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Some Properties of Cells and Gauges[1]

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The terminology and notation used in this paper are introduced in the following articles: [20], [25], [2], [7], [18], [21], [8], [3], [4], [16], [13], [23], [14], [17], [5], [11], [12], [1], [19], [6], [10], [15], [22], [24], and [9].

We adopt the following convention: C denotes a simple closed curve, i, j, n denote natural numbers, and p denotes a point of $\mathcal{E}^2_{\mathrm{T}}$.

The following propositions are true:

(1) BDD C is Bounded.

(2) If $\langle i, j\rangle \in$ the indices of Gauge(C,n) and $\langle i+1, j\rangle \in$ the indices of Gauge(C,n), then $\rho((\mathrm{Gauge}(C,n))_{1,1}, (\mathrm{Gauge}(C,n))_{2,1}) = |((\mathrm{Gauge}(C,n))_{i+1,j})_{\mathbf{1}} - ((\mathrm{Gauge}(C,n))_{i,j})_{\mathbf{1}}|$.

(3) If $\langle i, j\rangle \in$ the indices of Gauge(C,n) and $\langle i, j+1\rangle \in$ the indices of Gauge(C,n), then $\rho((\mathrm{Gauge}(C,n))_{1,1}, (\mathrm{Gauge}(C,n))_{1,2}) = |((\mathrm{Gauge}(C,n))_{i,j+1})_{\mathbf{2}} - ((\mathrm{Gauge}(C,n))_{i,j})_{\mathbf{2}}|$.

(4) For every subset S of $\mathcal{E}^2_{\mathrm{T}}$ such that S is Bounded holds $(\mathrm{proj1})^\circ S$ is bounded.

(5) Let C_1 be a non empty compact subset of $\mathcal{E}^2_{\mathrm{T}}$ and C_2, S be non empty subsets of $\mathcal{E}^2_{\mathrm{T}}$. If $S = C_1 \cup C_2$ and $(\mathrm{proj1})^\circ C_2$ is non empty and lower bounded, then W-bound $S = \min(\text{W-bound}\, C_1, \text{W-bound}\, C_2)$.

(6) For every subset X of $\mathcal{E}^2_{\mathrm{T}}$ such that $p \in X$ and X is Bounded holds W-bound $X \leqslant p_{\mathbf{1}}$ and $p_{\mathbf{1}} \leqslant$ E-bound X and S-bound $X \leqslant p_{\mathbf{2}}$ and $p_{\mathbf{2}} \leqslant$ N-bound X.

(7) $p \in$ WestHalfline p and $p \in$ EastHalfline p.

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(8) WestHalfline p is non Bounded.

(9) EastHalfline p is non Bounded.

(10) NorthHalfline p is non Bounded.

(11) SouthHalfline p is non Bounded.

(12) If UBD $C \neq \emptyset$, then UBD C is a component of $C^{\rm c}$.

(13) For every connected subset W_1 of $\mathcal{E}^2_{\rm T}$ such that W_1 is non Bounded and $W_1 \cap C = \emptyset$ holds $W_1 \subseteq$ UBD C.

(14) For every point p of $\mathcal{E}^2_{\rm T}$ such that WestHalfline $p \cap C = \emptyset$ holds WestHalfline $p \subseteq$ UBD C.

(15) For every point p of $\mathcal{E}^2_{\rm T}$ such that EastHalfline $p \cap C = \emptyset$ holds EastHalfline $p \subseteq$ UBD C.

(16) For every point p of $\mathcal{E}^2_{\rm T}$ such that SouthHalfline $p \cap C = \emptyset$ holds SouthHalfline $p \subseteq$ UBD C.

(17) For every point p of $\mathcal{E}^2_{\rm T}$ such that NorthHalfline $p \cap C = \emptyset$ holds NorthHalfline $p \subseteq$ UBD C.

(18) If BDD $C \neq \emptyset$, then W-bound $C \leqslant$ W-bound BDD C.

(19) If BDD $C \neq \emptyset$, then E-bound $C \geqslant$ E-bound BDD C.

(20) If BDD $C \neq \emptyset$, then S-bound $C \leqslant$ S-bound BDD C.

(21) If BDD $C \neq \emptyset$, then N-bound $C \geqslant$ N-bound BDD C.

(22) For every integer I such that $p \in$ BDD C and $I = \lfloor \frac{p_{\bf 1} - \text{W-bound}\,C}{\text{E-bound}\,C - \text{W-bound}\,C} \cdot 2^n + 2 \rfloor$ holds $1 < I$.

(23) For every integer I such that $p \in$ BDD C and $I = \lfloor \frac{p_{\bf 1} - \text{W-bound}\,C}{\text{E-bound}\,C - \text{W-bound}\,C} \cdot 2^n + 2 \rfloor$ holds $I + 1 \leqslant \text{len}\,\text{Gauge}(C, n)$.

(24) For every integer J such that $p \in$ BDD C and $J = \lfloor \frac{p_{\bf 2} - \text{S-bound}\,C}{\text{N-bound}\,C - \text{S-bound}\,C} \cdot 2^n + 2 \rfloor$ holds $1 < J$ and $J + 1 \leqslant \text{width}\,\text{Gauge}(C, n)$.

(25) For every integer I such that $I = \lfloor \frac{p_{\bf 1} - \text{W-bound}\,C}{\text{E-bound}\,C - \text{W-bound}\,C} \cdot 2^n + 2 \rfloor$ holds $\text{W-bound}\,C + \frac{\text{E-bound}\,C - \text{W-bound}\,C}{2^n} \cdot (I - 2) \leqslant p_{\bf 1}$.

(26) For every integer I such that $I = \lfloor \frac{p_{\bf 1} - \text{W-bound}\,C}{\text{E-bound}\,C - \text{W-bound}\,C} \cdot 2^n + 2 \rfloor$ holds $p_{\bf 1} < \text{W-bound}\,C + \frac{\text{E-bound}\,C - \text{W-bound}\,C}{2^n} \cdot (I - 1)$.

(27) For every integer J such that $J = \lfloor \frac{p_{\bf 2} - \text{S-bound}\,C}{\text{N-bound}\,C - \text{S-bound}\,C} \cdot 2^n + 2 \rfloor$ holds $\text{S-bound}\,C + \frac{\text{N-bound}\,C - \text{S-bound}\,C}{2^n} \cdot (J - 2) \leqslant p_{\bf 2}$.

(28) For every integer J such that $J = \lfloor \frac{p_{\bf 2} - \text{S-bound}\,C}{\text{N-bound}\,C - \text{S-bound}\,C} \cdot 2^n + 2 \rfloor$ holds $p_{\bf 2} < \text{S-bound}\,C + \frac{\text{N-bound}\,C - \text{S-bound}\,C}{2^n} \cdot (J - 1)$.

(29) Let C be a closed subset of $\mathcal{E}^2_{\rm T}$ and p be a point of $\mathcal{E}^2$. If $p \in$ BDD C, then there exists a real number r such that $r > 0$ and $\text{Ball}(p, r) \subseteq$ BDD C.

(30) Let p, q be points of $\mathcal{E}^2_{\rm T}$ and r be a real number. Suppose $\rho((\text{Gauge}(C, n))_{1,1}, (\text{Gauge}(C, n))_{1,2}) < r$ and $\rho((\text{Gauge}(C, n))_{1,1},$

$(\mathrm{Gauge}(C,n))_{2,1}) < r$ and $p \in \mathrm{cell}(\mathrm{Gauge}(C,n),i,j)$ and $q \in \mathrm{cell}(\mathrm{Gauge}(C,n),i,j)$ and $1 \leqslant i$ and $i+1 \leqslant \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 \leqslant j$ and $j+1 \leqslant \mathrm{width}\,\mathrm{Gauge}(C,n)$. Then $\rho(p,q) < 2 \cdot r$.

(31) If $p \in \mathrm{BDD}\,C$, then $p_{\mathbf{2}} \neq$ N-bound BDD C.

(32) If $p \in \mathrm{BDD}\,C$, then $p_{\mathbf{1}} \neq$ E-bound BDD C.

(33) If $p \in \mathrm{BDD}\,C$, then $p_{\mathbf{2}} \neq$ S-bound BDD C.

(34) If $p \in \mathrm{BDD}\,C$, then $p_{\mathbf{1}} \neq$ W-bound BDD C.

(35) Suppose $p \in \mathrm{BDD}\,C$. Then there exist natural numbers n, i, j such that $1 < i$ and $i < \mathrm{len}\,\mathrm{Gauge}(C,n)$ and $1 < j$ and $j < \mathrm{width}\,\mathrm{Gauge}(C,n)$ and $p_{\mathbf{1}} \neq ((\mathrm{Gauge}(C,n))_{i,j})_{\mathbf{1}}$ and $p \in \mathrm{cell}(\mathrm{Gauge}(C,n),i,j)$ and $\mathrm{cell}(\mathrm{Gauge}(C,n),i,j) \subseteq \mathrm{BDD}\,C$.

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