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Some Remarks on Clockwise Oriented Sequences on Go-boards[1]

Adam Naumowicz[2]
University of Białystok

Robert Milewski
University of Białystok

Summary. The main goal of this paper is to present alternative characterizations of clockwise oriented sequences on Go-boards.

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The articles [8], [21], [9], [2], [3], [26], [24], [4], [16], [18], [23], [14], [20], [19], [5], [7], [13], [1], [6], [12], [28], [15], [17], [25], [27], [22], [10], and [11] provide the terminology and notation for this paper.

1. Preliminaries

In this paper i, j, k, n denote natural numbers.

Next we state several propositions:

(1) For all subsets A, B of $\mathcal{E}^n_{\rm T}$ such that A is Bounded or B is Bounded holds $A \cap B$ is Bounded.

(2) For all subsets A, B of $\mathcal{E}^n_{\rm T}$ such that A is not Bounded and B is Bounded holds $A \setminus B$ is not Bounded.

(3) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\rm T}$ holds $(\text{W-min}\,\widetilde{\mathcal{L}}(\text{Cage}(C,n))) \leftarrow\!\!\!\!\!\!\; \text{Cage}(C,n) > 1$.

(4) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\rm T}$ holds $(\text{E-max}\,\widetilde{\mathcal{L}}(\text{Cage}(C,n))) \leftarrow\!\!\!\!\!\!\; \text{Cage}(C,n) > 1$.

(5) For every compact connected non vertical non horizontal subset C of $\mathcal{E}^2_{\rm T}$ holds $(\text{S-max}\,\widetilde{\mathcal{L}}(\text{Cage}(C,n))) \leftarrow\!\!\!\!\!\!\; \text{Cage}(C,n) > 1$.

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2. On Bounding Points of Circular Sequences

Next we state several propositions:

(6) Let f be a non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in \operatorname{rng} f$, then $\operatorname{leftcell}(f, p \hookleftarrow f) = \operatorname{leftcell}(f^p_{\circlearrowleft}, 1)$.

(7) Let f be a non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p \in \operatorname{rng} f$, then $\operatorname{rightcell}(f, p \hookleftarrow f) = \operatorname{rightcell}(f^p_{\circlearrowleft}, 1)$.

(8) For every compact connected non vertical non horizontal non empty subset C of $\mathcal{E}^2_{\rm T}$ holds $\text{W-min}\, C \in \operatorname{rightcell}((\operatorname{Cage}(C, n))^{\text{W-min}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))}_{\circlearrowleft}, 1)$.

(9) For every compact connected non vertical non horizontal non empty subset C of $\mathcal{E}^2_{\rm T}$ holds $\text{E-max}\, C \in \operatorname{rightcell}((\operatorname{Cage}(C, n))^{\text{E-max}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))}_{\circlearrowleft}, 1)$.

(10) For every compact connected non vertical non horizontal non empty subset C of $\mathcal{E}^2_{\rm T}$ holds $\text{S-max}\, C \in \operatorname{rightcell}((\operatorname{Cage}(C, n))^{\text{S-max}\, \widetilde{\mathcal{L}}(\operatorname{Cage}(C,n))}_{\circlearrowleft}, 1)$.

3. On Clockwise Oriented Sequences

One can prove the following propositions:

(11) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p_{\mathbf{1}} < \text{W-bound}\, \widetilde{\mathcal{L}}(f)$, then $p \in \operatorname{LeftComp}(f)$.

(12) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p_{\mathbf{1}} > \text{E-bound}\, \widetilde{\mathcal{L}}(f)$, then $p \in \operatorname{LeftComp}(f)$.

(13) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p_{\mathbf{2}} < \text{S-bound}\, \widetilde{\mathcal{L}}(f)$, then $p \in \operatorname{LeftComp}(f)$.

(14) Let f be a clockwise oriented non constant standard special circular sequence and p be a point of $\mathcal{E}^2_{\rm T}$. If $p_{\mathbf{2}} > \text{N-bound}\, \widetilde{\mathcal{L}}(f)$, then $p \in \operatorname{LeftComp}(f)$.

(15) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k + 1 \leqslant \operatorname{len} f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i + 1,\, j\rangle \in$ the indices of G and $f_k = G \circ (i + 1, j)$ and $f_{k+1} = G \circ (i, j)$. Then $j < \operatorname{width} G$.

(16) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k + 1 \leqslant \operatorname{len} f$ and $\langle i,\, j\rangle \in$ the indices of G and $\langle i,\, j + 1\rangle \in$ the indices of G and $f_k = G \circ (i, j)$ and $f_{k+1} = G \circ (i, j + 1)$. Then $i < \operatorname{len} G$.

(17) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k + 1 \leqslant$

len f and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i,j)$ and $f_{k+1} = G \circ (i+1,j)$. Then $j > 1$.

(18) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k+1 \leqslant$ len f and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i,j+1)$ and $f_{k+1} = G \circ (i,j)$. Then $i > 1$.

(19) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k+1 \leqslant$ len f and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i+1,j)$ and $f_{k+1} = G \circ (i,j)$. Then $(f_k)_{\mathbf{2}} \neq$ N-bound $\widetilde{\mathcal{L}}(f)$.

(20) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k+1 \leqslant$ len f and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i,j)$ and $f_{k+1} = G \circ (i,j+1)$. Then $(f_k)_{\mathbf{1}} \neq$ E-bound $\widetilde{\mathcal{L}}(f)$.

(21) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k+1 \leqslant$ len f and $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i,j)$ and $f_{k+1} = G \circ (i+1,j)$. Then $(f_k)_{\mathbf{2}} \neq$ S-bound $\widetilde{\mathcal{L}}(f)$.

(22) Let f be a clockwise oriented non constant standard special circular sequence and G be a Go-board. Suppose f is a sequence which elements belong to G. Let i, j, k be natural numbers. Suppose $1 \leqslant k$ and $k+1 \leqslant$ len f and $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i,j+1)$ and $f_{k+1} = G \circ (i,j)$. Then $(f_k)_{\mathbf{1}} \neq$ W-bound $\widetilde{\mathcal{L}}(f)$.

(23) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is a sequence which elements belong to G and $1 \leqslant k$ and $k+1 \leqslant$ len f and $f_k =$ W-min $\widetilde{\mathcal{L}}(f)$. Then there exist natural numbers i, j such that $\langle i, j\rangle \in$ the indices of G and $\langle i, j+1\rangle \in$ the indices of G and $f_k = G \circ (i,j)$ and $f_{k+1} = G \circ (i,j+1)$.

(24) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is a sequence which elements belong to G and $1 \leqslant k$ and $k+1 \leqslant$ len f and $f_k =$ N-min $\widetilde{\mathcal{L}}(f)$. Then there exist natural numbers i, j such that $\langle i, j\rangle \in$ the indices of G and $\langle i+1, j\rangle \in$ the indices of G and $f_k = G \circ (i,j)$ and $f_{k+1} = G \circ (i+1,j)$.

(25) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is

a sequence which elements belong to G and $1 \leqslant k$ and $k+1 \leqslant \operatorname{len} f$ and $f_k = \text{E-max}\,\widetilde{\mathcal{L}}(f)$. Then there exist natural numbers i, j such that $\langle i, j+1\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i, j+1)$ and $f_{k+1} = G \circ (i, j)$.

(26) Let f be a clockwise oriented non constant standard special circular sequence, G be a Go-board, and k be a natural number. Suppose f is a sequence which elements belong to G and $1 \leqslant k$ and $k+1 \leqslant \operatorname{len} f$ and $f_k = \text{S-max}\,\widetilde{\mathcal{L}}(f)$. Then there exist natural numbers i, j such that $\langle i+1, j\rangle \in$ the indices of G and $\langle i, j\rangle \in$ the indices of G and $f_k = G \circ (i+1, j)$ and $f_{k+1} = G \circ (i, j)$.

(27) Let f be a non constant standard special circular sequence. Then f is clockwise oriented if and only if $(f_{\circlearrowleft}^{\text{W-min}\,\widetilde{\mathcal{L}}(f)})_2 \in \text{W-most}\,\widetilde{\mathcal{L}}(f)$.

(28) Let f be a non constant standard special circular sequence. Then f is clockwise oriented if and only if $(f_{\circlearrowleft}^{\text{E-max}\,\widetilde{\mathcal{L}}(f)})_2 \in \text{E-most}\,\widetilde{\mathcal{L}}(f)$.

(29) Let f be a non constant standard special circular sequence. Then f is clockwise oriented if and only if $(f_{\circlearrowleft}^{\text{S-max}\,\widetilde{\mathcal{L}}(f)})_2 \in \text{S-most}\,\widetilde{\mathcal{L}}(f)$.

(30) Let C be a compact non vertical non horizontal non empty subset of $\mathcal{E}^2_{\mathrm{T}}$ satisfying conditions of simple closed curve and p be a point of $\mathcal{E}^2_{\mathrm{T}}$. Suppose $p_{\mathbf{1}} = \frac{\text{W-bound}\,C + \text{E-bound}\,C}{2}$ and $i > 0$ and $1 \leqslant k$ and $k \leqslant \operatorname{width} \operatorname{Gauge}(C, i)$ and $\operatorname{Gauge}(C, i) \circ (\operatorname{Center} \operatorname{Gauge}(C, i), k) \in \operatorname{UpperArc} \widetilde{\mathcal{L}}(\operatorname{Cage}(C, i))$ and $p_{\mathbf{2}} = \sup(\text{proj2}^\circ(\mathcal{L}(\operatorname{Gauge}(C, 1) \circ (\operatorname{Center} \operatorname{Gauge}(C, 1), 1), \operatorname{Gauge}(C, i) \circ (\operatorname{Center} \operatorname{Gauge}(C, i), k)) \cap \operatorname{LowerArc} \widetilde{\mathcal{L}}(\operatorname{Cage}(C, i))))$. Then there exists j such that $1 \leqslant j$ and $j \leqslant \operatorname{len} \operatorname{Gauge}(C, i)$ and $p = \operatorname{Gauge}(C, i) \circ (\operatorname{Center} \operatorname{Gauge}(C, i), j)$.

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