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Predicate Calculus for Boolean Valued Functions. Part XI

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of usual predicate logic.

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The terminology and notation used in this paper have been introduced in the following articles: [1], [2], [3], [4], and [5].

For simplicity, we adopt the following rules: Y is a non empty set, a is an element of BVF(Y), G is a subset of PARTITIONS(Y), and A, B, C are partitions of Y.

One can prove the following propositions:

(1) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\neg \exists_{a,A} G, B} G \Subset \exists_{\exists_{\neg a,B} G, A} G$.

(2) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \exists_{\exists_{\neg a,B} G, A} G$.

(3) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\neg \exists_{a,A} G, B} G \Subset \forall_{\exists_{\neg a,B} G, A} G$.

(4) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \forall_{\exists_{\neg a,B} G, A} G$.

(5) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \exists_{\forall_{\neg a,B} G, A} G$.

(6) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\neg \exists_{a,A} G, B} G \Subset \forall_{\forall_{\neg a,B} G, A} G$.

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(7) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(8) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(9) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\exists_{a,B}G,A}G$.

(10) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(11) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(12) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(13) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(14) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(15) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(16) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(17) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\neg\exists_{a,B}G,A}G$.

(18) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\neg\exists_{a,B}G,A}G$.

(19) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(21)[1] If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(22) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(23) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(24) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(25) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \exists_{\forall_{\neg a,B}G,A}G$.

(26) If G is a coordinate and $G = \{A, B, C\}$ and $A \neq B$ and $B \neq C$ and $C \neq A$, then $\forall_{\forall_{\neg a,A}G,B}G \Subset \forall_{\forall_{\neg a,B}G,A}G$.

[1]The proposition (20) has been removed.

References

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