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Six Variable Predicate Calculus for Boolean Valued Functions. Part I

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Summary. In this paper, we proved some elementary predicate calculus formulae containing the quantifiers of Boolean valued functions with respect to partitions. Such a theory is an analogy of ordinary predicate logic.

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The terminology and notation used in this paper are introduced in the following papers: [10], [4], [6], [1], [8], [7], [2], [3], [5], [11], and [9].

1. Preliminaries

For simplicity, we follow the rules: Y denotes a non empty set, a denotes an element of $\mathrm{BVF}(Y)$, G denotes a subset of $\mathrm{PARTITIONS}(Y)$, and A, B, C, D, E, F denote partitions of Y.

We now state a number of propositions:

(1) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(A, G) = B \wedge C \wedge D \wedge E \wedge F$.

(2) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(B, G) = A \wedge C \wedge D \wedge E \wedge F$.

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(3) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(C, G) = A \wedge B \wedge D \wedge E \wedge F$.

(4) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(D, G) = A \wedge B \wedge C \wedge E \wedge F$.

(5) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(E, G) = A \wedge B \wedge C \wedge D \wedge F$.

(6) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{CompF}(F, G) = A \wedge B \wedge C \wedge D \wedge E$.

(7) Let A, B, C, D, E, F be sets, h be a function, and A', B', C', D', E', F' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (A \dot{\longmapsto} A')$.
Then $h(A) = A'$ and $h(B) = B'$ and $h(C) = C'$ and $h(D) = D'$ and $h(E) = E'$ and $h(F) = F'$.

(8) Let A, B, C, D, E, F be sets, h be a function, and A', B', C', D', E', F' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (A \dot{\longmapsto} A')$.
Then $\mathrm{dom}\, h = \{A, B, C, D, E, F\}$.

(9) Let A, B, C, D, E, F be sets, h be a function, and A', B', C', D', E', F' be sets. Suppose that
$A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$ and $h = (B \dot{\longmapsto} B') +\cdot (C \dot{\longmapsto} C') +\cdot (D \dot{\longmapsto} D') +\cdot (E \dot{\longmapsto} E') +\cdot (F \dot{\longmapsto} F') +\cdot (A \dot{\longmapsto} A')$.

Then $\operatorname{rng} h = \{h(A), h(B), h(C), h(D), h(E), h(F)\}$.

(10) Let G be a subset of PARTITIONS(Y), A, B, C, D, E, F be partitions of Y, z, u be elements of Y, and h be a function. Suppose that G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\mathrm{EqClass}(u, B \wedge C \wedge D \wedge E \wedge F) \cap \mathrm{EqClass}(z, A) \neq \emptyset$.

(11) Let G be a subset of PARTITIONS(Y), A, B, C, D, E, F be partitions of Y, z, u be elements of Y, and h be a function. Suppose that G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$ and $\mathrm{EqClass}(z, C \wedge D \wedge E \wedge F) = \mathrm{EqClass}(u, C \wedge D \wedge E \wedge F)$. Then $\mathrm{EqClass}(u, \mathrm{CompF}(A, G)) \cap \mathrm{EqClass}(z, \mathrm{CompF}(B, G)) \neq \emptyset$.

2. Predicate Calculus

The following propositions are true:

(12) Suppose that G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\forall_{a,B}G,A}G$.

(13) Suppose that G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\forall_{\forall_{a,A}G,B}G = \forall_{\forall_{a,B}G,A}G$.

(14) Suppose that G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\exists_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(15) Suppose that G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\exists_{\exists_{a,B}G,A}G \Subset \exists_{\exists_{a,A}G,B}G$.

(16) Suppose that

G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\exists_{\exists_{a,A}G,B}G = \exists_{\exists_{a,B}G,A}G$.

(17) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\forall_{a,B}G,A}G$.

(18) $\forall_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(19) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\forall_{\forall_{a,A}G,B}G \Subset \forall_{\exists_{a,B}G,A}G$.

(20) $\forall_{\exists_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

(21) Suppose that
G is a coordinate and $G = \{A, B, C, D, E, F\}$ and $A \neq B$ and $A \neq C$ and $A \neq D$ and $A \neq E$ and $A \neq F$ and $B \neq C$ and $B \neq D$ and $B \neq E$ and $B \neq F$ and $C \neq D$ and $C \neq E$ and $C \neq F$ and $D \neq E$ and $D \neq F$ and $E \neq F$. Then $\exists_{\forall_{a,A}G,B}G \Subset \exists_{\exists_{a,B}G,A}G$.

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