C}G,A}G,B}G \Subset \forall_{\exists_{\exists_{a,C}G,B}G,A}G$.

(19) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\exists_{\forall_{a,C}G,A}G,B}G \Subset \exists_{\exists_{\forall_{a,C}G,B}G,A}G$.

(20) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\exists_{\exists_{a,C}G,A}G,B}G \Subset \exists_{\exists_{\exists_{a,C}G,B}G,A}G$.

(21) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\forall_{\forall_{a,C}G,A}G,B}G \Subset \exists_{\exists_{\forall_{a,C}G,B}G,A}G$.

(22) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\forall_{\exists_{a,C}G,A}G,B}G \Subset \exists_{\exists_{\exists_{a,C}G,B}G,A}G$.

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