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Five Variable Predicate Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The terminology and notation used here have been introduced in the following articles: [10], [4], [6], [1], [8], [7], [2], [3], [5], [11], and [9].

1. Preliminaries

For simplicity, we follow the rules: Y denotes a non empty set, a denotes an element of $\mathrm{BVF}(Y)$, G denotes a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C, D, E denote partitions of Y.

One can prove the following propositions:

(1) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\mathrm{CompF}(A, G) = B \wedge C \wedge D \wedge E$.

(2) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\mathrm{CompF}(B, G) = A \wedge C \wedge D \wedge E$.

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(3) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\mathrm{CompF}(C, G) = A \wedge B \wedge D \wedge E$.

(4) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\mathrm{CompF}(D, G) = A \wedge B \wedge C \wedge E$.

(5) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\mathrm{CompF}(E, G) = A \wedge B \wedge C \wedge D$.

(6) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. Suppose $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (E \dot{\longmapsto} E') {+\cdot} (A \dot{\longmapsto} A')$. Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$.

(7) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. Suppose $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (E \dot{\longmapsto} E') {+\cdot} (A \dot{\longmapsto} A')$. Then $\mathrm{dom}\, h = \{A, B, C, D, E\}$.

(8) Let A, B, C, D, E be sets, h be a function, and A', B', C', D', E' be sets. Suppose $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and $h = (B \dot{\longmapsto} B') {+\cdot} (C \dot{\longmapsto} C') {+\cdot} (D \dot{\longmapsto} D') {+\cdot} (E \dot{\longmapsto} E') {+\cdot} (A \dot{\longmapsto} A')$. Then $\mathrm{rng}\, h = \{h(A), h(B), h(C), h(D), h(E)\}$.

(9) Let G be a subset of PARTITIONS(Y), A, B, C, D, E be partitions of Y, z, u be elements of Y, and h be a function. Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(10) Let G be a subset of PARTITIONS(Y), A, B, C, D, E be partitions of Y, and z, u be elements of Y. Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$ and $\mathrm{EqClass}(z, C \wedge D \wedge E) = \mathrm{EqClass}(u, C \wedge D \wedge E)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G)) \cap \mathrm{EqClass}(z, \mathrm{CompF}(B, G)) \neq \emptyset$.

2. Predicate Calculus

One can prove the following propositions:

(11) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\forall_{a,B}G,A}G$.

(12) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(13) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(14) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\exists_{\exists_{a,B}G,A}G \Subset \exists_{\exists_{a,A}G,B}G$.

(15) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\exists_{\exists_{a,A}G,B}G = \exists_{\exists_{a,B}G,A}G$.

(16) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(17) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(18) $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(19) $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(20) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(22)[1] Suppose that

[1]The proposition (21) has been removed.

G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\exists_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

(23) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\neg\forall_{\forall_{a,A}G,B}G = \exists_{\neg\forall_{a,B}G,A}G$.

(24) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\neg\forall_{\forall_{a,A}G,B}G = \exists_{\exists_{\neg a,B}G,A}G$.

(25) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\forall_{\neg\forall_{a,A}G,B}G \Subset \neg\forall_{\forall_{a,B}G,A}G$.

(26) Suppose that
G is a coordinate and $G = \{A, B, C, D, E\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $C \neq D$ and $C \neq E$ and $D \neq E$. Then $\forall_{\neg\forall_{a,A}G,B}G \Subset \exists_{\exists_{\neg a,B}G,A}G$.

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