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Standard Ordering of Instruction Locations

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The notation and terminology used in this paper have been introduced in the following articles: [11], [15], [12], [18], [1], [3], [14], [4], [16], [6], [7], [8], [9], [2], [10], [5], [19], [20], [13], and [17].

1. Preliminaries

We use the following convention: x, X are sets, D is a non empty set, and k, m, n are natural numbers.

The following two propositions are true:

(1) For every real number r holds $\max\{r\} = r$.

(2) $\max\{n\} = n$.

One can verify that there exists a finite sequence which is non trivial.

The following proposition is true

(3) For every trivial finite sequence f of elements of D holds f is empty or there exists an element x of D such that $f = \langle x \rangle$.

Let x, y be sets. Note that $\langle x, y \rangle$ is non empty.

Let us observe that every binary relation has non empty elements.

One can prove the following proposition

(4) id_X is bijective.

Let A be a finite set and let B be a set. Observe that $A \longmapsto B$ is finite.

Let x, y be sets. One can check that $x \dot{\longmapsto} y$ is trivial.

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2. Restricted Concatenation

Let f_1 be a non empty finite sequence and let f_2 be a finite sequence. Observe that $f_1 \frown f_2$ is non empty.

The following propositions are true:

(5) Let f_1 be a non empty finite sequence of elements of D and f_2 be a finite sequence of elements of D. Then $(f_1 \frown f_2)_1 = (f_1)_1$.

(6) Let f_1 be a finite sequence of elements of D and f_2 be a non trivial finite sequence of elements of D. Then $(f_1 \frown f_2)_{\mathrm{len}(f_1 \frown f_2)} = (f_2)_{\mathrm{len}\, f_2}$.

(7) For every finite sequence f holds $f \frown \varepsilon = f$.

(8) For every finite sequence f holds $f \frown \langle x \rangle = f$.

(9) For all finite sequences f_1, f_2 of elements of D such that $1 \leqslant n$ and $n \leqslant \mathrm{len}\, f_1$ holds $(f_1 \frown f_2)_n = (f_1)_n$.

(10) For all finite sequences f_1, f_2 of elements of D such that $1 \leqslant n$ and $n < \mathrm{len}\, f_2$ holds $(f_1 \frown f_2)_{\mathrm{len}\, f_1 + n} = (f_2)_{n+1}$.

3. Ami-Struct

For simplicity, we adopt the following convention: N is a set with non empty elements, S is a von Neumann definite AMI over N, i is an instruction of S, l, l_1, l_2, l_3 are instruction-locations of S, and s is a state of S.

We now state the proposition

(11) Let S be a definite AMI over N, I be an instruction of S, and s be a state of S. Then $s+\cdot((\text{the instruction locations of } S) \longmapsto I)$ is a state of S.

Let N be a set and let S be an AMI over N. Observe that every finite partial state of S which is empty is also programmed.

Let N be a set and let S be an AMI over N. One can check that there exists a finite partial state of S which is empty.

Let N be a set with non empty elements and let S be a von Neumann definite AMI over N. Note that there exists a finite partial state of S which is non empty, trivial, and programmed.

Let N be a set with non empty elements, let S be an AMI over N, let i be an instruction of S, and let s be a state of S. One can verify that (the execution of $S)(i)(s)$ is function-like and relation-like.

Let N be a set and let S be an AMI over N.

(Def. 1) An element of the instruction codes of S is said to be an instruction type of S.

Let N be a set, let S be an AMI over N, and let I be an element of the instructions of S. The functor InsCode(I) yields an instruction type of S and is defined by:

(Def. 2) $\text{InsCode}(I) = I_{\mathbf{1}}$.

Let N be a set with non empty elements and let S be a steady-programmed von Neumann definite AMI over N. Observe that there exists a finite partial state of S which is non empty, trivial, autonomic, and programmed.

One can prove the following propositions:

(12) Let S be a steady-programmed von Neumann definite AMI over N, i_1 be an instruction-location of S, and I be an instruction of S. Then $i_1 \mapsto I$ is autonomic.

(13) Every steady-programmed von Neumann definite AMI over N is programmable.

Let N be a set with non empty elements. One can check that every von Neumann definite AMI over N which is steady-programmed is also programmable.

Let N be a set with non empty elements, let S be an AMI over N, and let T be an instruction type of S. We say that T is jump-only if and only if the condition (Def. 3) is satisfied.

(Def. 3) Let s be a state of S, o be an object of S, and I be an instruction of S. If $\text{InsCode}(I) = T$ and $o \neq \mathbf{IC}_S$, then $(\text{Exec}(I, s))(o) = s(o)$.

Let N be a set with non empty elements, let S be an AMI over N, and let I be an instruction of S. We say that I is jump-only if and only if:

(Def. 4) InsCode(I) is jump-only.

Let us consider N, S, i, l. The functor NIC(i, l) yielding a subset of the instruction locations of S is defined by:

(Def. 5) $\text{NIC}(i, l) = \{\mathbf{IC}_{\text{Following}(s)} : \mathbf{IC}_s = l \ \wedge \ s(l) = i\}$.

Let N be a set with non empty elements, let S be a realistic von Neumann definite AMI over N, let i be an instruction of S, and let l be an instruction-location of S. Note that NIC(i, l) is non empty.

Let us consider N, S, i. The functor JUMP(i) yields a subset of the instruction locations of S and is defined by:

(Def. 6) $\text{JUMP}(i) = \bigcap\{\text{NIC}(i, l)\}$.

Let us consider N, S, l. The functor SUCC(l) yielding a subset of the instruction locations of S is defined by:

(Def. 7) $\text{SUCC}(l) = \bigcup\{\text{NIC}(i, l) \setminus \text{JUMP}(i)\}$.

One can prove the following propositions:

(14) Let S be a von Neumann definite AMI over N and i be an instruction of S. Suppose the instruction locations of S are non trivial and for every instruction-location l of S holds $\text{NIC}(i, l) = \{l\}$. Then JUMP(i) is empty.

(15) Let S be a realistic von Neumann definite AMI over N, i_1 be an instruction-location of S, and i be an instruction of S. If i is halting, then $\mathrm{NIC}(i, i_1) = \{i_1\}$.

4. Ordering of Instruction Locations

Let us consider N, S, l_1, l_2. The predicate $l_1 \leqslant l_2$ is defined by the condition (Def. 8).

(Def. 8) There exists a non empty finite sequence f of elements of the instruction locations of S such that $f_1 = l_1$ and $f_{\mathrm{len}\, f} = l_2$ and for every n such that $1 \leqslant n$ and $n < \mathrm{len}\, f$ holds $f_{n+1} \in \mathrm{SUCC}(f_n)$.

Let us note that the predicate $l_1 \leqslant l_2$ is reflexive.

Next we state the proposition

(16) If $l_1 \leqslant l_2$ and $l_2 \leqslant l_3$, then $l_1 \leqslant l_3$.

Let us consider N, S. We say that S is InsLoc-antisymmetric if and only if:

(Def. 9) For all l_1, l_2 such that $l_1 \leqslant l_2$ and $l_2 \leqslant l_1$ holds $l_1 = l_2$.

Let us consider N, S. We say that S is standard if and only if the condition (Def. 10) is satisfied.

(Def. 10) There exists a function f from $\mathbb{N}$ into the instruction locations of S such that f is bijective and for all natural numbers m, n holds $m \leqslant n$ iff $f(m) \leqslant f(n)$.

One can prove the following three propositions:

(17) Let S be a von Neumann definite AMI over N and f_1, f_2 be functions from $\mathbb{N}$ into the instruction locations of S. Suppose that
- (i) f_1 is bijective,
- (ii) for all natural numbers m, n holds $m \leqslant n$ iff $f_1(m) \leqslant f_1(n)$,
- (iii) f_2 is bijective, and
- (iv) for all natural numbers m, n holds $m \leqslant n$ iff $f_2(m) \leqslant f_2(n)$.

Then $f_1 = f_2$.

(18) Let S be a von Neumann definite AMI over N and f be a function from $\mathbb{N}$ into the instruction locations of S. Suppose f is bijective. Then the following statements are equivalent
- (i) for all natural numbers m, n holds $m \leqslant n$ iff $f(m) \leqslant f(n)$,
- (ii) for every natural number k holds $f(k+1) \in \mathrm{SUCC}(f(k))$ and for every natural number j such that $f(j) \in \mathrm{SUCC}(f(k))$ holds $k \leqslant j$.

(19) Let S be a von Neumann definite AMI over N. Then S is standard if and only if there exists a function f from $\mathbb{N}$ into the instruction locations of S such that f is bijective and for every natural number k holds $f(k+1) \in$

SUCC($f(k)$) and for every natural number j such that $f(j) \in$ SUCC($f(k)$) holds $k \leqslant j$.

5. Standard Trivial Computer

Let N be a set with non empty elements. The functor STC(N) yielding a strict AMI over N is defined by the conditions (Def. 11).

(Def. 11) The objects of STC(N) $= \mathbb{N} \cup \{\mathbb{N}\}$ and the instruction counter of STC(N) $= \mathbb{N}$ and the instruction locations of STC(N) $= \mathbb{N}$ and the instruction codes of STC(N) $= \{0, 1\}$ and the instructions of STC(N) $= \{\langle 0, 0\rangle, \langle 1, 0\rangle\}$ and the object kind of STC(N) $= (\mathbb{N} \longmapsto \{\langle 1, 0\rangle, \langle 0, 0\rangle\}) +\cdot (\{\mathbb{N}\} \longmapsto \mathbb{N})$ and there exists a function f from $\prod$(the object kind of STC(N)) into $\prod$(the object kind of STC(N)) such that for every element s of $\prod$(the object kind of STC(N)) holds $f(s) = s +\cdot (\{\mathbb{N}\} \longmapsto \mathrm{succ}\, s(\mathbb{N}))$ and the execution of STC(N) $= (\{\langle 1, 0\rangle\} \longmapsto f) +\cdot (\{\langle 0, 0\rangle\} \longmapsto \mathrm{id}_{\prod(\text{the object kind of STC}(N))})$.

Let N be a set with non empty elements. Note that the instruction locations of STC(N) is infinite.

Let N be a set with non empty elements. Observe that STC(N) is von Neumann definite realistic steady-programmed and data-oriented.

Next we state several propositions:

(20) For every instruction i of STC(N) such that InsCode(i) $= 0$ holds i is halting.

(21) For every instruction i of STC(N) such that InsCode(i) $= 1$ holds i is non halting.

(22) For every instruction i of STC(N) holds InsCode(i) $= 1$ or InsCode(i) $= 0$.

(23) Every instruction of STC(N) is jump-only.

(24) For every instruction-location l of STC(N) such that $l = k$ holds SUCC(l) $= \{k, k + 1\}$.

Let N be a set with non empty elements. Observe that STC(N) is standard.

Let N be a set with non empty elements. Observe that STC(N) is halting.

Let N be a set with non empty elements. One can check that there exists a von Neumann definite AMI over N which is standard, halting, realistic, steady-programmed, and programmable.

Let N be a set with non empty elements, let S be a standard von Neumann definite AMI over N, and let k be a natural number. The functor $\mathrm{il}_S(k)$ yields an instruction-location of S and is defined by the condition (Def. 12).

(Def. 12) There exists a function f from $\mathbb{N}$ into the instruction locations of S such that f is bijective and for all natural numbers m, n holds $m \leqslant n$ iff $f(m) \leqslant f(n)$ and $\mathrm{il}_S(k) = f(k)$.

We now state two propositions:

(25) Let S be a standard von Neumann definite AMI over N and k_1, k_2 be natural numbers. If $\mathrm{il}_S(k_1) = \mathrm{il}_S(k_2)$, then $k_1 = k_2$.

(26) Let S be a standard von Neumann definite AMI over N and l be an instruction-location of S. Then there exists a natural number k such that $l = \mathrm{il}_S(k)$.

Let N be a set with non empty elements, let S be a standard von Neumann definite AMI over N, and let l be an instruction-location of S. The functor $\mathrm{locnum}(l)$ yields a natural number and is defined as follows:

(Def. 13) $\mathrm{il}_S(\mathrm{locnum}(l)) = l$.

One can prove the following propositions:

(27) Let S be a standard von Neumann definite AMI over N and l_1, l_2 be instruction-locations of S. If $\mathrm{locnum}(l_1) = \mathrm{locnum}(l_2)$, then $l_1 = l_2$.

(28) Let S be a standard von Neumann definite AMI over N and k_1, k_2 be natural numbers. Then $\mathrm{il}_S(k_1) \leqslant \mathrm{il}_S(k_2)$ if and only if $k_1 \leqslant k_2$.

(29) Let S be a standard von Neumann definite AMI over N and l_1, l_2 be instruction-locations of S. Then $\mathrm{locnum}(l_1) \leqslant \mathrm{locnum}(l_2)$ if and only if $l_1 \leqslant l_2$.

(30) If S is standard, then S is InsLoc-antisymmetric.

Let us consider N. Observe that every von Neumann definite AMI over N which is standard is also InsLoc-antisymmetric.

Let N be a set with non empty elements, let S be a standard von Neumann definite AMI over N, let f be an instruction-location of S, and let k be a natural number. The functor $f + k$ yielding an instruction-location of S is defined by:

(Def. 14) $f + k = \mathrm{il}_S(\mathrm{locnum}(f) + k)$.

Next we state three propositions:

(31) For every standard von Neumann definite AMI S over N and for every instruction-location f of S holds $f + 0 = f$.

(32) Let S be a standard von Neumann definite AMI over N and f, g be instruction-locations of S. If $f + k = g + k$, then $f = g$.

(33) For every standard von Neumann definite AMI S over N and for every instruction-location f of S holds $\mathrm{locnum}(f) + k = \mathrm{locnum}(f + k)$.

Let N be a set with non empty elements, let S be a standard von Neumann definite AMI over N, and let f be an instruction-location of S. The functor $\mathrm{NextLoc}\, f$ yields an instruction-location of S and is defined as follows:

(Def. 15) $\mathrm{NextLoc}\, f = f + 1$.

The following propositions are true:

(34) For every standard von Neumann definite AMI S over N and for every instruction-location f of S holds $\mathrm{NextLoc}\, f = \mathrm{il}_S(\mathrm{locnum}(f) + 1)$.

(35) For every standard von Neumann definite AMI S over N and for every instruction-location f of S holds $f \neq \mathrm{NextLoc}\, f$.

(36) Let S be a standard von Neumann definite AMI over N and f, g be instruction-locations of S. If $\mathrm{NextLoc}\, f = \mathrm{NextLoc}\, g$, then $f = g$.

(37) $\mathrm{il}_{\mathrm{STC}(N)}(k) = k$.

(38) For every instruction i of $\mathrm{STC}(N)$ and for every state s of $\mathrm{STC}(N)$ such that $\mathrm{InsCode}(i) = 1$ holds $(\mathrm{Exec}(i, s))(\mathbf{IC}_{\mathrm{STC}(N)}) = \mathrm{NextLoc}\, \mathbf{IC}_s$.

(39) For every instruction-location l of $\mathrm{STC}(N)$ and for every instruction i of $\mathrm{STC}(N)$ such that $\mathrm{InsCode}(i) = 1$ holds $\mathrm{NIC}(i, l) = \{\mathrm{NextLoc}\, l\}$.

(40) For every instruction-location l of $\mathrm{STC}(N)$ holds $\mathrm{SUCC}(l) = \{l, \mathrm{NextLoc}\, l\}$.

Let N be a set with non empty elements, let S be a standard von Neumann definite AMI over N, and let i be an instruction of S. We say that i is sequential if and only if:

(Def. 16) For every state s of S holds $(\mathrm{Exec}(i, s))(\mathbf{IC}_S) = \mathrm{NextLoc}\, \mathbf{IC}_s$.

The following propositions are true:

(41) Let S be a standard realistic von Neumann definite AMI over N, i_1 be an instruction-location of S, and i be an instruction of S. If i is sequential, then $\mathrm{NIC}(i, i_1) = \{\mathrm{NextLoc}\, i_1\}$.

(42) Let S be a realistic standard von Neumann definite AMI over N and i be an instruction of S. If i is sequential, then i is non halting.

Let us consider N and let S be a realistic standard von Neumann definite AMI over N. Observe that every instruction of S which is sequential is also non halting and every instruction of S which is halting is also non sequential.

One can prove the following proposition

(43) Let S be a standard von Neumann definite AMI over N and i be an instruction of S. If $\mathrm{JUMP}(i)$ is non empty, then i is non sequential.

6. Closedness of Finite Partial States

Let N be a set with non empty elements, let S be a von Neumann definite AMI over N, and let F be a finite partial state of S. We say that F is closed if and only if:

(Def. 17) For every instruction-location l of S such that $l \in \mathrm{dom}\, F$ holds $\mathrm{NIC}(\pi_l F, l) \subseteq \mathrm{dom}\, F$.

We say that F is really-closed if and only if:

(Def. 18) For every state s of S such that $F \subseteq s$ and $\mathbf{IC}_s \in \operatorname{dom} F$ and for every natural number k holds $\mathbf{IC}_{(\mathrm{Computation}(s))(k)} \in \operatorname{dom} F$.

Let N be a set with non empty elements, let S be a standard von Neumann definite AMI over N, and let F be a finite partial state of S. We say that F is para-closed if and only if:

(Def. 19) For every state s of S such that $F \subseteq s$ and $\mathbf{IC}_s = \mathrm{il}_S(0)$ and for every natural number k holds $\mathbf{IC}_{(\mathrm{Computation}(s))(k)} \in \operatorname{dom} F$.

The following propositions are true:

(44) Let S be a standard steady-programmed von Neumann definite AMI over N and F be a finite partial state of S. If F is really-closed and $\mathrm{il}_S(0) \in \operatorname{dom} F$, then F is para-closed.

(45) Let S be a von Neumann definite steady-programmed AMI over N and F be a finite partial state of S. If F is closed, then F is really-closed.

Let N be a set with non empty elements and let S be a von Neumann definite steady-programmed AMI over N. One can verify that every finite partial state of S which is closed is also really-closed.

We now state the proposition

(46) For every standard realistic halting von Neumann definite AMI S over N holds $\mathrm{il}_S(0) \dot{\longmapsto} \mathbf{halt}_S$ is closed.

Let N be a set with non empty elements, let S be a von Neumann definite AMI over N, and let F be a finite partial state of S. We say that F is lower if and only if the condition (Def. 20) is satisfied.

(Def. 20) Let l be an instruction-location of S. Suppose $l \in \operatorname{dom} F$. Let m be an instruction-location of S. If $m \leqslant l$, then $m \in \operatorname{dom} F$.

The following proposition is true

(47) For every von Neumann definite AMI S over N holds every empty finite partial state of S is lower.

Let N be a set with non empty elements and let S be a von Neumann definite AMI over N. Observe that every finite partial state of S which is empty is also lower.

The following proposition is true

(48) For every standard von Neumann definite AMI S over N and for every instruction i of S holds $\mathrm{il}_S(0) \dot{\longmapsto} i$ is lower.

Let N be a set with non empty elements and let S be a standard von Neumann definite AMI over N. Note that there exists a finite partial state of S which is lower, non empty, trivial, and programmed.

We now state two propositions:

(49) Let S be a standard von Neumann definite AMI over N and F be a lower non empty programmed finite partial state of S. Then $\mathrm{il}_S(0) \in \mathrm{dom}\, F$.

(50) Let N be a set with non empty elements, S be a standard von Neumann definite AMI over N, and P be a lower programmed finite partial state of S. Then $m < \mathrm{card}\, P$ if and only if $\mathrm{il}_S(m) \in \mathrm{dom}\, P$.

Let N be a set with non empty elements, let S be a standard von Neumann definite AMI over N, and let F be a non empty programmed finite partial state of S. The functor $\mathrm{LastLoc}\, F$ yields an instruction-location of S and is defined by the condition (Def. 21).

(Def. 21) There exists a finite non empty subset M of $\mathbb{N}$ such that $M = \{\mathrm{locnum}(l); l$ ranges over elements of the instruction locations of S: $l \in \mathrm{dom}\, F\}$ and $\mathrm{LastLoc}\, F = \mathrm{il}_S(\max M)$.

We now state several propositions:

(51) Let S be a standard von Neumann definite AMI over N and F be a non empty programmed finite partial state of S. Then $\mathrm{LastLoc}\, F \in \mathrm{dom}\, F$.

(52) Let S be a standard von Neumann definite AMI over N and F, G be non empty programmed finite partial states of S. If $F \subseteq G$, then $\mathrm{LastLoc}\, F \leqslant \mathrm{LastLoc}\, G$.

(53) Let S be a standard von Neumann definite AMI over N, F be a non empty programmed finite partial state of S, and l be an instruction-location of S. If $l \in \mathrm{dom}\, F$, then $l \leqslant \mathrm{LastLoc}\, F$.

(54) Let S be a standard von Neumann definite AMI over N, F be a lower non empty programmed finite partial state of S, and G be a non empty programmed finite partial state of S. If $F \subseteq G$ and $\mathrm{LastLoc}\, F = \mathrm{LastLoc}\, G$, then $F = G$.

(55) Let N be a set with non empty elements, S be a standard von Neumann definite AMI over N, and F be a lower non empty programmed finite partial state of S. Then $\mathrm{LastLoc}\, F = \mathrm{il}_S(\mathrm{card}\, F -' 1)$.

Let N be a set with non empty elements and let S be a standard steady-programmed von Neumann definite AMI over N. Note that every finite partial state of S which is really-closed, lower, non empty, and programmed is also para-closed.

Let N be a set with non empty elements, let S be a standard halting von Neumann definite AMI over N, and let F be a non empty programmed finite partial state of S. We say that F is halt-ending if and only if:

(Def. 22) $F(\mathrm{LastLoc}\, F) = \mathbf{halt}_S$.

We say that F is unique-halt if and only if:

(Def. 23) For every instruction-location f of S such that $F(f) = \mathbf{halt}_S$ and $f \in \mathrm{dom}\, F$ holds $f = \mathrm{LastLoc}\, F$.

Let N be a set with non empty elements and let S be a standard halting von Neumann definite AMI over N. One can check that there exists a lower non empty programmed finite partial state of S which is halt-ending, unique-halt, and trivial.

Let N be a set with non empty elements and let S be a standard halting realistic von Neumann definite AMI over N. One can check that there exists a finite partial state of S which is trivial, closed, lower, non empty, and programmed.

Let N be a set with non empty elements and let S be a standard halting realistic von Neumann definite AMI over N. Observe that there exists a lower non empty programmed finite partial state of S which is halt-ending, unique-halt, trivial, and closed.

Let N be a set with non empty elements and let S be a standard halting realistic steady-programmed von Neumann definite AMI over N. Observe that there exists a lower non empty programmed finite partial state of S which is halt-ending, unique-halt, autonomic, trivial, and closed.

Let N be a set with non empty elements and let S be a standard halting von Neumann definite AMI over N.

(Def. 24) A halt-ending unique-halt lower non empty programmed finite partial state of S is said to be a pre-Macro of S.

Let N be a set with non empty elements and let S be a standard realistic halting von Neumann definite AMI over N. One can verify that there exists a pre-Macro of S which is closed.

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