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Gauges and Cages. Part II[1]

Artur Korniłowicz
University of Białystok

Robert Milewski
University of Białystok

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The articles [16], [7], [17], [8], [2], [15], [4], [19], [3], [6], [11], [1], [13], [5], [10], [21], [14], [20], [18], [9], and [12] provide the terminology and notation for this paper.

1. Preliminaries

For simplicity, we use the following convention: a, b, i, k, m, n are natural numbers, r, s are real numbers, D is a non empty subset of $\mathcal{E}^2_{\rm T}$, and C is a compact connected non vertical non horizontal subset of $\mathcal{E}^2_{\rm T}$.

Next we state the proposition

(1) For all sets A, B such that for every set x such that $x \in A$ there exists a set K such that $K \subseteq B$ and $x \subseteq \bigcup K$ holds $\bigcup A \subseteq \bigcup B$.

Let m be an even integer. Note that $m+2$ is even.

Let m be an odd integer. Observe that $m+2$ is odd.

Let m be a non empty natural number. Observe that 2^m is even.

Let n be an even natural number and let m be a non empty natural number. Note that n^m is even.

We now state several propositions:

(2) If $r \neq 0$, then $\frac{1}{r} \cdot r^{i+1} = r^i$.

(3) If $\frac{r}{s}$ is an integer and $s \neq 0$, then $-\lfloor \frac{r}{s} \rfloor = \lfloor \frac{-r}{s} \rfloor + 1$.

(4) If $\frac{r}{s}$ is an integer, then $-\lfloor \frac{r}{s} \rfloor = \lfloor \frac{-r}{s} \rfloor$.

(5) If $n > 0$ and $k \bmod n \neq 0$, then $-(k \div n) = (-k \div n) + 1$.

(6) If $n > 0$ and $k \bmod n = 0$, then $-(k \div n) = -k \div n$.

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2. Gauges and Cages

We now state a number of propositions:

(7) If $2 \leqslant m$ and $m < \operatorname{len}\operatorname{Gauge}(D,i)$ and $1 \leqslant a$ and $a \leqslant \operatorname{len}\operatorname{Gauge}(D,i)$ and $1 \leqslant b$ and $b \leqslant \operatorname{len}\operatorname{Gauge}(D,i+1)$, then $((\operatorname{Gauge}(D,i))_{m,a})_{\mathbf{1}} = ((\operatorname{Gauge}(D,i+1))_{2\cdot m -' 2,b})_{\mathbf{1}}$.

(8) If $2 \leqslant n$ and $n < \operatorname{len}\operatorname{Gauge}(D,i)$ and $1 \leqslant a$ and $a \leqslant \operatorname{len}\operatorname{Gauge}(D,i)$ and $1 \leqslant b$ and $b \leqslant \operatorname{len}\operatorname{Gauge}(D,i+1)$, then $((\operatorname{Gauge}(D,i))_{a,n})_{\mathbf{2}} = ((\operatorname{Gauge}(D,i+1))_{b,2\cdot n -' 2})_{\mathbf{2}}$.

(9) Let D be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $2 \leqslant m$ and $m+1 < \operatorname{len}\operatorname{Gauge}(D,i)$ and $2 \leqslant n$ and $n+1 < \operatorname{len}\operatorname{Gauge}(D,i)$. Then $\operatorname{cell}(\operatorname{Gauge}(D,i),m,n) = \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2\cdot m -' 2, 2\cdot n -' 2) \cup \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2\cdot m -' 1, 2\cdot n -' 2) \cup \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2\cdot m -' 2, 2\cdot n -' 1) \cup \operatorname{cell}(\operatorname{Gauge}(D,i+1), 2\cdot m -' 1, 2\cdot n -' 1)$.

(10) Let D be a compact non vertical non horizontal subset of $\mathcal{E}^2_{\mathrm{T}}$ and k be a natural number. Suppose $2 \leqslant m$ and $m+1 < \operatorname{len}\operatorname{Gauge}(D,i)$ and $2 \leqslant n$ and $n+1 < \operatorname{len}\operatorname{Gauge}(D,i)$. Then $\operatorname{cell}(\operatorname{Gauge}(D,i),m,n) = \bigcup\{\operatorname{cell}(\operatorname{Gauge}(D,i+k),a,b)$; a ranges over natural numbers, b ranges over natural numbers: $(2^k \cdot m - 2^{k+1}) + 2 \leqslant a \ \wedge\ a \leqslant (2^k \cdot m - 2^k) + 1 \ \wedge\ (2^k \cdot n - 2^{k+1}) + 2 \leqslant b \ \wedge\ b \leqslant (2^k \cdot n - 2^k) + 1\}$.

(11) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and N-max $C \in$ right_cell$(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(12) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and N-max $C \in$ rightcell$(\operatorname{Cage}(C,n), i)$.

(13) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and E-min $C \in$ right_cell$(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(14) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and E-min $C \in$ rightcell$(\operatorname{Cage}(C,n), i)$.

(15) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and E-max $C \in$ right_cell$(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(16) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and E-max $C \in$ rightcell$(\operatorname{Cage}(C,n), i)$.

(17) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and S-min $C \in$ right_cell$(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(18) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and S-min $C \in$ rightcell$(\operatorname{Cage}(C,n), i)$.

(19) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C,n)$ and S-max $C \in$ right_cell$(\operatorname{Cage}(C,n), i, \operatorname{Gauge}(C,n))$.

(20) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C, n)$ and $\text{S-max}\, C \in \operatorname{rightcell}(\operatorname{Cage}(C, n), i)$.

(21) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C, n)$ and $\text{W-min}\, C \in \text{right\_cell}(\operatorname{Cage}(C, n), i, \operatorname{Gauge}(C, n))$.

(22) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C, n)$ and $\text{W-min}\, C \in \operatorname{rightcell}(\operatorname{Cage}(C, n), i)$.

(23) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C, n)$ and $\text{W-max}\, C \in \text{right\_cell}(\operatorname{Cage}(C, n), i, \operatorname{Gauge}(C, n))$.

(24) There exists a natural number i such that $1 \leqslant i$ and $i < \operatorname{len}\operatorname{Cage}(C, n)$ and $\text{W-max}\, C \in \operatorname{rightcell}(\operatorname{Cage}(C, n), i)$.

(25) There exists a natural number i such that $1 \leqslant i$ and $i \leqslant \operatorname{len}\operatorname{Gauge}(C, n)$ and $\text{N-min}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{i, \operatorname{width}\operatorname{Gauge}(C,n)}$.

(26) There exists a natural number i such that $1 \leqslant i$ and $i \leqslant \operatorname{len}\operatorname{Gauge}(C, n)$ and $\text{N-max}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{i, \operatorname{width}\operatorname{Gauge}(C,n)}$.

(27) There exists a natural number i such that $1 \leqslant i$ and $i \leqslant \operatorname{len}\operatorname{Gauge}(C, n)$ and $(\operatorname{Gauge}(C, n))_{i, \operatorname{width}\operatorname{Gauge}(C,n)} \in \operatorname{rng}\operatorname{Cage}(C, n)$.

(28) There exists a natural number j such that $1 \leqslant j$ and $j \leqslant \operatorname{width}\operatorname{Gauge}(C, n)$ and $\text{E-min}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{\operatorname{len}\operatorname{Gauge}(C,n), j}$.

(29) There exists a natural number j such that $1 \leqslant j$ and $j \leqslant \operatorname{width}\operatorname{Gauge}(C, n)$ and $\text{E-max}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{\operatorname{len}\operatorname{Gauge}(C,n), j}$.

(30) There exists a natural number j such that $1 \leqslant j$ and $j \leqslant \operatorname{width}\operatorname{Gauge}(C, n)$ and $(\operatorname{Gauge}(C, n))_{\operatorname{len}\operatorname{Gauge}(C,n), j} \in \operatorname{rng}\operatorname{Cage}(C, n)$.

(31) There exists a natural number i such that $1 \leqslant i$ and $i \leqslant \operatorname{len}\operatorname{Gauge}(C, n)$ and $\text{S-min}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{i,1}$.

(32) There exists a natural number i such that $1 \leqslant i$ and $i \leqslant \operatorname{len}\operatorname{Gauge}(C, n)$ and $\text{S-max}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{i,1}$.

(33) There exists a natural number i such that $1 \leqslant i$ and $i \leqslant \operatorname{len}\operatorname{Gauge}(C, n)$ and $(\operatorname{Gauge}(C, n))_{i,1} \in \operatorname{rng}\operatorname{Cage}(C, n)$.

(34) There exists a natural number j such that $1 \leqslant j$ and $j \leqslant \operatorname{width}\operatorname{Gauge}(C, n)$ and $\text{W-min}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{1,j}$.

(35) There exists a natural number j such that $1 \leqslant j$ and $j \leqslant \operatorname{width}\operatorname{Gauge}(C, n)$ and $\text{W-max}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C, n)) = (\operatorname{Gauge}(C, n))_{1,j}$.

(36) There exists a natural number j such that $1 \leqslant j$ and $j \leqslant \operatorname{width}\operatorname{Gauge}(C, n)$ and $(\operatorname{Gauge}(C, n))_{1,j} \in \operatorname{rng}\operatorname{Cage}(C, n)$.

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