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Four Variable Predicate Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The terminology and notation used here have been introduced in the following articles: [10], [4], [6], [1], [8], [7], [2], [3], [5], [11], and [9].

1. Preliminaries

For simplicity, we follow the rules: Y is a non empty set, a is an element of $\mathrm{BVF}(Y)$, G is a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C, D are partitions of Y.

One can prove the following propositions:

(1) Let h be a function and A', B', C', D' be sets. Suppose $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (A \dot{\longmapsto} A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$.

(2) Let A, B, C, D be sets, h be a function, and A', B', C', D' be sets. If $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (A \dot{\longmapsto} A')$, then $\mathrm{dom}\, h = \{A, B, C, D\}$.

(3) For every function h and for all sets A', B', C', D' such that $G = \{A, B, C, D\}$ and $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (A \dot{\longmapsto} A')$ holds $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D)\}$.

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(4) Let z, u be elements of Y and h be a function. Suppose G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(5) Let z, u be elements of Y. Suppose G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$ and $\mathrm{EqClass}(z, C \wedge D) = \mathrm{EqClass}(u, C \wedge D)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G)) \cap \mathrm{EqClass}(z, \mathrm{CompF}(B, G)) \neq \emptyset$.

2. Four Variable Predicate Calculus

Next we state a number of propositions:

(6) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\forall_{a,B}G,A}G$.

(7) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(8) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(9) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\exists_{a,B}G,A}G \Subset \exists_{\exists_{a,A}G,B}G$.

(10) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\exists_{a,B}G,A}G = \exists_{\exists_{a,A}G,B}G$.

(11) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(12) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(13) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(14) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(15) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(16) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(17) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(18) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(19) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\forall_{\exists_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(20) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(21) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\forall_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(22) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\neg\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(23) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\exists_{\exists_{\neg a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(24) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(25) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\exists_{\forall_{a,B}G,A}G$.

(26) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\exists_{\exists_{a,B}G,A}G$.

(27) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(28) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(29) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(30) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(31) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\forall_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(32) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(33) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg\forall_{a,B}G,A}G$.

(34) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(35) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\neg\forall_{a,B}G,A}G$.

(36) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\neg\exists_{a,B}G,A}G$.

(37) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\neg\exists_{a,B}G,A}G$.

(38) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(39) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(40) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\forall_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

(41) If G is a coordinate and $G = \{A, B, C, D\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $B \neq C$ and $B \neq D$ and $C \neq D$, then $\neg\exists_{\exists_{a,A}G,B}G \Subset \forall_{\exists_{\neg a,B}G,A}G$.

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