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Predicate Calculus for Boolean Valued Functions. Part VIII

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The terminology and notation used here are introduced in the following articles: [1], [2], [3], [4], and [5].

In this paper Y is a non empty set.

We now state a number of propositions:

(1) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of PARTITIONS(Y) and for all partitions A, B of Y holds $\neg\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(2) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(3) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(4) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(5) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of PARTITIONS(Y), and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

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(6) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(7) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B, C of Y holds $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(8) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B, C of Y holds $\forall_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(9) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\forall_{\exists_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(10) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(11) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(12) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(13) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\exists_{\exists_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(14) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(15) Let a be an element of $\mathrm{BVF}(Y)$, G be a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C be partitions of Y. Suppose G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$. Then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(16) For every element a of $\mathrm{BVF}(Y)$ and for every subset G of $\mathrm{PARTITIONS}(Y)$ and for all partitions A, B, C of Y holds $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

References

[1] Shunichi Kobayashi and Kui Jia. A theory of Boolean valued functions and partitions. *Formalized Mathematics*, 7(**2**):249–254, 1998.

[2] Shunichi Kobayashi and Yatsuka Nakamura. A theory of Boolean valued functions and quantifiers with respect to partitions. *Formalized Mathematics*, 7(**2**):307–312, 1998.

[3] Konrad Raczkowski and Paweł Sadowski. Equivalence relations and classes of abstraction. *Formalized Mathematics*, 1(**3**):441–444, 1990.

[4] Andrzej Trybulec. Semilattice operations on finite subsets. *Formalized Mathematics*, 1(**2**):369–376, 1990.
[5] Zinaida Trybulec. Properties of subsets. *Formalized Mathematics*, 1(**1**):67–71, 1990.

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