

# On the Instructions of $\mathbf{SCM}_{\text{FSA}}$ <sup>1</sup>

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MML Identifier:  $\mathbf{SCM}_{\text{FSA}}10$ .

The articles [18], [10], [11], [12], [22], [5], [14], [3], [6], [20], [7], [8], [9], [4], [19], [1], [2], [23], [24], [17], [16], [13], [21], and [15] provide the terminology and notation for this paper.

For simplicity, we use the following convention:  $a, b$  are integer locations,  $f$  is a finite sequence location,  $i_1, i_2, i_3$  are instruction-locations of  $\mathbf{SCM}_{\text{FSA}}$ ,  $T$  is an instruction type of  $\mathbf{SCM}_{\text{FSA}}$ , and  $k$  is a natural number.

Next we state two propositions:

- (1) For every function  $f$  and for all sets  $a, A, b, B, c, C$  such that  $a \neq b$  and  $a \neq c$  holds  $(f + \cdot (a \mapsto A) + \cdot (b \mapsto B) + \cdot (c \mapsto C))(a) = A$ .
- (2) For all sets  $a, b$  holds  $\langle a \rangle + \cdot (1, b) = \langle b \rangle$ .

Let  $l_1, l_2$  be integer locations and let  $a, b$  be integers. Then  $[l_1 \mapsto a, l_2 \mapsto b]$  is a finite partial state of  $\mathbf{SCM}_{\text{FSA}}$ .

One can prove the following propositions:

- (3)  $a \notin$  the instruction locations of  $\mathbf{SCM}_{\text{FSA}}$ .
- (4)  $f \notin$  the instruction locations of  $\mathbf{SCM}_{\text{FSA}}$ .
- (5)  $\text{Data-Loc}_{\mathbf{SCM}_{\text{FSA}}} \neq$  the instruction locations of  $\mathbf{SCM}_{\text{FSA}}$ .
- (6)  $\text{Data}^*\text{-Loc}_{\mathbf{SCM}_{\text{FSA}}} \neq$  the instruction locations of  $\mathbf{SCM}_{\text{FSA}}$ .
- (7) Let  $o$  be an object of  $\mathbf{SCM}_{\text{FSA}}$ . Then
  - (i)  $o = \mathbf{IC}_{\mathbf{SCM}_{\text{FSA}}}$ , or
  - (ii)  $o \in$  the instruction locations of  $\mathbf{SCM}_{\text{FSA}}$ , or
  - (iii)  $o$  is an integer location or a finite sequence location.
- (8) If  $i_2 \neq i_3$ , then  $\text{Next}(i_2) \neq \text{Next}(i_3)$ .
- (9)  $a := b = \langle 1, \langle a, b \rangle \rangle$ .
- (10)  $\text{AddTo}(a, b) = \langle 2, \langle a, b \rangle \rangle$ .

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<sup>1</sup>This work has been partially supported by TYPES grant IST-1999-29001.

- (11)  $\text{SubFrom}(a, b) = \langle 3, \langle a, b \rangle \rangle$ .
- (12)  $\text{MultBy}(a, b) = \langle 4, \langle a, b \rangle \rangle$ .
- (13)  $\text{Divide}(a, b) = \langle 5, \langle a, b \rangle \rangle$ .
- (14)  $\text{goto } i_1 = \langle 6, \langle i_1 \rangle \rangle$ .
- (15) **if**  $a = 0$  **goto**  $i_1 = \langle 7, \langle i_1, a \rangle \rangle$ .
- (16) **if**  $a > 0$  **goto**  $i_1 = \langle 8, \langle i_1, a \rangle \rangle$ .
- (17)  $\text{AddressPart}(\mathbf{halts}_{\text{SCM}_{\text{FSA}}}) = \emptyset$ .
- (18)  $\text{AddressPart}(a:=b) = \langle a, b \rangle$ .
- (19)  $\text{AddressPart}(\text{AddTo}(a, b)) = \langle a, b \rangle$ .
- (20)  $\text{AddressPart}(\text{SubFrom}(a, b)) = \langle a, b \rangle$ .
- (21)  $\text{AddressPart}(\text{MultBy}(a, b)) = \langle a, b \rangle$ .
- (22)  $\text{AddressPart}(\text{Divide}(a, b)) = \langle a, b \rangle$ .
- (23)  $\text{AddressPart}(\text{goto } i_2) = \langle i_2 \rangle$ .
- (24)  $\text{AddressPart}(\mathbf{if } a = 0 \mathbf{ goto } i_2) = \langle i_2, a \rangle$ .
- (25)  $\text{AddressPart}(\mathbf{if } a > 0 \mathbf{ goto } i_2) = \langle i_2, a \rangle$ .
- (26)  $\text{AddressPart}(b:=f_a) = \langle b, f, a \rangle$ .
- (27)  $\text{AddressPart}(f_a:=b) = \langle b, f, a \rangle$ .
- (28)  $\text{AddressPart}(a:=\text{len } f) = \langle a, f \rangle$ .
- (29)  $\text{AddressPart}(f:=\underbrace{\langle 0, \dots, 0 \rangle}_a) = \langle a, f \rangle$ .
- (30) If  $T = 0$ , then  $\text{AddressParts } T = \{0\}$ .

Let us consider  $T$ . Observe that  $\text{AddressParts } T$  is non empty.

Next we state a number of propositions:

- (31) If  $T = 1$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (32) If  $T = 2$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (33) If  $T = 3$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (34) If  $T = 4$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (35) If  $T = 5$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (36) If  $T = 6$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1\}$ .
- (37) If  $T = 7$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (38) If  $T = 8$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (39) If  $T = 9$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2, 3\}$ .
- (40) If  $T = 10$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2, 3\}$ .
- (41) If  $T = 11$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (42) If  $T = 12$ , then  $\text{dom } \prod_{\text{AddressParts } T} = \{1, 2\}$ .
- (43)  $\prod_{\text{AddressParts } \text{InsCode}(a:=b)}(1) = \text{Data-Loc}_{\text{SCM}_{\text{FSA}}}$ .
- (44)  $\prod_{\text{AddressParts } \text{InsCode}(a:=b)}(2) = \text{Data-Loc}_{\text{SCM}_{\text{FSA}}}$ .

- (45)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{AddTo}(a,b))(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (46)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{AddTo}(a,b))(2) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (47)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{SubFrom}(a,b))(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (48)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{SubFrom}(a,b))(2) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (49)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{MultBy}(a,b))(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (50)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{MultBy}(a,b))(2) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (51)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{Divide}(a,b))(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (52)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{Divide}(a,b))(2) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (53)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{goto } i_2)(1) = \text{the instruction locations of } \mathbf{SCM}_{\text{FSA}}$ .
- (54)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{if } a=0 \text{ goto } i_2)(1) = \text{the instruction locations of } \mathbf{SCM}_{\text{FSA}}$ .
- (55)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{if } a=0 \text{ goto } i_2)(2) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (56)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{if } a>0 \text{ goto } i_2)(1) = \text{the instruction locations of } \mathbf{SCM}_{\text{FSA}}$ .
- (57)  $\prod_{\text{AddressParts}} \text{InsCode}(\text{if } a>0 \text{ goto } i_2)(2) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (58)  $\prod_{\text{AddressParts}} \text{InsCode}(b:=f_a)(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (59)  $\prod_{\text{AddressParts}} \text{InsCode}(b:=f_a)(2) = \text{Data}^*\text{-LocSCM}_{\text{FSA}}$ .
- (60)  $\prod_{\text{AddressParts}} \text{InsCode}(b:=f_a)(3) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (61)  $\prod_{\text{AddressParts}} \text{InsCode}(f_a:=b)(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (62)  $\prod_{\text{AddressParts}} \text{InsCode}(f_a:=b)(2) = \text{Data}^*\text{-LocSCM}_{\text{FSA}}$ .
- (63)  $\prod_{\text{AddressParts}} \text{InsCode}(f_a:=b)(3) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (64)  $\prod_{\text{AddressParts}} \text{InsCode}(a:=\text{len } f)(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (65)  $\prod_{\text{AddressParts}} \text{InsCode}(a:=\text{len } f)(2) = \text{Data}^*\text{-LocSCM}_{\text{FSA}}$ .
- (66)  $\prod_{\text{AddressParts}} \text{InsCode}(f:=\underbrace{(0, \dots, 0)}_a)(1) = \text{Data-LocSCM}_{\text{FSA}}$ .
- (67)  $\prod_{\text{AddressParts}} \text{InsCode}(f:=\underbrace{(0, \dots, 0)}_a)(2) = \text{Data}^*\text{-LocSCM}_{\text{FSA}}$ .
- (68)  $\text{NIC}(\mathbf{halts}_{\text{SCM}_{\text{FSA}}}, i_1) = \{i_1\}$ .

One can verify that  $\text{JUMP}(\mathbf{halts}_{\text{SCM}_{\text{FSA}}})$  is empty.

We now state the proposition

- (69)  $\text{NIC}(a:=b, i_1) = \{\text{Next}(i_1)\}$ .

Let us consider  $a, b$ . Note that  $\text{JUMP}(a:=b)$  is empty.

One can prove the following proposition

- (70)  $\text{NIC}(\text{AddTo}(a, b), i_1) = \{\text{Next}(i_1)\}$ .

Let us consider  $a, b$ . Note that  $\text{JUMP}(\text{AddTo}(a, b))$  is empty.

Next we state the proposition

- (71)  $\text{NIC}(\text{SubFrom}(a, b), i_1) = \{\text{Next}(i_1)\}$ .

Let us consider  $a, b$ . Note that  $\text{JUMP}(\text{SubFrom}(a, b))$  is empty.

One can prove the following proposition

$$(72) \quad \text{NIC}(\text{MultBy}(a, b), i_1) = \{\text{Next}(i_1)\}.$$

Let us consider  $a, b$ . Note that  $\text{JUMP}(\text{MultBy}(a, b))$  is empty.

Next we state the proposition

$$(73) \quad \text{NIC}(\text{Divide}(a, b), i_1) = \{\text{Next}(i_1)\}.$$

Let us consider  $a, b$ . One can verify that  $\text{JUMP}(\text{Divide}(a, b))$  is empty.

We now state two propositions:

$$(74) \quad \text{NIC}(\text{goto } i_2, i_1) = \{i_2\}.$$

$$(75) \quad \text{JUMP}(\text{goto } i_2) = \{i_2\}.$$

Let us consider  $i_2$ . One can verify that  $\text{JUMP}(\text{goto } i_2)$  is non empty and trivial.

We now state two propositions:

$$(76) \quad i_2 \in \text{NIC}(\text{if } a = 0 \text{ goto } i_2, i_1) \text{ and } \text{NIC}(\text{if } a = 0 \text{ goto } i_2, i_1) \subseteq \{i_2, \text{Next}(i_1)\}.$$

$$(77) \quad \text{JUMP}(\text{if } a = 0 \text{ goto } i_2) = \{i_2\}.$$

Let us consider  $a, i_2$ . One can check that  $\text{JUMP}(\text{if } a = 0 \text{ goto } i_2)$  is non empty and trivial.

One can prove the following two propositions:

$$(78) \quad i_2 \in \text{NIC}(\text{if } a > 0 \text{ goto } i_2, i_1) \text{ and } \text{NIC}(\text{if } a > 0 \text{ goto } i_2, i_1) \subseteq \{i_2, \text{Next}(i_1)\}.$$

$$(79) \quad \text{JUMP}(\text{if } a > 0 \text{ goto } i_2) = \{i_2\}.$$

Let us consider  $a, i_2$ . Note that  $\text{JUMP}(\text{if } a > 0 \text{ goto } i_2)$  is non empty and trivial.

The following proposition is true

$$(80) \quad \text{NIC}(a := f_b, i_1) = \{\text{Next}(i_1)\}.$$

Let us consider  $a, b, f$ . Observe that  $\text{JUMP}(a := f_b)$  is empty.

Next we state the proposition

$$(81) \quad \text{NIC}(f_b := a, i_1) = \{\text{Next}(i_1)\}.$$

Let us consider  $a, b, f$ . One can check that  $\text{JUMP}(f_b := a)$  is empty.

The following proposition is true

$$(82) \quad \text{NIC}(a := \text{len } f, i_1) = \{\text{Next}(i_1)\}.$$

Let us consider  $a, f$ . Observe that  $\text{JUMP}(a := \text{len } f)$  is empty.

The following proposition is true

$$(83) \quad \text{NIC}(f := \underbrace{\langle 0, \dots, 0 \rangle}_a, i_1) = \{\text{Next}(i_1)\}.$$

Let us consider  $a, f$ . Note that  $\text{JUMP}(f := \underbrace{\langle 0, \dots, 0 \rangle}_a)$  is empty.

The following two propositions are true:

(84)  $\text{SUCC}(i_1) = \{i_1, \text{Next}(i_1)\}$ .

(85) Let  $f$  be a function from  $\mathbb{N}$  into the instruction locations of  $\mathbf{SCM}_{\text{FSA}}$ .

Suppose that for every natural number  $k$  holds  $f(k) = \text{insloc}(k)$ . Then

(i)  $f$  is bijective, and

(ii) for every natural number  $k$  holds  $f(k+1) \in \text{SUCC}(f(k))$  and for every natural number  $j$  such that  $f(j) \in \text{SUCC}(f(k))$  holds  $k \leq j$ .

Let us observe that  $\mathbf{SCM}_{\text{FSA}}$  is standard.

The following propositions are true:

(86)  $\text{ils}_{\mathbf{SCM}_{\text{FSA}}}(k) = \text{insloc}(k)$ .

(87)  $\text{Next}(\text{ils}_{\mathbf{SCM}_{\text{FSA}}}(k)) = \text{ils}_{\mathbf{SCM}_{\text{FSA}}}(k+1)$ .

(88)  $\text{Next}(i_1) = \text{NextLoc } i_1$ .

Let us mention that  $\text{InsCode}(\mathbf{halts}_{\mathbf{SCM}_{\text{FSA}}})$  is jump-only.

Let us mention that  $\mathbf{halts}_{\mathbf{SCM}_{\text{FSA}}}$  is jump-only.

Let us consider  $i_2$ . One can verify that  $\text{InsCode}(\text{goto } i_2)$  is jump-only.

Let us consider  $i_2$ . Observe that  $\text{goto } i_2$  is jump-only non sequential and non instruction location free.

Let us consider  $a, i_2$ . One can check that  $\text{InsCode}(\mathbf{if } a = 0 \mathbf{ goto } i_2)$  is jump-only and  $\text{InsCode}(\mathbf{if } a > 0 \mathbf{ goto } i_2)$  is jump-only.

Let us consider  $a, i_2$ . Observe that  $\mathbf{if } a = 0 \mathbf{ goto } i_2$  is jump-only non sequential and non instruction location free and  $\mathbf{if } a > 0 \mathbf{ goto } i_2$  is jump-only non sequential and non instruction location free.

Let us consider  $a, b$ . One can verify the following observations:

- \*  $\text{InsCode}(a:=b)$  is non jump-only,
- \*  $\text{InsCode}(\text{AddTo}(a, b))$  is non jump-only,
- \*  $\text{InsCode}(\text{SubFrom}(a, b))$  is non jump-only,
- \*  $\text{InsCode}(\text{MultBy}(a, b))$  is non jump-only, and
- \*  $\text{InsCode}(\text{Divide}(a, b))$  is non jump-only.

Let us consider  $a, b$ . One can verify the following observations:

- \*  $a:=b$  is non jump-only and sequential,
- \*  $\text{AddTo}(a, b)$  is non jump-only and sequential,
- \*  $\text{SubFrom}(a, b)$  is non jump-only and sequential,
- \*  $\text{MultBy}(a, b)$  is non jump-only and sequential, and
- \*  $\text{Divide}(a, b)$  is non jump-only and sequential.

Let us consider  $a, b, f$ . One can check that  $\text{InsCode}(b:=f_a)$  is non jump-only and  $\text{InsCode}(f_a:=b)$  is non jump-only.

Let us consider  $a, b, f$ . Observe that  $b:=f_a$  is non jump-only and sequential and  $f_a:=b$  is non jump-only and sequential.

Let us consider  $a, f$ . One can check that  $\text{InsCode}(a:=\text{len } f)$  is non jump-only and  $\text{InsCode}(f:=\underbrace{\langle 0, \dots, 0 \rangle}_a)$  is non jump-only.

Let us consider  $a, f$ . Note that  $a:=\text{len } f$  is non jump-only and sequential and  $f:=\underbrace{\langle 0, \dots, 0 \rangle}_a$  is non jump-only and sequential.

One can verify that  $\mathbf{SCM}_{\text{FSA}}$  is homogeneous and has explicit jumps and no implicit jumps.

Let us note that  $\mathbf{SCM}_{\text{FSA}}$  is regular.

The following propositions are true:

$$(89) \quad \text{IncAddr}(\text{goto } i_2, k) = \text{goto } \text{il}_{\mathbf{SCM}_{\text{FSA}}}(\text{locnum}(i_2) + k).$$

$$(90) \quad \text{IncAddr}(\text{if } a = 0 \text{ goto } i_2, k) = \text{if } a = 0 \text{ goto } \text{il}_{\mathbf{SCM}_{\text{FSA}}}(\text{locnum}(i_2) + k).$$

$$(91) \quad \text{IncAddr}(\text{if } a > 0 \text{ goto } i_2, k) = \text{if } a > 0 \text{ goto } \text{il}_{\mathbf{SCM}_{\text{FSA}}}(\text{locnum}(i_2) + k).$$

Let us note that  $\mathbf{SCM}_{\text{FSA}}$  is IC-good and Exec-preserving.

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*Received May 8, 2001*

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